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HOMOMORPHISM-HOMOGENEOUS MONOUNARY ALGEBRAS

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ABSTRACT. In 2006, P. J. Cameron and J. Nešetřil introduced the following variant of homogeneity: we say that a structure is homomorphism-homogeneous if every homomorphism between finitely generated substructures of the structure extends to an endomorphism of the structure. In several recent papers homomorphism-homogeneous objects in some well-known classes of algebras have been investigated (e.g. lattices and semilattices), while finite homomorphism-homogeneous groups were described in 1979 under the name of finite quasi-injective groups. In this paper we characterize homomorphism-homogenous monounary algebras of arbitrary cardinalities.

1. Introduction

In their paper [3], P. Cameron and J. Nešetřil propose the study of homomorphism-homogeneous structures. This new property is closely related to, but is not a generalization of, the model-theoretic notion of ultrahomogeneity. Recall that a first-order structure $\mathcal{A}$ is called *ultrahomogeneous* if every isomorphism between finitely generated substructures of $\mathcal{A}$ extends to an automorphism of $\mathcal{A}$. As well, using the notion of isomorphism, several types of homogeneity were studied for various algebraic structures; for monounary algebras see [6–8]. Analogously, we say that a first-order structure $\mathcal{A}$ is *homomorphism-homogeneous* [3] if every homomorphism between finitely generated substructures of $\mathcal{A}$ extends to an endomorphism of $\mathcal{A}$.

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In several recent papers [2, 5, 10, 12] homomorphism-homogeneous relational structures such as graphs, tournaments and partially ordered sets have been investigated. As far as algebras are concerned, [4] contains the description of homomorphism-homogeneous lattices, together with some initial results concerning homomorphism-homogeneous semilattices. Interestingly, finite homomorphism-homogeneous groups were described in 1979 in [1] under the name of *finite quasi-injective groups*.

What makes the classification problem particularly interesting is a result presented in [12] where the authors show that deciding homomorphism-homogeneity for finite graphs with loops allowed is coNP-complete. Hence, there exist classes of finite structures where no feasible characterization of homomorphism-homogeneous objects is possible (unless P = coNP).

In this paper we characterize homomorphism-homogeneous monounary algebras (of arbitrary cardinalities) and the effective classification we obtain ensures that deciding homomorphism-homogeneity of finite monounary algebras is in P. On the other hand, [11] shows that deciding homomorphism-homogeneity for finite algebras with finitely many fundamental operations and with at least one at least binary fundamental operation is coNP-complete. Currently, we do not know the status of the decision problem in case of finite unary algebras with at least two unary operations. Our feeling is that the problem should be computationally hard.

2. Preliminaries

Let us recall some basic notions concerning monounary algebras (see [9]). For a monounary algebra $\mathcal{A} = (A, \alpha)$ and $a, b \in A$ let $a \rightsquigarrow b$ denote that $\alpha^k(a) = b$ for some $k \geqslant 0$. We define a binary relation $\sim$ on A as follows: $a \sim b$ if there exists a $c \in A$ such that $a \rightsquigarrow c$ and $b \rightsquigarrow c$. It is easy to show that $\sim$ is an equivalence relation and elements of $A/\sim$ are referred to as *connected components* of $\mathcal{A}$. An $a \in A$ is *cyclic* if there exists a $k \geqslant 1$ such that $\alpha^k(a) = a$. Otherwise, a is said to be *acyclic*. The set of all cyclic elements in a connected component $S \subseteq A$ is called *the cycle of* S. It may happen that a connected component does not have a cycle. For an acyclic element $a \in A$ let $\mathrm{ht}(a)$, the *height of* a, denote the least $k \geqslant 1$ such that $\alpha^k(a)$ is a cyclic element. If no such k exists, we set $\mathrm{ht}(a) = \infty$. For a connected component $S \subseteq A$ let $\mathrm{cn}(S)$, the *cycle number of* S, denote the length of the cycle in S. If S does not have a cycle, we set $\mathrm{cn}(S) = \infty$.

Following [3], we say that a monounary algebra $\mathcal{A} = (A, \alpha)$ is *homomorphism-homogeneous* if every homomorphism $f\colon \mathcal{B} \to \mathcal{A}$ from a finitely generated subalgebra $\mathcal{B}$ of $\mathcal{A}$ into $\mathcal{A}$ extends to an endomorphism of $\mathcal{A}$. On the other hand, an algebra $\mathcal{A} = (A, \alpha)$ is said to be *quasi-injective* if every homomorphism

$f\colon \mathcal{B} \to \mathcal{A}$ from an *arbitrary* subalgebra $\mathcal{B}$ of $\mathcal{A}$ into $\mathcal{A}$ extends to an endomorphism of $\mathcal{A}$. In case of monounary algebras these two notions coincide, as the following lemma demonstrates.

Lemma 1. *Let $\mathcal{A}$ be a monounary algebra. The following are equivalent:*

(1) *$\mathcal{A}$ is homomorphism-homogeneous;*

(2) *for every homomorphism $f\colon \mathcal{B} \to \mathcal{A}$ from a proper subalgebra $\mathcal{B}$ of $\mathcal{A}$ into $\mathcal{A}$ there is a subalgebra $\mathcal{C}$ of $\mathcal{A}$ and a homomorphism $f_1\colon \mathcal{C} \to \mathcal{A}$ such that $\mathcal{C}$ properly contains $\mathcal{B}$ and f_1 extends f;*

(3) *$\mathcal{A}$ is quasi-injective.*

Proof.

(2) $\Longrightarrow$ (3) follows easily by transfinite induction, while (3) $\Longrightarrow$ (1) is trivial.

Let us show (1) $\Longrightarrow$ (2). Let $\mathcal{A} = (A, \alpha)$ be a monounary algebra, let $\mathcal{B} = (B, \alpha|_B)$ be a proper subalgebra of $\mathcal{A}$, that is $B \subsetneqq A$, and let $f\colon \mathcal{A} \to \mathcal{B}$ be a homomorphism. Take any $a \in A \setminus B$ and let $\langle a \rangle_{\mathcal{A}} = \{\alpha^k(a) : \ k \geqslant 0\}$ be a subalgebra of $\mathcal{A}$ generated by a.

If $B \cap \langle a \rangle_{\mathcal{A}} = \emptyset$, take $C = B \cup \langle a \rangle_{\mathcal{A}}$ and let $\mathcal{C} = (C, \alpha|_C)$. Clearly, $f_1\colon C \to A$ defined by

$$f_1(x) = \begin{cases} f(x), & x \in B \\ x, & x \in \langle a \rangle_{\mathcal{A}} \end{cases}$$

is a homomorphism from $\mathcal{C}$ to $\mathcal{A}$ which extends f.

Assume, now, that $B \cap \langle a \rangle_{\mathcal{A}} \neq \emptyset$, and choose a $b \in B$ in such a way that $\langle b \rangle_{\mathcal{A}} = B \cap \langle a \rangle_{\mathcal{A}}$. Let $g = f|_{\langle b \rangle_{\mathcal{A}}}$. Since $\mathcal{A}$ is homomorphism-homogeneous and $\langle b \rangle_{\mathcal{A}}$ is finitely generated, there is an endomorphism $g^*\colon \mathcal{A} \to \mathcal{A}$ which extends g. Take $C = B \cup \langle a \rangle_{\mathcal{A}}$ and let $\mathcal{C} = (C, \alpha|_C)$. It is easy to show that $f_1\colon C \to A$ defined by

$$f_1(x) = \begin{cases} f(x), & x \in B \\ g^*(x), & x \in \langle a \rangle_{\mathcal{A}} \setminus B \end{cases}$$

is a homomorphism from $\mathcal{C}$ to $\mathcal{A}$ which extends f. $\square$

3. The characterization

In order to present a characterization of homomorphism-homogeneous monounary algebras, we need a few more technical notions. We say that an $a \in A$ is a *leaf* in $\mathcal{A} = (A, \alpha)$ if $b \rightsquigarrow a$ implies $b = a$, or, equivalently, $\alpha^{-1}(a) = \emptyset$. A *branch* in a monounary algebra $\mathcal{A}$ is a finite or infinite sequence $a_1, a_2, a_3, \dots$ such that

- $a_i = \alpha(a_{i+1})$ for all $i \geqslant 1$,
- a_i is acyclic for all $i \geqslant 1$, and
- if the branch is finite, that is if it has the form $a_1, a_2, \ldots, a_n$, then a_n is a leaf in $\mathcal{A}$.

We say that the branch $a_1, a_2, a_3, \ldots$ *starts at* a_1. A connected component $S \subseteq A$ is said to be *regular* if

- the set of cyclic elements of S is nonempty, and
- either every branch in S is infinite, or every branch in S is finite and $\mathrm{ht}(a) = \mathrm{ht}(b)$ for all leaves a, b in S.

Note that a connected component consisting of cyclic elements only is regular. If $S \subseteq A$ is a regular connected component of $\mathcal{A}$, then the *height* of S, denoted by $\mathrm{ht}(S)$, is defined as follows:

- $\mathrm{ht}(S) = 0$ if there are no acyclic elements in S;
- $\mathrm{ht}(S) = \infty$ if every branch starting at a cyclic element of S is infinite;
- otherwise, $\mathrm{ht}(S)$ denotes the common height of all the leaves in S.

Lemma 2. *Let $\mathcal{A}$ be a homomorphism-homogeneous monounary algebra with a connected component in which every element is acyclic. Then every branch in $\mathcal{A}$ is infinite.*

Proof. Let S be a connected component of $\mathcal{A}$ in which every element is acyclic. Let us first show that every branch in S is infinite. Assume that there is a finite branch $(a_1, \ldots, a_n)$ in S. Then a_n must be a leaf in $\mathcal{A}$. Let $a_0 = \alpha(a_1)$, $a_{-1} = \alpha(a_0)$, $a_{-2} = \alpha(a_{-1})$ etc. Since every element of S is acyclic, the mapping $f\colon \langle a_{n-1}\rangle_{\mathcal{A}} \to A$: $a_k \mapsto a_{k+1}$ is a homomorphism which, by the homogeneity of $\mathcal{A}$, extends to an endomorphism f^* of $\mathcal{A}$. Then $\alpha(a_n) = a_{n-1}$ implies $\alpha(f^*(a_n)) = f^*(a_{n-1}) = a_n$, which contradicts the fact that a_n is a leaf in $\mathcal{A}$.

Assume now that there is a finite branch $(a_1, \ldots, a_n)$ where $a_1, \ldots, a_n \in A \setminus S$. Again, a_n is a leaf in $\mathcal{A}$. Let $a_0 = \alpha(a_1)$, $a_{-1} = \alpha(a_0)$, $a_{-2} = \alpha(a_{-1})$ etc, let $(s_1, s_2, \ldots)$ be an infinite branch in S and let $s_0 = \alpha(s_1)$, $s_{-1} = \alpha(s_0)$, $s_{-2} = \alpha(s_{-1})$ etc. Since every element of S is acyclic, the mapping $f\colon \langle s_n\rangle_{\mathcal{A}} \to A$: $s_k \mapsto a_k$ is a homomorphism which, by the homogeneity of $\mathcal{A}$, extends to an endomorphism f^* of $\mathcal{A}$. Then $\alpha(s_{n+1}) = s_n$ implies $\alpha(f^*(s_{n+1})) = f^*(s_n) = a_n$, which contradicts the fact that a_n is a leaf in $\mathcal{A}$. □

Lemma 3. *Let $\mathcal{A} = (A, \alpha)$ be a monounary algebra such that every branch in $\mathcal{A}$ is infinite. Then $\mathcal{A}$ is homomorphism-homogeneous.*

Proof. By Lemma 1 it suffices to show that for every homomorphism $f\colon \mathcal{B} \to \mathcal{A}$ from a proper subalgebra $\mathcal{B}$ of $\mathcal{A}$ into $\mathcal{A}$ there is a subalgebra $\mathcal{C}$ of $\mathcal{A}$ and a homomorphism $f_1\colon \mathcal{C} \to \mathcal{A}$ such that $\mathcal{C}$ properly contains $\mathcal{B}$ and f_1 extends f.

Let $\mathcal{B}$ be a proper subalgebra of $\mathcal{A}$, let B be the base set of $\mathcal{B}$, and let $f\colon \mathcal{B} \to \mathcal{A}$ be a homomorphism. Take any $a \in A \setminus B$.

If $B \cap \langle a\rangle_{\mathcal{A}} = \emptyset$, take $C_1 = B \cup \langle a\rangle_{\mathcal{A}}$ and let $\mathcal{C}_1 = (C_1, \alpha|_{C_1})$. Clearly, $f_1\colon C_1 \to A$ defined by

$$f_1(x) = \begin{cases} f(x), & x \in B \\ x, & x \in \langle a\rangle_{\mathcal{A}} \end{cases}$$

is a homomorphism from $\mathcal{C}_1$ to $\mathcal{A}$ which extends f.

Assume, now, that $B \cap \langle a\rangle_{\mathcal{A}} \neq \emptyset$. Let $a_1 = a$, let $a_{i+1} = \alpha(a_i)$ for $i \geqslant 1$, and let k be the least integer j such that $a_j \in B$. Then $k \geqslant 2$ and $\{a_1, \dots, a_{k-1}\} \cap B = \emptyset$. If $f(a_k)$ is a cyclic element, choose cyclic elements $c_1, \dots, c_{k_1}$ in such a way that $\alpha(c_{k-1}) = f(a_k)$ while $\alpha(c_i) = c_{i+1}$ for $i \in \{1, \dots, k-2\}$, let $C_2 = B \cup \langle a\rangle_{\mathcal{A}}$ and let $\mathcal{C}_2 = (C_2, \alpha|_{C_2})$. Clearly, $f_2\colon C_2 \to A$ defined by

$$f_2(x) = \begin{cases} f(x), & x \in B \\ c_i, & x = a_i \ \text{ for } \ i \in \{1, \dots, k-1\} \end{cases}$$

is a homomorphism from $\mathcal{C}_2$ to $\mathcal{A}$ which extends f.

Finally, if $f(a_k)$ is not a cyclic element, recall that every branch in $\mathcal{A}$ is infinite and take any branch $(b_0, b_1, b_2, \dots)$ starting at $b_0 = f(a_k)$. Let $C_3 = B \cup \langle a\rangle_{\mathcal{A}}$ and let $\mathcal{C}_3 = (C_3, \alpha|_{C_3})$. Clearly, $f_3\colon C_3 \to A$ defined by

$$f_3(x) = \begin{cases} f(x), & x \in B \\ b_{k-i}, & x = a_i \ \text{ for } \ i \in \{1, \dots, k-1\} \end{cases}$$

is a homomorphism from $\mathcal{C}_3$ to $\mathcal{A}$ which extends f. □

Lemma 4. *Let $\mathcal{A}$ be a monounary algebra in which every connected component has a cyclic element. Then $\mathcal{A}$ is homomorphism-homogeneous if and only if*

- *every connected component in $\mathcal{A}$ is regular, and*
- *for any two connected components $S_1, S_2 \subseteq A$, if $\operatorname{cn}(S_1) | \operatorname{cn}(S_2)$ then $\operatorname{ht}(S_1) \geqslant \operatorname{ht}(S_2)$ or $\operatorname{ht}(S_1) = 0$.*

Proof.

($\Longrightarrow$) Let S be a connected component in $\mathcal{A}$ and assume that there is an infinite branch $(a_1, a_2, \dots)$ in S. Let us show that in this case every branch in S is infinite. Let $a_0 = \alpha(a_1)$, $a_{-1} = \alpha(a_0)$, $a_{-2} = \alpha(a_{-1})$ etc, and let $k \geqslant 0$ be the smallest nonnegative integer such that a_{-k} is a cyclic element. Assume that there is a finite branch $(b_1, \dots, b_n)$ in S. Then b_n must be a leaf in $\mathcal{A}$. Again, let $b_0 = \alpha(b_1)$, $b_{-1} = \alpha(b_0)$ etc, and let $m \geqslant 0$ be the smallest nonnegative integer such that b_{-m} is a cyclic element. The mapping f: $\begin{pmatrix} a_{-k} & a_{-k+1} & \dots & a_{n-k+m} \\ b_{-m} & b_{-m+1} & \dots & b_n \end{pmatrix}$ can be extended to all the cyclic elements of S in an obvious way, and thus turned into a homomorphism $f_1\colon \langle a_{n-k+m}\rangle_{\mathcal{A}} \to \langle b_n\rangle_{\mathcal{A}}$. By the homogeneity requirement, f_1 then extends to an endomorphism f_1^* of $\mathcal{A}$. Now, $\alpha(a_{n-k+m+1}) = a_{n-k+m}$ implies $\alpha(f_1^*(a_{n-k+m+1})) = f_1^*(a_{n-k+m}) = b_n$, which contradicts the fact that b_n is a leaf in $\mathcal{A}$.

If there are no infinite branches in S, we take two arbitrary leaves a and b and, using essentially the same idea, show that $\mathrm{ht}(a) > \mathrm{ht}(b)$ leads to a contradiction.

Let S_1 and S_2 be two connected components of $\mathcal{A}$ such that $\mathrm{cn}(S_1) | \mathrm{cn}(S_2)$, and assume that $0 < \mathrm{ht}(S_1) < \mathrm{ht}(S_2)$. There exists a leaf $a \in S_1$ such that $\mathrm{ht}(a) = \mathrm{ht}(S_1)$. Since $\mathrm{ht}(S_2) \geqslant \mathrm{ht}(S_1) + 1 = \mathrm{ht}(a) + 1$ there is a $b \in S_2$ such that $\mathrm{ht}(b) = \mathrm{ht}(a) + 1$. The condition $\mathrm{cn}(S_1) | \mathrm{cn}(S_2)$ yields that there exists a homomorphism $f\colon \langle \alpha(b)\rangle_{\mathcal{A}} \to \langle a\rangle_{\mathcal{A}}$ that takes $\alpha(b)$ to a. By the homogeneity requirement, f extends to an endomorphism f^* of $\mathcal{A}$. Now, $\alpha(f^*(b)) = f^*(\alpha(b)) = a$, which contradicts the fact that a is a leaf in $\mathcal{A}$.

($\Longleftarrow$) Again, we use Lemma 1. Let $\mathcal{B}$ be a proper subalgebra of $\mathcal{A}$, let B be the base set of $\mathcal{B}$, and let $f\colon \mathcal{B} \to \mathcal{A}$ be a homomorphism. Take any $a \in A \setminus B$. If $B \cap \langle a\rangle_{\mathcal{A}} = \emptyset$, take $C_1 = B \cup \langle a\rangle_{\mathcal{A}}$ and note that f_1 constructed as in the proof of Lemma 3 is a homomorphism from $\mathcal{C}_1$ to $\mathcal{A}$ which extends f.

Assume, next, that $B \cap \langle a\rangle_{\mathcal{A}} \neq \emptyset$. Let $a_1 = a$, let $a_{i+1} = \alpha(a_i)$ for $i \geqslant 1$, and let k be the least integer j such that $a_j \in B$. Then $k \geqslant 2$ and $\{a_1, \dots, a_{k-1}\} \cap B = \emptyset$. If $f(a_k)$ is a cyclic element, then $f_2\colon \mathcal{C}_2 \to \mathcal{A}$ constructed as in the proof of Lemma 3 is a homomorphism from $\mathcal{C}_2$ to $\mathcal{A}$ which extends f.

Finally, assume that $f(a_k)$ is not a cyclic element and take any branch $\beta = (b_0, b_1, b_2, \dots)$ starting at $b_0 = f(a_k)$. Let S_1 be the connected component of $\mathcal{A}$ that contains b_0 and S_2 the connected component of $\mathcal{A}$ that contains a_k. Note that $\mathrm{ht}(S_1) \neq 0$ because $f(a_k) \in S_1$ is not a cyclic element. Since f maps homomorphically $\langle a_k\rangle_{\mathcal{A}} \subseteq S_2$ into $\langle b_0\rangle_{\mathcal{A}} \subseteq S_1$, it follows that $\mathrm{cn}(S_1) | \mathrm{cn}(S_2)$, hence $\mathrm{ht}(S_2) \leqslant \mathrm{ht}(S_1)$ according to the assumption. Consequently, there are at least k elements in the sequence β. This ensures that $f_3\colon \mathcal{C}_3 \to \mathcal{A}$ constructed as in the proof of Lemma 3 is a homomorphism from $\mathcal{C}_3$ to $\mathcal{A}$ which extends f. □

Finally, we have the following characterization of homomorphsim-homogeneous monounary algebras:

Theorem 5. *Let $\mathcal{A}$ be a monounary algebra. Then $\mathcal{A}$ is homomorphism-homogeneous if and only if $\mathcal{A}$ belongs to one of the following classes:*

(1) *every branch in $\mathcal{A}$ is infinite;*

(2) *every connected component in $\mathcal{A}$ is regular, and for any two connected components $S_1, S_2 \subseteq A$, if $\mathrm{cn}(S_1) | \mathrm{cn}(S_2)$ then $\mathrm{ht}(S_1) \geqslant \mathrm{ht}(S_2)$ or $\mathrm{ht}(S_1) = 0$.*

P r o o f.

($\Longleftarrow$) Follows from Lemmas 3 and 4.

($\Longrightarrow$) Let $\mathcal{A}$ be a homomorphism-homogeneous monounary algebra. If $\mathcal{A}$ has a connected component in which every element is acyclic, then every branch in $\mathcal{A}$ is infinite by Lemma 2 and we have case (1). If, however, every connected component of $\mathcal{A}$ has a cyclic element, then by Lemma 4 we have case (2). □

4. Concluding remarks

Let $\Delta = (r_1, r_2, \dots, r_n)$ be a finite algebraic type, where $r_1, r_2, \dots, r_n \geqslant 0$ are nonnegative integers. We say that Δ is *at least binary* if $r_i \geqslant 2$ for some $i \in \{1, \dots, n\}$. Let HH-FINALG(Δ) denote the following decision problem: given a finite algebra $\mathcal{A}$ of type Δ, decide whether $\mathcal{A}$ is homomorphism-homogeneous.

From the explicit description of homomorphism-homogeneous monounary algebras (Theorem 5) it easily follows that HH-FINALG(1) is in P. On the other hand, it was shown in [11] that HH-FINALG(Δ) is coNP-complete whenever Δ is at least binary. Currently, we do not know the status of HH-FINALG($1, \dots, 1, 0, \dots, 0$), where at least two unary operations appear in the signature. Our feeling is that the problem should be computationally hard.

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