Erratum to “On the strong metric dimension of the strong products of graphs”

Abstract: The original version of the article was published in Open Mathematics (formerly Central European Journal of Mathematics) 13 (2015) 64–74. Unfortunately, the original version of this article contains a mistake: in Lemma 2.17 appears that for any $C_1$-graph $G$ and any graph $H$, $\beta(G \boxtimes H) \leq \beta(G)(\beta(H) + 1)$, while should be $\beta(G \boxtimes H) \leq \beta(H)(\beta(G) + 1)$. In this erratum we correct the lemma, its proof and some of its consequences.

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We use herein all the notions, notations and terminology of the work [1]. Moreover, all the cross-references to theorems, equations and other results are done according to such a work [1]. Lemma 2.17 contains a misunderstanding in the last line of the proof. It appears there $\beta(G \boxtimes H) = |S| = \sum_{i=1}^{\beta(G)} |S_i| + |S_B| = \sum_{i=1}^{\beta(G)} |P_H(S_i)| + |P_H(S_B)| \leq \beta(G) \cdot \beta(H) + \beta(H) = \beta(G)(\beta(H) + 1)$, which is clearly not correct in the last equality. Thus the Lemma is corrected as follows.

Lemma 2.17. For any $C_1$-graph $G$ and any graph $H$,

$$\beta(G \boxtimes H) \leq \beta(H)(\beta(G) + 1).$$

Proof. Let $A_1, A_2, \ldots, A_{\beta(G)}$, $B$ be a partition of $V(G)$ such that $A_i$ is a clique for every $i \in \{1, 2, \ldots, \beta(G)\}$ and $B = \{b\}$, where $b$ is an isolated vertex. Let $S$ be an $\beta(G) \boxtimes H$-set and let $S_i = S \cap (A_i \times V_2)$ and $i \in \{1, 2, \ldots, \beta(G)\}$. Let $S_B = S \cap (B \times V_2)$. By using analogous procedures as in proof of Lemma 2.14 we can show that for every $i \in \{1, 2, \ldots, \beta(G)\}$, $P_H(S_i)$ is an independent set in $H$ and $|S_i| = |P_H(S_i)|$. Moreover, since $|B| = 1$ we have that $P_H(S_B)$ is an independent set in $H$ and $|S_B| = |P_H(S_B)|$. Thus, we obtain the following

$$\beta(G \boxtimes H) = |S| = \sum_{i=1}^{\beta(G)} |S_i| + |S_B| = \sum_{i=1}^{\beta(G)} |P_H(S_i)| + |P_H(S_B)| \leq \beta(G) \cdot \beta(H) + \beta(H)$$

$$= \beta(H)(\beta(G) + 1).$$

The lemma above was used in the proofs of some other results of [1] (namely, Theorems 2.18 and 2.19). Next we make a correct deduction of these theorems according to the corrected Lemma 2.17.

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Theorem 2.18. Let $G$ and $H$ be two connected nontrivial graphs of order $n_1, n_2$, respectively. If $G_{SR}$ is a $C_1$-graph, then
\[ \dim_s(G \boxtimes H) \geq n_2 \dim_s(G) - 1 + \dim_s(H)(n_1 - \dim_s(G) + 1). \]

Proof. By using Corollary 2.8 we have that $\beta(G_{SR} \boxtimes H_{SR}) \geq \beta((G \boxtimes H)_{SR})$. Hence, from equality (1) and Lemma 2.17 we have
\[ \dim_s(G \boxtimes H) = n_1 \cdot n_2 - \beta((G \boxtimes H)_{SR}) \]
\[ \geq n_1 \cdot n_2 - \beta(G_{SR} \boxtimes H_{SR}) \]
\[ \geq n_1 \cdot n_2 - \beta(H_{SR})(\beta(G_{SR}) + 1) \]
\[ = n_1 \cdot n_2 - (n_2 - \dim_s(H)) \cdot (n_1 - \dim_s(G) + 1) \]
\[ = n_2(\dim_s(G) - 1) + \dim_s(H)(n_1 - \dim_s(G) + 1). \]

Theorem 2.19. Let $H$ be a connected nontrivial graph of order $n$ and $r \geq 1$. Then
\[ nr + \dim_s(H)(r + 1) \leq \dim_s(C_{2r+1} \boxtimes H) \leq n(r + 1) + r \cdot \dim_s(H). \]

Next we give a corrected version of Theorem 2.21, which is deduced from Theorem 2.19.

Theorem 2.21. For $1 \leq r \leq t$,
\[ 3rt + 2r + 2t + 1 - \left\lfloor \frac{r}{2} \right\rfloor \leq \dim_s(C_{2r+1} \boxtimes C_{2t+1}) \leq 3rt + 2r + 2t + 1. \]

Proof. By using Theorem 2.7 we have that $G_{SR} \boxtimes H_{SR} \subseteq (G \boxtimes H)_{SR}$. Thus, $\beta(G_{SR} \boxtimes H_{SR}) \geq \beta((G \boxtimes H)_{SR})$. Hence, from equality (1) and Theorem 2.20 we have
\[ \dim_s(C_{2r+1} \boxtimes C_{2t+1}) = (2r + 1) \cdot (2t + 1) - \beta((C_{2r+1} \boxtimes C_{2t+1})_{SR}) \]
\[ \geq (2r + 1) \cdot (2t + 1) - \beta((C_{2r+1})_{SR} \boxtimes (C_{2t+1})_{SR}) \]
\[ = (2r + 1) \cdot (2t + 1) - \beta(C_{2r+1} \boxtimes C_{2t+1}) \]
\[ = (2r + 1) \cdot (2t + 1) - r \cdot t - \left\lfloor \frac{r}{2} \right\rfloor \]
\[ = 3rt + 2r + 2t + 1 - \left\lfloor \frac{r}{2} \right\rfloor. \]

The upper bound is a direct consequence of Theorem 2.19.

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References