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The Epistemological Import of Euclidean Diagrams (in a non-Euclidean world)

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Abstract In this paper I concentrate on Euclidean diagrams, namely on those diagrams that are licensed by the rules of Euclid's plane geometry. I shall overview some philosophical stances that have recently been proposed in philosophy of mathematics to account for the role of such diagrams in mathematics, and more particularly in Euclid's *Elements*. Furthermore, I shall provide an original analysis of the epistemic role that Euclidean diagrams may (and, indeed) have in empirical sciences, more specifically in physics. I shall claim that, although the world we live in is not Euclidean, Euclidean diagrams permit to obtain knowledge of the world through a pecific mechanism of inference I shall call inheritance.

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1. Introduction

The fate of Euclidean geometry, similarly to that of the Newtonian's conception of space and time (which was, indeed, strongly affected by the former), is very peculiar. On one hand, Euclidean geometry has lost the status of paradigm of epistemic certainty that it had before the advent of non-Euclidean geometries and formal axiomatizations programs.[1] On the other hand, however, Euclidean geometry is still alive and continues to be used in mathematics and in empirical science. And this despite the advent of more sophisticated, non-Euclidean, mathematical concepts and formal systems. This is especially true in physics, where Euclidean geometry, in tandem with Newtonian mechanics, continues to be an essential instrument of analysis insofar as "the actual physical three-dimensional space we live in is curved and matter, as Einstein told us, is the source of the curvature; therefore the rules of Euclidean geometry break down if we want to correctly describe the physical space; however, if the local curvature of our three-dimensional space is very small, we can still use Euclidean geometry and consider that it provides a good approximation of physical reality".

The previous considerations prompt a very simple question: what is the role of Euclidean geometry in science (i.e. contemporary mathematics and empirical science)? This is, of course, a difficult question to answer, and it is plausible to think that providing a satisfying answer to this question would require a huge variety of analysis, depending on the domain of science that is considered. For instance, we may think that there are parts of modern mathematics (e.g. set theory) in which Euclidean geometry has no role at all, while in others (e.g. linear algebra) it is still involved and used (though implicitly or embedded in the formalism). As the philosopher of mathematics Ken Manders has pointed out: "modern mathematics *subsumes* Euclid's geometrical conclusions in real analytic geometry, real analysis, and functional analysis" and "Euclid's Elements already sets patterns that are extrapolated by more abstract 20th-century mathematics" (Manders, 2008a, p. 67). And the same can be told of physics, where we still use Euclidean geometry in mechanics, in educational or even research contexts, while Euclidean geometry disappears in quantum mechanics and is replaced by hyperbolic geometry in special relativity. Therefore, whether there is no doubt that most of (if not all) the advances in modern mathematics (and in empirical science

1 There are many books that trace the history of Euclidean geometry before and after the advent of non-Euclidean geometries due to the work of Gauss, Bolyai and Lobachevsky in differential geometry. The historian of mathematics Geremy Gray offers an excellent (historical and mathematical) treatment in his Gray (1989).

as well) are the result of Euclidean geometry and its study, the relevance of Euclidean geometry to modern mathematics and empirical science is very difficult to estimate.

In this paper I address one specific aspect of the question concerning the role played by Euclidean geometry in contemporary science. I shall focus on the epistemic role of Euclidean diagrams (henceforth ED), which I take to be those diagrams than can be constructed and "read" using the rules of Euclid's plane geometry (henceforth EPG). ED are therefore specific kind of diagrams, being the recipient and the artifact of EPG. And it is important to stress from the beginning that I am not considering here diagrams *tout court*. Indeed, not all diagrams are Euclidean. For instance, I consider that a diagram representing the Möbius strip, a one-sided non-orientable surface obtained by cutting a closed band into a single strip, is not Euclidean because it cannot be obtained and interpreted using EPG. A diagram representing what I am seeing out of my window is, arguably, a non Euclidean diagram because it cannot be obtained using the resources of EPG alone. On the other hand, a diagram representing two straight lines or a triangle as the result of three lines that intersect in three distinct points, constructed according to the possibilities admitted by Euclidean geometry, is an Euclidean diagram.

Various studies of Euclidean diagrams have been proposed in philosophy of mathematics (I will survey some of these below). For the most, these analyses have investigated the role played by ED in mathematics, more specifically in Euclid's *Elements*, and little attention has been devoted to the function of ED in the empirical sciences. This is why I shall restrict my attention to physics and I shall analyze ED in this context. Using some notions that have been proposed by philosophers of mathematics in the context of the analysis of ED in mathematics, I shall propose the idea that there exists a pattern of reasoning which is employed when a physical situation is represented and studied through an Euclidean diagram: manipulating mentally the diagram we transfer information from an higher (i.e. more abstract) structure to a lower (i.e. less abstract) structure, namely the physical system. Thus the physical structure inherits the information from the geometrical structure. This is a form of inference and I shall call it "inheritance". The epistemic outcome of this process is knowledge concerning the physical system under scrutiny.

In the next section I shall give a quick overview of some philosophical analysis devoted to visual reasoning and the role of Euclidean diagrams in mathematics. This section will be instrumental in developing my analysis of ED in physics, which will be the content of section three. In section three I shall also provide a very simple example of how inheritance works in a concrete case. Finally, I will report my conclusions.

Rather than offering a comprehensive analysis of the role played by ED in empirical science, the purpose of this study is to contribute to a young but very vibrant debate, namely that concerning the understanding of the function of diagrams and diagrammatic reasoning in science, and to underline the import of Euclid's geometry in modern science.

2. The philosophical analysis of Euclidean diagrams

The philosophical investigation of Euclidean diagrams has been carried out along two main research paths, both of which have received a lot of attention in philosophy of mathematics during the last two decades. This interest is also due to the fact that the study of the function of mathematical diagrams is a topic that interests different domains of philosophy of mathematics (logic, epistemology, metaphysics, proof theory, mathematics education) and it also has far-reaching ramifications for many areas of philosophy, including, in addition to philosophy of mathematics, epistemology, cognitive sciences, and philosophy of science.[2]

The first line of investigation concerns the analysis of visual reasoning, and more particularly the study of diagrammatic reasoning (i.e. visual reasoning on diagrams), in mathematics. This is a vast area of study, which encompasses research questions that may be very varied and that have been also prompted by studies in domains which are far from traditional philosophy of mathematics, as for instance studies in visual imagery in cognitive psychology (cf. Shepard and Cooper 1982; Kosslyn 1994). The main goal of this research path can be stated as follows: have a better understanding of the ways in which visual reasoning on a diagram may produce mathematical beliefs and be a resource for justification, discovery or even proof, thus playing a genuine epistemic role in

2 Mancosu (2005) provides a detailed survey.

mathematics. To what extent, if any, diagrams and visual reasoning can provide us with mathematical knowledge? In this context, visual reasoning is generally taken as the thought process through which we form an inner, mental image, of a situation and we perform a mental operation, as for instance an inference, on this mental image (Giaquinto 2007; cf. also studies in Mancosu et al. 2005).[3] Thus, whether diagrammatic reasoning has an epistemic import in mathematical activity, the utility of diagrams is not only psychological and merely heuristic, as a time-honoured view (still prevalent) has maintained, but also epistemological.[4] For instance, Marcus Giaquinto has claimed that diagrammatic reasoning can play an epistemic role in mathematical practice which goes beyond the mere heuristic role attributed to it in the past, and more precisely that it plays the role of "trigger" for belief-forming dispositions which in turn give us geometrical knowledge (Giaquinto 2008). Barwise and Etchemendy have considered diagrams as "essential and legitimate components in valid deductive reasoning" (Barwise and Etchemendy, 1996, p. 12). A rare

3 Note that 'visualization' and 'visual reasoning' are distinct notions. Visualization is the act of visualizing, literally 'seeing something', while visual reasoning requires more. To reason visually on a situation we need to 'see' it and, furthermore, to perform a mental operation on the mental image of that situation. For instance, we can just see (visualize) that the chair is close to the couch. But if we close the eyes for a moment we may still be able to form a mental image of that situation and performing a mental operation (Thagard, 2005, p. 98). An example of non-mental image is given by the words on this page. An example of mental image is given by the layout of the university campus that we form in our mind for getting to building to building.

4 The time-honoured view has many eminent partisans in past and present mathematics and philosophy of mathematics. Leibniz wrote that "it is not the figures which furnish the proof with geometers, though the style of the exposition may make you think so. The force of the demonstration is independent of the figure drawn, which is drawn only to facilitate the knowledge of our meaning" (Leibniz 1949, p. 403; on Leibniz's project of putting Euclidean geometry on secure grounds, and more particularly on his works on the parallel postulate, see De Risi 2016). Frege's position, for instance, was that the only role of perception or an experience of visual imaging is that of providing evidence for a further generalization. According to Frege, indeed, psychological activity cannot warrant the generalizations drawn from it and therefore in this way one could not account for the objectivity of mathematics. The modern attitude towards diagrams and reasoning on them is to consider that diagrams are at best an heuristic aid. This viewpoint is well summarized by the logician Neil Tennant: "[The diagram] is only an heuristic to prompt certain trains of inference; [...] it is dispensable as a proof-theoretic device; indeed, [...] it has no proper place in the proof as such" (Tennant, 1986, p. 304).

view, endorsed by the philosopher of mathematics James Robert Brown (Brown, 1999, Ch. 3), is that in some cases a diagram alone *is* a proof.

The first research path that I shortly reviewed in the previous paragraph concerns ED only partly, and this simply because not all diagrams are Euclidean. Nevertheless, there is a second line of investigation in philosophy of mathematics that is related to the former but is more focused on ED. Indeed, it specifically deals with the role of diagrams in Euclid's *Elements* and Euclid's geometry in general. Research questions addressed in this area generally have to do with the argumentative structure of the *Elements* and the thesis that many of Euclid's geometric arguments are (or are not) diagram-based. In his studies on Greek mathematics, Reviel Netz maintained that diagrams were for the Greeks the *object* of mathematics, and the object of the proof was logically determined by the diagram and not by the text alone (Netz 1998, 1999). Apart from the historical import of Netz' analysis, his studies have prompted new research questions on the role of diagrams in the *Elements* and a large number of philosophers of mathematics are now interested in this topic. In his book *After Euclid: Visual Reasoning and the Epistemology of Diagrams*, Jesse Norman argues that diagrams play a genuine justificatory role in traditional Euclidean arguments and that certain Euclidean arguments (such as Proposition I.32, namely the internal-angle-sum theorem) require inferences from diagrams (Norman, 2006). Other philosophers, as for instance Nathaniel Miller (Miller, 2008) and John Mumma (Mumma 2006; cf. also Avigad et al. 2009 and Mumma 2010), have offered formal systems for Euclidean geometry to analyze the methods of inference that are employed in the *Elements*. Both Miller and Mumma argue that the role played by diagrams in the proofs of the *Elements* is essential, and both address the issue of how reasoning based on a particular diagram can secure general conclusions (though they develop different formal systems). A different analysis has been advanced by Marco Panza, who has proposed an alternative way to account for the relation that (abstract) geometrical objects have with diagrams in Euclid's geometry (Panza, 2012). Panza distinguishes between diagrams that are taken to display some properties and relations of some other objects (possibly abstract ones) which are associated to them, and diagrams that do not. Euclidean diagrams *represent* (abstract) geometrical objects like for instance points or circles, which are associated to them, but arguments in EPG are about (abstract) geometrical objects. In order arguments in EPG be diagram-based, diagrams should have an appropriate relation with these objects and Panza's analysis is aimed to account for this relation. Another important contribution to the study of the role of ED in Euclid's geometry has been put forward by Ken Manders. Manders has proposed the idea that in a Euclidean proof diagrams are only used to infer "co-exact" (regional/topological) information (such as incidence or intersection) while "exact" (metric) information, like congruence, is always made explicit in the text (Manders, 2008a,b).[5]

In the next section I shall profit from some of the analysis advanced by philosophers of mathematics engaged in the analysis of diagrams and visual reasoning. More precisely, I shall provide a novel, though very primitive and basic, analysis of the role of ED in physics and, more generally, of the epistemic role that ED may play when used in empirical science.

3. Euclidean diagrams and inheritance

Scientists extensively use diagrams in their educational and research practice. They use diagrams to perform very different tasks, for instance to model and represent a physical setting, to convey knowledge (hypotheses, methods, findings) within a scientific community or educational context, to calculate, to analyze data, to resume some basic information about a process or an experiment or even to design and analyze novel experimental settings. Feynman diagrams, for example, are used to perform calculations in sub-atomic physics. Minkowski diagrams, originally used by Minkowski himself to obtain local geometrical axioms (cf. Smadja 2012), are commonly used to illustrate the properties of space and time in the special theory of relativity. Although the previous examples come from physics, diagrams are used in other empirical sciences as well. For instance, biologists make a great use of diagrams (Sheredos et al., 2013).

5 As expected, I am not exhausting here all the types of studies that have been proposed in this research context. Nevertheless, the works summarized here offer a good overview of this area and I take them to be representative of the research field that concerns the study of ED in Euclid's geometry. Furthermore, some of these studies are instrumental to the analysis I shall offer in the next section.

Many of these diagrams used in science, though not all, are ED. But what is the epistemic import of using an Euclidean diagram in science? While there have been several attempts to make sense of the role of Euclidean diagrams in mathematics, in what follows I focus on the epistemic import of using an ED in science.

Consider two finite straight lines AB and CD (‘segments’ in modern terms) that intersect in point E on a plane (Figure 1). The resulting diagram is Euclidean. Indeed, it is a very simple one and it can be constructed very easily using the rules of EPG. Consider now that the two segments ‘represent’ the rectilinear motions, i.e. the trajectories, of two bodies *a* and *b* along paths AB and CD respectively. Intersection point E on the diagram ‘represents’ a point in real space where both the trajectories have gone through.[6] There is, of course, a process of idealization (i.e. falsification of some of the actual properties of the concrete system) behind the step in which we consider the two bodies *as* points and the two

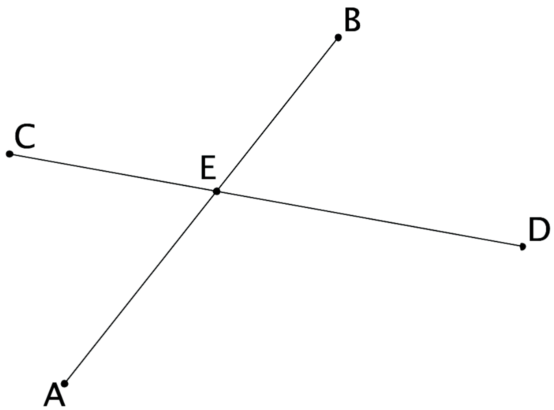

Fig. 1: ED representing two trajectories in a small region of space.

segments *as* their rectilinear trajectories in real space. Nevertheless, this process concerns the physics and it comes before the tracing of the

6 It may be noted that E could also represent a point in real space where both the trajectories will go through (in the future). Nevertheless, let me point out that time-dimension is not relevant for my analysis. Therefore, in what follows, I will refer to the two trajectories as the trajectories that have gone through a specific point in space, represented by E in the diagram.

diagram and our reasoning on it. The same, indeed, can be told of the fact that the trajectories in real space are considered rectilinear in a local (small enough) region of space, which is of course not true because we know that for the two bodies the curvature of space time is always present (although negligible for our analysis). All these are *physical* assumptions we make and therefore they do not affect the fact that the diagram can be *now* (i.e. after the idealization was made and a proper mapping has been established between the concrete system and the geometrical elements of the diagram) seen as representing a local region of space with two bodies moving along their rectilinear paths.

Let's now focus on the elements of the Euclidean diagram that appear in Figure 1, and more particularly on the intersection point E. Ken Manders distinguishes between two types of assertions that can be made of the geometric configurations arising in Euclid's proofs: "co-exact attributions", which describe general topological properties of the diagram configuration that are stable under perturbations of the diagram, and "exact attributions", which describe properties of the diagram configuration that are not stable under (even small) variations of the diagram (Manders, 2008b, p. 92). Intersections and incidence of points and lines are co-exact attributions, while proportionalities or congruence of segments and angles

constitute examples of exact attributions (the latter exact conditions, in fact, would fail immediately upon almost any diagram variation). Moreover, as Manders points out, co-exact attributions may be licensed directly by what is seen in the diagram (rather than by what appears in the discursive text of the *Elements*):

Co-exact attributions either arise by suitable entries in the discursive text (the setting-out of a claim, the application of a prior result or a postulate, such as that licensing entry of a circle in the proof of I.1); or *are licensed directly by the diagram*; for example, an intersection point of the two circles in Euclid I.1. This poses no immediate threat of disarray, because co-exact attributes (again, by definition) are 'locally invariant' under variation of the diagram: they are shared by a range of perturbed diagrams (Manders, 2008b, p. 94)

Let's now "perturb" the diagram of Figure 1 by considering a continuous variation of the initial diagram that results in the diagram of Figure 2. In this variation, the straight line CD has remained the same, point A has not moved, however point B has been moved to B′ so that AB′ is equal to AB. The effect of this variation, which is licensed by the rules of EPG, is that the intersection point E has now moved to E′. Note that the intersection point has followed the diagram-pertubation continuously, namely it has continuously moved from E till E' in the passage from Figure 1 to Figure 2. Thus a specific co-exact condition, given by the intersection of AB and CD, has resulted insensitive to the effects of a range of variation in the diagram entries, namely to the effects of the variation B → B′. In fact, although point B has now moved to B′, AB and CD still have an intersection point (now E′) on the diagram.

The fact that there exists a co-exact condition, and this condition has remained stable under a specific perturbation of our initial diagram, is particularly important for the point I want to make in this section. Indeed, if we focus on the ED after the perturbation (Figure 2) *and* we consider that it still represent the trajectories of the same bodies, we can now infer that changing the trajectory of body *a* has not affected (or won't affect, if we consider a future setting) the fact that the trajectories of the two bodies have intersected (or will intersect, in the future) at some point in real space. And this holds for a limited range of variations of the initial diagram (I shall say more on these variations below). This inference, though not particularly illuminating and interesting from a scientific point of view, produces knowledge about the world and therefore has epistemic value. Consider, for instance, that it gives an answer to the very simple question: "Would the trajectories of bodies *a* and *b* have intersected if the trajectory of *a* had been changed within a specific range of variation?". Furthermore, observe that the inference can be obtained by reasoning visually on the original diagram, namely that of Figure 1, and this simply varying it within a specific range of perturbations. Due to its co-exact character, the intersection point will always be preserved within a range of continuous variations (Figure 2 gives only the result of a particular perturbation/variation, however we can 'mentally track' a full range of continuous variations in which the intersection point will always exist).

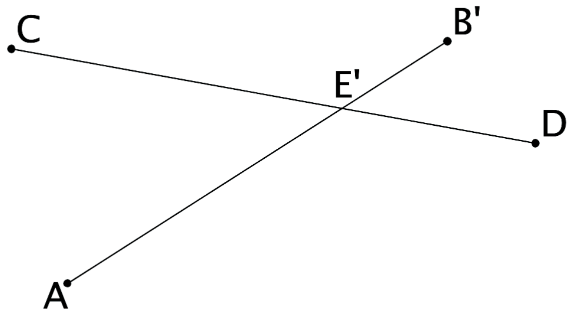

Fig. 2: Perturbed ED representing the two trajectories.

Not all thinking is a matter of making inferences in the way that logic -based systems do. And the specific form of inference that is involved in visual thinking on a diagram, as in the example above, is exactly of this kind. This is a form of inference I call "inheritance".[7] Manipulating mentally the diagram, i.e. reasoning visually on it, we can keep track of the local invariance of certain features of the diagram (co-exact attributes as the intersection point in our case) and therefore transfer information from an higher (i.e. more abstract, geometrical) structure to a lower (i.e. less abstract) structure, namely the physical system.[8] Thus the physical structure "inherits" the information from the geometrical (abstract) structure. The epistemic outcome of this process is knowledge concerning the physical system under scrutiny. In this process, moreover, it is important to note that our (mental or graphical) manipulations on the diagram are licensed by the rules of EPG.

7 Inheritance is a form of inference studied in cognitive science (Thagard, 2005, p. 63). Inference by inheritance is also used in object-oriented programming, when a class is defined in terms of another class. Although I share with cognitive scientists and informaticians the idea that inference via inheritance obtains when we transfer information from an higher structure to a lower structure, I consider here that the structures involved are essentially different. I take the higher structure as the 'more abstract', i.e. the mathematical structure, while the lower structure is the 'less abstract', namely the actual world.

8 *En passant*, let me note that the cognitive process through which we form a mental image and we perform a mental operation on this image is not relevant to the issue at hand.

A straightforward objection to my argument above would be that not every variation of B preserves intersection and therefore, in order the inheritance inference may be valid, I should define its conditions of validity, namely specify the family of variations that preserve intersection. Nevertheless, this could be made only considering metric information which is not available (not attributable) in a Euclidean diagram. Indeed, metric information is shown by, but not *determined* by, the ED. For instance, in our example we might choose B′ in a way that AB′ and CD do not intersect anymore because they became parallel or they became too short, and these variations belong to a family of variations that can be defined only using metrical considerations. This objection, which is central to the debate on the role played by diagrams in Euclidean geometry, can be replied adding further considerations about the role played by the rules and instruments of EPG in my account.[9] Consider Figure 3, where I show how using the rules of EPG we can define a specific range of variations that arises from varying the position of B as that the intersection point is still subject to a topological constraint. More precisely, we consider the circle of center A and radius AB. Next, varying the position of B along the arc GF, we obtain point B′ (as well as points B′′ or B′′′) and a corresponding

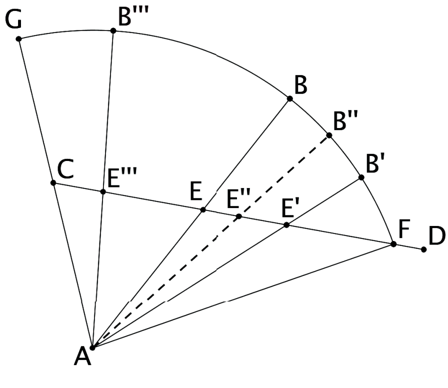

Fig. 3: Range of variations in which intersection is preserved.

9 Although I consider here only one response to this objection, my feeling is that my claims above can be defended adopting other strategies as well. Nevertheless, an illustration of these strategies would stretch the present paper far beyond its scope. This is why I leave it for future work.

intersection point E′ (as well as intersection points E′′ or E′′′). This defines the range of variations that preserve intersection and therefore the conditions of validity of the inheritance inference for this particular case. According to this reply, diagrams should be read in tandem with EPG. This is, in fact, what I have in mind when I stress that the diagrammatic representations are constructed using the rules of EPG. It should be observed, however, that these considerations do not amount to say that diagrams are only heuristic devices. Indeed, they permit us to infer information about the physical system. Of course, the conditions of validity of the inheritance inference are not given here at the same level of generality that we require in pure mathematics (this simply because in pure mathematics we want general conditions in order a proof to be valid; representation should be representation of a full range of geometrical configurations and conditions of validity of an inference should be provided for all these possible configurations). But this is not a problem for physics, where we are considering a particular representation and we might ask for more relaxed conditions of validity (in our example this translates into the claim that the two trajectories will still intersect in a point E′ or E′′ if topology of Figure 2 arises when B is moved to B′ or B′′).[10]

Even if my aim here is not to give a comprehensive characterization of the inferences that be obtained when reasoning visually on a ED, and at this stage my claim that Euclidean diagrams have epistemic value in science is conditional to the presence of co-exact attributes in such diagrams, I want to close this section with some remarks that may be fruitful for future investigation.

First of all, I want to consider again the mechanism of idealization that is involved in the scenario I considered above. Idealizations are often (if not always) used in physics, and once an idealization has been put in place we analyze the physical system through our mathematical machi-

10 It is not the goal of the present paper to discuss the role of generality in mathematics and in the empirical sciences but is well know that the standards of rigor required and used in mathematics are quite different from those of the empirical sciences. As Feynman pointed out: “The mathematical rigor of great precision is not very useful in physics. But one should not criticize the mathematicians on this score ... They are doing their own job” (Feynman, 1965, p. 56).

nery. This machinery may be formal (for instance, a set of equations) or not (as in the case of the Euclidean diagram we used), but in both cases the analysis is made on the idealization itself. Therefore the fact that an idealization step already be present does not detract from the fact that the successive analysis be formal or diagrammatic. In our example above, for instance, we considered that space is locally flat and there is no curvature. Moreover, the two bodies are points that move in rectilinear motion. All this is physical idealization, and therefore (once assumed) it legitimates the use of Euclid's (flat) geometry and the acceptance of an analysis performed through its instruments, including diagrams. Moreover, I think we can push further my analysis above and consider a different setting in which another sort of idealization is employed. We can consider, for instance, an ED that represents a physical system in which no idealization has been made concerning the space-time properties of the physical system under study. The example I have in mind is that of two thermodynamic systems that are described by thermodynamic state variables such as temperature, entropy, internal energy and pressure. Straight lines in Figure 1 may represent different states of the system during two transformations, with each point of a line representing a precise state of the system. The intersection point represents a common state, in which the two thermodynamic states of the systems are the same. Although I won't pursue here an analysis of this scenario, and I suppose that similar examples can be offered within other empirical sciences as well, it is *prima facie* plausible to think that we can get some knowledge of the system by reasoning on the Euclidean diagram in a way very similar to that we used in the case of the two trajectories.

There is another topic that is closely related to the analysis of diagrammatic reasoning in science. This topic concerns the applicability of mathematics in science, namely the research on *how* mathematics applies successfully to the world (cf. Molinini and Panza 2014 for a survey on the applicability problem). Indeed, if diagrammatic reasoning has an epistemic import in empirical science, therefore we should say something about the way in which we apply geometrical diagrams and get interesting conclusions about our concrete non-flat world. The applicability problem covers a huge area of research in philosophy of mathematics, and here I only want to show that the considerations I put forward above can be potentially accommodated by one influential account of applicability, the so-called "inferential conception" of the application of mathematics proposed by Otávio Bueno and Mark Colyvan (Bueno and Colyvan, 2011). According to Bueno and Colyvan, the process of applying mathematics to the world can be accounted through a three -steps analysis: the immersion step, in which we specify a mapping from some relevant aspects of the empirical domain to a mathematical structure; the derivation step, also called deduction step, which takes place entirely within the mathematical domain and in which we realize mathematical inferences, licensed by the mathematical theory, about the immersed structure; the interpretation step, in which the consequences found in the derivation step are mapped back to the empirical domain *via* an appropriate mapping (in this step we interpret the results of our mathematical investigation). Within this framework I see the possibility to accommodate the inheritance form of inference. Diagrammatic reasoning on ED, indeed, operates at level of the deduction step, in which deductions are licensed by the rules of EPG. Before this step, a mapping is established between the physical system and the diagram (immersion step), while after the derivation step the consequences found on the diagram are mapped back to the empirical domain (interpretation step).

4. Conclusions

On an older view, which dates back to Kant, diagrams have epistemic value in mathematics via a dubious appeal to a postulated faculty of "intuition". Nevertheless, this appeal to intuition lost its appeal by the late 19th century, when mathematicians such as Dedekind, Pasch and Hilbert regarded the use of geometric intuitions and figures in basic infinitesimal analysis and geometry as unreliable. This anti-diagrammatic standpoint was not shared by some mathematicians, however it led to an attitude which is still predominant today: mathematicians and philosopher of mathematics usually consider that diagrams only have heuristic value and could not bear any epistemological weight. Trying to reverse the latter standpoint, a recent trend in philosophy of mathematics has given great importance to the epistemic role that, according to some, diagrams and visual reasoning may have in mathematics.

Although the interest in the function of diagrams and visual reasoning has grown among philosophers of mathematics, the contemporary discussion on diagrammatic reasoning has remained almost entirely confined to the role of diagrams in proofs and in the argumentative structure of the *Elements*. In this paper I have followed a different research path. I have considered Euclidean diagrams, namely those diagrams that can be constructed and read using the rules of Euclid's plane geometry, and I have provided a primitive analysis of their role in science, more particularly in physics. In my analysis I have considered a very basic scenario in which an Euclidean diagram represents a concrete physical system (via an idealization and a suitable mapping), claiming that the epistemic role of the Euclidean diagram is disclosed once we analyze a specific form of inference. I called inheritance this form of inference. Inheritance is licensed by the fact that we can reason visually on a diagram and some of its co-exact properties (therefore co-exact properties should be present in the diagram). This shows how manipulations and visual reasoning with diagrams can shed light on the concrete realm of physical objects, and how Euclidean diagrams may have epistemic value in science.

Even in light of the attenuated role of Euclidean diagrams in modern mathematics and empirical science, understanding their nature and function is still an important part of understanding how modern science works. The modest aim of the present study was to make a step towards such understanding.

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