Cent. Eur. J. Math. • 11(10) • 2013 • 1725-1731
DOI: 10.2478/s11533-013-0280-2

VERSITA

Central European Journal of **Mathematics**

Completely normal elements in some finite abelian extensions

Research Article

Ja Kyung Koo[1]*, Dong Hwa Shin[2]†

1 Department of Mathematical Sciences, KAIST, Daejeon 305-701, Republic of Korea

2 Department of Mathematics, Hankuk University of Foreign Studies, Yongin-si, Gyeonggi-do 449-791, Republic of Korea

Received 23 February 2012; accepted 12 November 2012

Abstract: We present some completely normal elements in the maximal real subfields of cyclotomic fields over the field of rational numbers, relying on the criterion for normal element developed in [Jung H.Y., Koo J.K., Shin D.H., Normal bases of ray class fields over imaginary quadratic fields, Math. Z., 2012, 271(1-2), 109–116]. And, we further find completely normal elements in certain abelian extensions of modular function fields in terms of Siegel functions.

MSC: 11R18, 11F03, 12F05

Keywords: Cyclotomic extensions • Modular functions • Normal bases

© *Versita Sp. z o.o.*

1. Introduction

Let L be a finite Galois extension of a field K. We know by the normal basis theorem [13, Section 8.11] that there exists an element $x \in L$ such that $\{x^\gamma : \gamma \in \mathrm{Gal}(L/K)\}$ forms a K-basis of L, a so-called *normal basis* of L/K. Such an element x is also said to be *normal* in L/K. Moreover, if x is normal in L/F for any intermediate field F, then x is said to be *completely normal* in L/K. Faith [3] proved in 1957 that every finite Galois extension L of an infinite field K contains a completely normal element. Blessenohl and Johnson [1] showed in 1986 that this strengthening of the normal basis theorem holds for every finite extension L of a finite field K. It is usually very difficult, however, to determine completely normal elements in a finite Galois extension L/K. Hachenberger [5] provided explicit constructions of completely normal elements in prime-power cyclotomic fields over the rational field $\mathbb{Q}$, but such constructions are not known for general number fields.

* *E-mail: jkkoo@math.kaist.ac.kr*
† *E-mail: dhshin@hufs.ac.kr*

Unauthenticated
Download Date | 8/24/19 2:43 PM

Now, we let $\zeta_\ell = e^{2\pi i/\ell}$ be a primitive ℓth root of unity for a positive integer ℓ and $\mathbb{Q}(\zeta_\ell)^+$ be the maximal real subfield of the ℓth cyclotomic field $\mathbb{Q}(\zeta_\ell)$. Okada proved in [11] that if k and ℓ, $\ell > 2$, are positive integers with k odd and T is a set of representatives (mod ℓ) such that $(\mathbb{Z}/\ell\mathbb{Z})^\times = T \cup (-T)$, then the numbers $(1/\pi^k)(d/dz)^k(\cot \pi z)|_{z=a/\ell}$ for $a \in T$ form a normal basis of $\mathbb{Q}(\zeta_\ell)^+/\mathbb{Q}$, which is an extension of Chowla's result [2] when $k = 1$. He utilized the partial fractional decomposition of $(d/dz)^k \cot \pi z$ and the Frobenius determinant relation [10, Chapter 21, Theorem 5]. And, when $\mathcal{N}$ is the set of positive integers which are either odd or divisible by 4, we let $\ell \in \mathcal{N}$. Hachenberger constructed in [5] an element $w \in \mathbb{Q}(\zeta_\ell)$ which is simultaneously normal in $\mathbb{Q}(\zeta_\ell)/\mathbb{Q}(\zeta_n)$ for each $n \in \mathcal{N}$ dividing ℓ by making use of the notion of cyclic submodules in $\mathbb{Q}(\zeta_\ell)/\mathbb{Q}(\zeta_n)$ [4].

On the other hand, for an integer ℓ, $\ell \geq 2$, let $K_{(\ell)}$ be the ray class field of an imaginary quadratic field K $(\neq \mathbb{Q}(\sqrt{-1}), \mathbb{Q}(\sqrt{-3}))$ modulo ℓ. Jung et al. [7] recently showed that the singular value of a certain Siegel function is normal in $K_{(\ell)}/K$. To this end they derived a useful criterion for determining a normal element in a finite abelian extension of a number field from the Frobenius determinant relation.

In this paper we shall first improve this criterion to determine completely normal elements in finite abelian extensions (Theorem 2.2), and then use this new criterion to present an infinite family of completely normal elements in $\mathbb{Q}(\zeta_\ell)^+/\mathbb{Q}$ for an integer ℓ, $\ell \geq 5$, (Theorems 3.1 and 3.2). It is expressed in terms of the cosine function, which is simple but different from Okada's normal element [11]. We shall further find completely normal elements in certain abelian extensions of modular function fields in view of Siegel functions (Theorem 4.2).

2. A criterion for completely normal elements

Throughout this section we let L be a finite abelian extension of a field K of characteristic zero with $[L\!:\!K] = n$, $n \geq 2$, and $G = \mathrm{Gal}(L/K)$. Further, we let $|\cdot|$ be a valuation on the field $L(\mu_n)$, where μ_n is the set of all nth roots of unity in a fixed algebraic closure of L. Then $|\cdot|$ satisfies the triangle inequality, namely $|x+y| \leq |x| + |y|$ for all $x, y \in L(\mu_n)$. And, we see that

$$|x| - |y| \leq |x+y| \qquad \text{for all} \quad x, y \in L(\mu_n). \tag{1}$$

In particular, if $|\cdot|$ is nonarchimedean, then $|x+y| \leq \max\{|x|, |y|\}$, from which one can readily deduce the fact that

$$|x|^m - |y|^m \leq |x+y|^m \qquad \text{for all} \quad x, y \in L(\mu_n) \quad \text{and any positive real number } m \tag{2}$$

[6, Chapter II, Section 1].

Lemma 2.1 ([7, Proposition 2.3]).
An element $x \in L$ is normal in L/K if and only if

$$\sum_{\gamma \in G} \chi(\gamma^{-1}) x^\gamma \neq 0 \qquad (\text{in } L(\mu_n)) \quad \text{for all characters } \chi \text{ on } G.$$

Theorem 2.2.
Assume that there exists an element $x \in L$ such that $|x^\gamma/x| < 1$ for all $\gamma \in G - \{\mathrm{Id}\}$. Let m be any positive integer such that

$$\left|\frac{x^\gamma}{x}\right|^m \leq \frac{1}{n} \qquad \text{for all} \quad \gamma \in G - \{\mathrm{Id}\}. \tag{3}$$

Then x^m is completely normal in L/K. In particular, if $|\cdot|$ is nonarchimedean, then any positive power of x is completely normal in L/K.

Unauthenticated
Download Date | 8/24/19 2:43 PM

Proof. Let F be an intermediate field of L/K with $\ell = [L:F]$ and $H = \mathrm{Gal}(L/F) \leq G$. For any character χ on H we get that

$$\begin{aligned}
\left|\sum_{\gamma\in H}\chi(\gamma^{-1})(x^m)^\gamma\right| &\geq |x^m|\left(1-\sum_{\gamma\in H-\{\mathrm{Id}\}}|(x^m)^\gamma/x^m|\right) && \text{by (1)}\\
&\geq |x^m|\left(1-\frac{1}{n}(\ell-1)\right) && \text{by (3)}\\
&= |x^m|\,\frac{n-\ell+1}{n} > 0 && \text{because } \ell\leq n.
\end{aligned}$$

This shows that x^m is normal in L/F by Lemma 2.1; hence x^m is completely normal in L/K as desired. Furthermore, if $|\cdot|$ is nonarchimedean, then we derive for any positive integer t that

$$\begin{aligned}
\left|\sum_{\gamma\in H}\chi(\gamma^{-1})(x^t)^\gamma\right|^m &\geq |x^t|^m\left(1-\sum_{\gamma\in H-\{\mathrm{Id}\}}|(x^t)^\gamma/x^t|^m\right) && \text{by (2)}\\
&\geq |x^t|^m\left(1-\left(\frac{1}{n}\right)^t(\ell-1)\right) && \text{by (3)}\\
&\geq |x^t|^m\left(1-\frac{1}{n}(\ell-1)\right) = |x^t|^m\,\frac{n-\ell+1}{n} > 0 && \text{because } \ell\leq n.
\end{aligned}$$

Hence x^t is completely normal in L/K again by Lemma 2.1. □

Corollary 2.3.

Let L be an abelian extension of a number field K with $[L:K] = n$. Assume that we have an element $x \in L$ satisfying

(i) *x is an algebraic integer,*

(ii) *$L = K(x)$,*

(iii) *x^γ are real for all $\gamma \in G$.*

If a and b are nonzero integers such that $2 < |a/b|$ where $|\cdot|$ is the usual absolute value on $\mathbb{C}$, then a high power of $ax + b$ is completely normal in L/K.

Proof. Suppose that there are distinct elements γ and δ of G such that $|ax^\gamma + b| = |ax^\delta + b|$. Since x^γ and x^δ are real by the assumption (iii), we get $ax^\gamma + b = \pm(ax^\delta + b)$. Moreover, since $x^\gamma \neq x^\delta$ by the assumption (ii) and the fact $\gamma \neq \delta$, we obtain $ax^\gamma + b = -(ax^\delta + b)$, from which it follows that $x^\gamma + x^\delta = -2b/a$. Note that $x^\gamma + x^\delta$ is an algebraic integer by the assumption (i), but $-2b/a$ is a rational number such that $0 < |2b/a| < 1$, which yields a contradiction.

Thus, for each intermediate field F of L/K with $[L:F] \geq 2$, there is a unique element $\gamma_F \in \mathrm{Gal}(L/F)$ such that $|ax^\gamma + b| < |ax^{\gamma_F} + b|$ for all $\gamma \in \mathrm{Gal}(L/F) - \{\gamma_F\}$ by the preceding argument. Now, take a suitably large positive integer m_F so that $|(ax^\gamma + b)/(ax^{\gamma_F} + b)|^{m_F} \leq 1/[L:F]$, and set $m = \max\{m_F : F\}$. Then $(ax^{\gamma_F} + b)^m$ is completely normal in L/F by Theorem 2.2. In particular, the set $\{((ax^{\gamma_F} + b)^m)^\gamma : \gamma \in \mathrm{Gal}(L/F)\}$ is a normal basis of L over F, and hence $((ax^{\gamma_F} + b)^m)^{\gamma_F^{-1}} = (ax + b)^m$ is normal in L/F. This implies that $(ax + b)^m$ is completely normal in L/K because F is arbitrary. □

Remark 2.4.

When L is totally real, there always exists such an element $x \in L$ that satisfies the assumptions of Corollary 2.3. Therefore, in this case, one can get an explicit algorithm which gives infinitely many completely normal elements in L/K.

Unauthenticated
Download Date | 8/24/19 2:43 PM

3. Maximal real subfields of cyclotomic fields

Let ℓ be a positive integer. As is well known, $\mathbb{Q}(\zeta_\ell)^+ = \mathbb{Q}(\zeta_\ell + \zeta_\ell^{-1})$ and $\mathrm{Gal}(\mathbb{Q}(\zeta_\ell)^+/\mathbb{Q}) \simeq (\mathbb{Z}/\ell\mathbb{Z})^\times/\{\pm 1\}$, whose actions are given as follows: if $t \in (\mathbb{Z}/\ell\mathbb{Z})^\times/\{\pm 1\}$, then $(\zeta_\ell + \zeta_\ell^{-1})^t = \zeta_\ell^t + \zeta_\ell^{-t}$. Denote the number of positive integers relatively prime to ℓ by $\phi(\ell)$. Then we have (see [14, Chapter 2])

$$[\mathbb{Q}(\zeta_\ell)^+ : \mathbb{Q}] = \begin{cases} 1, & \text{if} \quad \ell = 1, 2, 3, 4, 6, \\ \dfrac{\phi(\ell)}{2} \ (\geq 2), & \text{otherwise.} \end{cases}$$

Let $|\cdot|$ be the usual absolute value on $\mathbb{C}$.

Theorem 3.1.
Let ℓ, $\ell \neq 1, 2, 3, 4, 6$, be a positive integer. If m is any positive integer such that

$$\left(\frac{\cos(4\pi/\ell) + 1}{\cos(2\pi/\ell) + 1} \right)^m \leq \frac{2}{\phi(\ell)},$$

then $(\cos(2\pi/\ell) + 1)^m$ is completely normal in $\mathbb{Q}(\zeta_\ell)^+/\mathbb{Q}$.

Proof. Set $x = (\zeta_\ell + \zeta_\ell^{-1})/2 + 1 = \cos(2\pi/\ell) + 1$. If $\gamma \in \mathrm{Gal}(\mathbb{Q}(\zeta_\ell)^+/\mathbb{Q}) - \{\mathrm{Id}\}$, then $x^\gamma = (\zeta^t + \zeta^{-t})/2 + 1$ for some integer t with $\gcd(\ell, t) = 1$ and $1 < t \leq [\ell/2]$. Here $[\cdot]$ stands for the Gauss symbol. So, we achieve that

$$\left| \frac{x^\gamma}{x} \right| = \left| \frac{(\zeta^t + \zeta^{-t})/2 + 1}{(\zeta + \zeta^{-1})/2 + 1} \right| = \left| \frac{\cos(2t\pi/\ell) + 1}{\cos(2\pi/\ell) + 1} \right| \leq \left| \frac{\cos(4\pi/\ell) + 1}{\cos(2\pi/\ell) + 1} \right|,$$

which is less than 1. The conclusion follows from Theorem 2.2. □

Theorem 3.2.
Let ℓ, $\ell \geq 5$, be an odd integer. If m is any positive integer such that

$$\left(\frac{\cos(2\pi/\ell)}{\cos(\pi/\ell)} \right)^m \leq \frac{2}{\phi(\ell)},$$

then $\cos^m(\pi/\ell)$ is completely normal in $\mathbb{Q}(\zeta_\ell)^+/\mathbb{Q}$.

Proof. Let $x = -(\zeta^{(\ell-1)/2} + \zeta^{-(\ell-1)/2})/2$. Since $1 \cdot \ell + (-2) \cdot (\ell - 1)/2 = 1$, we get $\gcd(\ell, (\ell-1)/2) = 1$, which implies that x is a conjugate of $-(\zeta + \zeta^{-1})/2$. Hence, if $\gamma \in \mathrm{Gal}(\mathbb{Q}(\zeta_\ell)^+/\mathbb{Q}) - \{\mathrm{Id}\}$, then $x^\gamma = -(\zeta^t + \zeta^{-t})/2$ for some integer t with $\gcd(\ell, t) = 1$ and $1 \leq t < (\ell - 1)/2$. And, we establish that

$$\left| \frac{x^\gamma}{x} \right| = \left| \frac{-(\zeta^t + \zeta^{-t})/2}{-(\zeta^{(\ell-1)/2} + \zeta^{-(\ell-1)/2})/2} \right| = \left| \frac{-\cos(2t\pi/\ell)}{-\cos(\pi - \pi/\ell)} \right| = \frac{|\cos(2t\pi/\ell)|}{\cos(\pi/\ell)} \leq \frac{\cos(2\pi/\ell)}{\cos(\pi/\ell)},$$

which is less than 1. We then obtain the assertion again by Theorem 2.2. □

Lemma 3.3 ([8, p. 227]).
Let L_1 and L_2 be finite Galois extensions of a number field K such that $L_1 \cap L_2 = K$. If $x_i \in L_i$ is normal in L_i/K, $i = 1, 2$, then $x_1 x_2$ is normal in $L_1 L_2/K$.

Unauthenticated
Download Date | 8/24/19 2:43 PM

Lemma 3.4 ([6, Theorem 11.1]).
Let $t = 4$ or an odd prime p such that $p \equiv 3 \pmod 4$. Then $\mathbb{Q}(\zeta_t)$ contains a unique quadratic extension of $\mathbb{Q}$, namely $\mathbb{Q}(\sqrt{-t})$.

Theorem 3.5.
Let $t = 4$ or an odd prime p such that $p \equiv 3 \pmod 4$. Let ℓ, $\ell \neq 1, 2, 3, 4, 6$, be a positive integer. If m is any positive integer such that

$$\left(\frac{\cos(4\pi/t\ell)+1}{\cos(2\pi/t\ell)+1}\right)^m \leq \frac{2}{\phi(t\ell)}, \tag{4}$$

then $(\sqrt{-t}+1)(\cos(2\pi/t\ell)+1)^m$ is normal in $\mathbb{Q}(\zeta_{t\ell})/\mathbb{Q}$.

Proof. One can readily see that $\sqrt{-t}+1$ is normal in $\mathbb{Q}(\sqrt{-t})/\mathbb{Q}$. And, if m is any positive integer satisfying the condition (4), then $(\cos(2\pi/t\ell)+1)^m$ is normal in $\mathbb{Q}(\zeta_{t\ell})^+/\mathbb{Q}$ by Theorem 3.1. On the other hand, since $\mathbb{Q}(\sqrt{-t})$ is an imaginary quadratic field contained in $\mathbb{Q}(\zeta_t)$ ($\subset \mathbb{Q}(\zeta_{t\ell})$) by Lemma 3.4, we have $\mathbb{Q}(\sqrt{-t}) \cap \mathbb{Q}(\zeta_{t\ell})^+ = \mathbb{Q}$ and $\mathbb{Q}(\sqrt{-t})\mathbb{Q}(\zeta_{t\ell})^+ = \mathbb{Q}(\zeta_{t\ell})$. Therefore, the theorem follows from Lemma 3.3. □

4. Fields of modular functions

Let $\mathbb{H} = \{\tau \in \mathbb{C} : \operatorname{Im}\tau > 0\}$ be the complex upper half-plane. For a positive integer N we let $\Gamma(N)$ be the principal congruence subgroup of level N, that is

$$\Gamma(N) = \{\gamma \in \mathrm{SL}_2(\mathbb{Z}) : \gamma \equiv I_2 \bmod N\}.$$

Then the quotient group $\overline{\Gamma}(N) = \pm\Gamma(N)/\{\pm I_2\}$ acts on $\mathbb{H}^* = \mathbb{H} \cup \mathbb{Q} \cup \{\infty\}$ as fractional linear transformations. A (meromorphic) *modular function of level N* is defined to be a function $f\colon \mathbb{H} \to \mathbb{C} \cup \{\infty\}$ which satisfies the following three conditions:

- f is meromorphic on $\mathbb{H}$,
- f is invariant under $\overline{\Gamma}(N)$,
- f is meromorphic at the cusps $\mathbb{Q} \cup \{\infty\}$,

[12, Definition 2.1]. It is essentially a meromorphic function on the compact Riemann surface $\overline{\Gamma}(N) \setminus \mathbb{H}^*$ [12, p. 30]. We denote by $\mathbb{C}(X(N))$ the field of all modular functions of level N. As is well known, $\mathbb{C}(X(N))$ is a Galois extension of $\mathbb{C}(X(1))$ whose Galois group is isomorphic to $\overline{\Gamma}(1)/\overline{\Gamma}(N) \simeq \mathrm{SL}_2(\mathbb{Z}/N\mathbb{Z})/\{\pm I_2\}$ [10, Chapter 6].

For a pair $(r_1, r_2) \in \mathbb{Q}^2 - \mathbb{Z}^2$, the *Siegel function* $g_{(r_1,r_2)}(\tau)$ on $\mathbb{H}$ is defined by the infinite product

$$g_{(r_1,r_2)}(\tau) = -q^{(1/2)\mathbf{B}_2(r_1)} e^{\pi i r_2(r_1-1)} \left(1-q^{r_1}e^{2\pi i r_2}\right) \prod_{n=1}^{\infty}\left(1-q^{n+r_1}e^{2\pi i r_2}\right)\left(1-q^{n-r_1}e^{-2\pi i r_2}\right),$$

where $q = e^{2\pi i\tau}$ and $\mathbf{B}_2(X) = X^2 - X + 1/6$ is the second Bernoulli polynomial. For $X \in \mathbb{R}$ we let $\langle X\rangle$ be the fractional part of X in the interval $[0, 1)$.

Lemma 4.1 ([9, Chapter 2, Proposition 1.3 and p. 31]).
Let N, $N \geq 2$, be an integer and $(r_1, r_2) \in (1/N)\mathbb{Z}^2 - \mathbb{Z}^2$.

(i) *$g_{(r_1,r_2)}(\tau)^{12N}$ is a modular function of level N.*

Unauthenticated
Download Date | 8/24/19 2:43 PM

(ii) *For* $\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}_2(\mathbb{Z})$ *we have the transformation formula*

$$g_{(r_1,r_2)}(\tau)^{12N} \circ \gamma = g_{(r_1,r_2)\gamma}(\tau)^{12N} = g_{(r_1a+r_2c, r_1b+r_2d)}(\tau)^{12N}.$$

(iii) $\mathrm{ord}_q g_{(r_1,r_2)}(\tau)^{12N} = 6N\mathbf{B}_2(\langle r_1 \rangle)$.

For a positive integer N we define a congruence subgroup $\Gamma^0(N)$ of $\mathrm{SL}_2(\mathbb{Z})$ by

$$\Gamma^0(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}_2(\mathbb{Z}) : b \equiv 0 \bmod N \right\}.$$

Theorem 4.2.
Let N, $N \geq 4$, *be an integer and* Γ *be a subgroup of* $\mathrm{SL}_2(\mathbb{Z})$ *such that*

(i) $\Gamma(N) < \Gamma \leq \Gamma^0(N)$,

(ii) $\overline{\Gamma}/\overline{\Gamma}(N)$ *is a nontrivial abelian group, where* $\overline{\Gamma} = \pm\Gamma/\{\pm I_2\}$.

Let $L = \mathbb{C}(X(N))$ *and* K *be the subfield of* L (over $\mathbb{C}(X(1))$) *fixed by* $\overline{\Gamma}/\overline{\Gamma}(N)$ *elementwise. Then, any positive power of* $(g_{(0,1/N)}(\tau)g_{(1/N,0)}(\tau))^{-12N}$ *is completely normal in* L/K.

Proof. First we note that $\mathrm{Gal}(L/K) \simeq \overline{\Gamma}/\overline{\Gamma}(N)$ by the Galois theory. Consider the nonarchimedean valuation $|\cdot|$ on L defined by

$$|\cdot| : L \to \mathbb{R}_{\geq 0}, \qquad f \mapsto |f| = \exp(-\mathrm{ord}_q f).$$

Put $x = (g_{(0,1/N)}(\tau)g_{(1/N,0)}(\tau))^{-12N}$, which belongs to L by Lemma 4.1 (i). Let $\gamma = \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right)$ be an element of $\pm\Gamma$ that represents a nonidentity element of $\mathrm{Gal}(L/K)$. Since $b \equiv 0 \bmod N$ by the assumption (i), we get

$$a \not\equiv 0 \bmod N. \tag{5}$$

Further, we deduce from the fact $\det\gamma = ad - bc = 1$ that

$$a \not\equiv \pm 1 \bmod N \qquad \text{or} \qquad c \not\equiv 0 \bmod N, \tag{6}$$

otherwise γ belongs to $\pm\Gamma(N)$ and gives rise to the identity of $\mathrm{Gal}(L/K)$. Now we achieve that

$$\begin{aligned} \left|\frac{x^\gamma}{x}\right| &= \exp\left(-\mathrm{ord}_q \frac{x^\gamma}{x}\right) = \exp\left(-\mathrm{ord}_q \frac{(g_{(c/N,d/N)}(\tau)g_{(a/N,b/N)}(\tau))^{-12N}}{(g_{(0,1/N)}(\tau)g_{(1/N,0)}(\tau))^{-12N}}\right) && \text{by Lemma 4.1 (ii)} \\ &= \exp\left\{6N\left(\mathbf{B}_2(\langle c/N\rangle) + \mathbf{B}_2(\langle a/N\rangle) - \mathbf{B}_2(0) - \mathbf{B}_2(\langle 1/N\rangle)\right)\right\} && \text{by Lemma 4.1 (iii)} \\ &< \exp(0) = 1 \end{aligned}$$

by (5) and (6) and the fact that $\mathbf{B}_2(0) > \mathbf{B}_2(\langle \pm 1/N\rangle) > \mathbf{B}_2(\langle \pm 2/N\rangle) > \ldots > \mathbf{B}_2(\langle \pm [N/2]/N\rangle)$. Therefore, any positive power of x is completely normal in L/K by the assumption (ii) and Theorem 2.2. □

Acknowledgements

The first named author was partially supported by the NRF of Korea grant funded by MEST (2012-0000798). The corresponding author was supported by Hankuk University of Foreign Studies Research Fund of 2012.

Unauthenticated
Download Date | 8/24/19 2:43 PM

References

[1] Blessenohl D., Johnsen K., Eine Verschärfung des Satzes von der Normalbasis, J. Algebra, 1986, 103(1), 141–159

[2] Chowla S., The nonexistence of nontrivial linear relations between the roots of a certain irreducible equation, J. Number Theory, 1970, 2(1), 120–123

[3] Faith C.C., Extensions of normal bases and completely basic fields, Trans. Amer. Math. Soc., 1957, 85(2), 406–427

[4] Hachenberger D., Finite Fields, Kluwer Internat. Ser. Engrg. Comput. Sci., 390, Kluwer, Boston, 1997

[5] Hachenberger D., Universal normal bases for the abelian closure of the field of rational numbers, Acta Arith., 2000, 93(4), 329–341

[6] Janusz G.J., Algebraic Number Fields, 2nd ed., Grad. Stud. Math., 7, American Mathematical Society, Providence, 1996

[7] Jung H.Y., Koo J.K., Shin D.H., Normal bases of ray class fields over imaginary quadratic fields, Math. Z., 2012, 271(1-2), 109–116

[8] Kawamoto F., On normal integral bases, Tokyo J. Math., 1984, 7(1), 221–231

[9] Kubert D., Lang S., Modular Units, Grundlehren Math. Wiss., 244, Spinger, New York–Berlin, 1981

[10] Lang S., Elliptic Functions, 2nd ed., Grad. Texts in Math., 112, Spinger, New York, 1987

[11] Okada T., On an extension of a theorem of S. Chowla, Acta Arith., 1980/81, 38(4), 341–345

[12] Shimura G., Introduction to the Arithmetic Theory of Automorphic Functions, Publ. Math. Soc. Japan, 11, Iwanami Shoten/Princeton University Press, Tokyo/Princeton, 1971

[13] van der Waerden B.L., Algebra I, Springer, New York, 1991

[14] Washington L.C., Introduction to Cyclotomic Fields, Grad. Texts in Math., 83, Springer, New York, 1982

Unauthenticated
Download Date | 8/24/19 2:43 PM