A Design of Finite Memory Residual Generation Filter for Sensor Fault Detection

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In the current paper, a residual generation filter with finite memory structure is proposed for sensor fault detection. The proposed finite memory residual generation filter provides the residual by real-time filtering of fault vector using only the most recent finite measurements and inputs on the window. It is shown that the residual given by the proposed residual generation filter provides the exact fault for noise-free systems. The proposed residual generation filter is specified to the digital filter structure for the amenability to hardware implementation. Finally, to illustrate the capability of the proposed residual generation filter, extensive simulations are performed for the discretized DC motor system with two types of sensor faults, incipient soft bias-type fault and abrupt bias-type fault. In particular, according to diverse noise levels and windows lengths, meaningful simulation results are given for the abrupt bias-type fault.

Keywords: Fault diagnosis, Fault detection, Residual generation filter, Finite memory structure, Kalman filter.

1. INTRODUCTION

As most dynamic process plants become more complex, there has been a growing demand for fault diagnosis. Fault diagnosis is the prompt indication of incipient and abrupt faults, it can help avoid major plant breakdowns and permit appropriate actions that maintain the operation. Thus, fault diagnosis is an important and challenging problem for disciplines such as chemical engineering, nuclear engineering, aerospace engineering, and automotive systems [1]-[3].

Generally, model-based fault diagnosis algorithms consist of fault detection and fault isolation. Fault detection is used to detect malfunctions in real time, as soon and as certainly as possible. Fault isolation is used to determine the root cause by isolating the system component(s) whose operation mode is not nominal. The essential step for fault diagnosis is the generation of a set of variables known as residuals using one or more residual generation filters. That is, residuals are signals, often time-varying, that are used as fault detectors. The residuals used for fault detection should ideally be zero or zero mean) under no-fault conditions. In practical applications, the residuals are corrupted by the presence of noise, unknown disturbances, and system model uncertainties. Hence, the residual should be insensitive to noise, disturbances, and model uncertainties while maximally sensitive to faults to be useful in practical applications.

Kalman filters have been adopted as residual generation filters in stochastic cases where noise has to be considered [4]-[9]. Their compact representation and efficient manner mean that the Kalman filter has been successfully applied in various areas including fault detection. However, the Kalman filter has an infinite memory structure that utilizes all observations, as accomplished by equal weighting, and has a recursive formulation. Thus, the Kalman filter tends to accumulate filtering errors as time goes by and can even show divergence phenomenon for temporary modeling uncertainties and round-off errors [10]-[13]. This inherent property of the Kalman filter has been shown in sensor application areas [14], [15]. In addition, long past measurements are not useful for fault detection with unknown occurrence times. Moreover, it is also known that the increase of the number of measurements for detection decisions will increase the detection latency in a system that detects a signal with an unknown occurrence time.

Therefore, an alternative residual generation filter with finite memory structure is proposed in the current paper for sensor fault detection. The proposed finite memory residual generation filter provides the residual via the real-time filtering of fault vector using only the most recent finite measurements and inputs in the window. It is shown that the residual given by the proposed residual generation filter provides the exact fault required for noise-free systems. The proposed residual generation filter is specified to the digital filter structure for its amenability to hardware implementation.

Finally, extensive simulations are performed for a discretized DC motor system to illustrate the capability of the proposed residual generation filter. Two types of sensor
fault are considered: incipient soft bias-type fault and abrupt bias-type fault. According to the diverse noise levels and windows lengths, simulation results can be shown to be meaningful for abrupt bias-type fault.

2. BASIC CONCEPT OF RESIDUAL GENERATION FILTER IN FAULT DIAGNOSIS

In general, the fault model can be represented by the following discrete-time state space model with unknown sensor faults as well as noises:

\[
\begin{align*}
    x(i+1) &= \Phi x(i) + D u(i) + \Delta_1 f(i) + E \epsilon(i), \\
    y(i) &= C x(i) + \Delta_2 f(i) + v(i),
\end{align*}
\]

where \( x(i) \in \mathbb{R}^p \) the state vector, \( u(i) \in \mathbb{R}^q \) and \( y(i) \in \mathbb{R}^r \) are the input vector and the measurement vector. The covariances of the system noise \( \epsilon(i) \in \mathbb{R}^p \) and the measurement noise \( v(i) \in \mathbb{R}^r \) are \( Q_\epsilon \) and \( R \), respectively. The fault vector \( f(i) \in \mathbb{R}^f \) in the system under consideration are to be represented by random-walk processes as

\[
f(i+1) = f(i) + \delta(i),
\]

where \( f(i) = f_1(i) f_2(i) \Lambda f_q(i) \) and \( \delta(i) \in \mathbb{R}^f \) is a zero-mean white Gaussian random process with covariance \( Q_\delta \). It is noted that the random-walk process provides a general and useful tool for the analysis of unknown time-varying parameters and has been widely used in the detection and estimation area.

The fault model (1) can be rewritten as an augmented state model as

\[
\begin{bmatrix}
    x(i+1) \\
    f(i+1)
\end{bmatrix} =
\begin{bmatrix}
    \Phi & \Delta_1 \\
    0 & I
\end{bmatrix}
\begin{bmatrix}
    x(i) \\
    f(i)
\end{bmatrix} +
\begin{bmatrix}
    D \\
    0
\end{bmatrix}
\begin{bmatrix}
    \epsilon(i) \\
    \delta(i)
\end{bmatrix},
\]

\[
y(i) = H \begin{bmatrix}
    x(i) \\
    f(i)
\end{bmatrix} + v(i),
\]

where

\[
\begin{align*}
A &= \begin{bmatrix}
\Phi & \Delta_1 \\
0 & I
\end{bmatrix},
B = \begin{bmatrix}
D \\
0
\end{bmatrix},
G = \begin{bmatrix}
E & 0 \\
0 & I
\end{bmatrix},
H = \begin{bmatrix}
C & \Delta_2
\end{bmatrix},
\end{align*}
\]

and the noise system term \( \begin{bmatrix}
\epsilon^T(i) & \delta^T(i)
\end{bmatrix}^T \approx w(i) \) is a zero-mean white Gaussian random process with covariance \( Q = \text{diag}(Q_\epsilon, Q_\delta) \).

The essential step in fault diagnosis is to generate a set of variables known as residuals by using one or more residual generation filters. A residual is an often time-varying signal that is used as a fault detector. In general, the residual used for fault detection is defined by the signal generated based on the measurement vector \( y(i) \) and input vector \( u(i) \). This residual should ideally be zero (or have zero mean) under no-fault conditions. In practical applications, the residuals are corrupted by the presence of noise, unknown disturbances, and uncertainties in the system model. Hence, the residual should be insensitive to noise, disturbances, and model uncertainties while maximally sensitive to faults to be useful in practical applications.

The main contribution of the current paper is to design and analyze an alternative residual generation filter. That is, the current paper focuses on fault detection that declares quickly which sensor is faulty by estimating size and type of each sensor faults. Although fault isolation is not the current paper's topic, the residual generation filter designed in the following section can provide the means for the isolation of faults using at least three different approaches can be distinguished: fixed direction residuals, structured residuals, and structured hypothesis tests.

3. ALTERNATIVE RESIDUAL GENERATION FILTER FOR FAULT DETECTION

In the current paper, an alternative residual generation filter is designed with finite memory structure. The proposed finite memory residual generation filter provides the residual \( r(i) \) by real-time filtering of fault vector \( f(i) \) using only the most recent finite measurements \( Y(i) \) and inputs \( U(i) \) on the window \([i-M,i]\) as follows

\[
r(i) \equiv \hat{r}(i) \equiv \mathcal{R}[Y(i) - \Omega U(i)],
\]

where \( \hat{r}(i) \) is the filtered estimate of \( f(i) \) and \( \mathcal{R} \) is the filter gain matrix. The term \( Y(i) - \Omega U(i) \) in (3) with the most recent finite measurements \( Y(i) \) and inputs \( U(i) \) can be represented by the following regression form on the window \([i-M,i]\):

\[
Y(i) - \Omega U(i) = \Gamma \begin{bmatrix}
    x(i) \\
    f(i)
\end{bmatrix} + \Lambda W(i) + V(i),
\]

where

\[
Y(i) \equiv \begin{bmatrix}
    y^T(i-M) \\
    y^T(i-M+1) \Lambda y^T(i-1)
\end{bmatrix},
\]

and \( U(i) \), \( W(i) \), \( V(i) \) have the same form as (5) for \( u(i) \), \( w(i) \), \( v(i) \), respectively, and matrices \( \Omega \), \( \Gamma \), \( \Lambda \) are as follows:

\[
\Omega = \begin{bmatrix}
H A^{-1} B & H A^{-2} B \Lambda H A^{M+1} B & H A^M B \\
0 & H A^{-1} B & H A^{M+2} B \\
M & M \Lambda M & M
\end{bmatrix},
\]

\[
\Lambda = \begin{bmatrix}
0 & 0 & \Lambda & 0 & H A^{-1} B
\end{bmatrix}.
\]
Each residual \( r_s(i) \) in the residual \( r(i) \) (8) can be obtained by
\[
r_s(i) = \mathcal{H}_s [Y(i) - \Omega U(i)],
\]
where \( s = 1, 2, \ldots, q \) and \( \mathcal{H}_s \) is the \( s \)th row of the \( \mathcal{H} \) (7).

Several inherent properties of the proposed finite memory residual generation filter are described.
As shown in the following Theorem, the residual \( r(i) \) given by the proposed residual generation filter on the window \([i-M,i]\) provides the exact fault \( f(i) \) when there are no noises.

**Theorem 3.1.** When \( M \geq n \), the residual \( r(i) \) given by the proposed finite memory residual generation filter on the window \([i-M,i]\) provides the exact fault \( f(i) \) for noise-free systems.

**Proof:** When there are no noises on the window \([i-M,i]\) for the discrete-time state space model (2) as follows:
\[
\begin{bmatrix}
\dot{x}(i+1) \\
\end{bmatrix} =
\begin{bmatrix}
A & B \\
\end{bmatrix}
\begin{bmatrix}
x(i) \\
f(i) \\
\end{bmatrix},
\]
\[
y(i) = \begin{bmatrix}
H_1 \\
H_2 \\
\end{bmatrix} \begin{bmatrix}
x(i) \\
f(i) \\
\end{bmatrix}
\]
the most recent finite measurements and inputs \( Y(i) - \Omega U(i) \) are determined from (4) as follows:
\[
Y(i) - \Omega U(i) = \begin{bmatrix}
x(i) \\
f(i) \\
\end{bmatrix}.
\]

Therefore, the following is true:
\[
\begin{bmatrix}
\hat{x}(i) \\
\hat{f}(i) \\
\end{bmatrix} = \mathcal{H} [Y(i) - \Omega U(i)]
\]
system (10) has an additive system and measurement noises, \( w(i) \) and \( v(i) \) as the fault model (1). This property indicates a finite convergence time and the fast tracking ability of the residual given by the proposed residual generation filter. Thus, it can be expected that the proposed residual generation filter might be appropriate for fast detection.

The filter gain matrix \( \mathbf{H} \) in (9) only requires computation on the interval \([i-M,i]\) once and is time-invariant for all windows. This means that the proposed finite memory residual generation filter is time-invariant. However, discrete time-varying state space models can be often used for many practical and real-time applications. Thus, a finite memory residual generation filter for discrete time-varying systems is necessary. In this case, the computation for the filter gain matrix \( \mathbf{H} \) might become burdensome. Hence, the iterative strategy of [16], [17] can be applied on the window \([i-M,i]\) to overcome the computational burden.

The window length \( M \) can be a useful design parameter for the proposed finite memory residual generation filter. Thus, the important issue here is how to choose an appropriate window length \( M \) that makes the residual performance as good as possible. The noise suppression of the proposed residual generation filter might be closely related to the window length \( M \), and it can have greater noise suppression as the window length \( M \) increases, which improves the residual performance. However, the convergence time of a filtered residual becomes longer as the window length increases. This illustrates the proposed finite memory residual generation filter's compromise between noise suppression and tracking ability. Since \( M \) is an integer, fine adjustment of the properties with \( M \) is difficult. Moreover, it is difficult to determine the window length systematically. In applications, one method of determining the window length is to take the appropriate value that can provide sufficient noise suppression. Therefore, it can be stated from the above discussions that the window length \( M \) can be considered a useful parameter to make the residual performance of the proposed finite memory residual generation filter as good as possible.

4. SPECIFIED RESIDUAL FOR HARDWARE IMPLEMENTATION

In practice, it should be required that a fault detection system can be implemented with a discrete-time analog or digital hardware. In this case, the fault detection system should be specified to an algorithm or structure that can be realized in the desired technology. Thus, the proposed finite memory residual generation filter is specified to the well-known digital filter structure in [18] for its amenability to hardware implementation.

The filter gain matrix \( \mathbf{H}_s \) for the \( s \)th residual \( r_s(i) \) in (9) can be defined by

\[
\mathbf{H}_s = \begin{bmatrix} h_s(M-1) & h_s(M-2) & \cdots & h_s(0) \end{bmatrix}
\]

Then, the proposed residual generation filter for the \( s \)th residual \( r_s(i) \) is given by

\[
r_s(i) = \sum_{j=0}^{M-1} h_s(j)y(i-j) - \sum_{j=0}^{M-1} \mathbf{H}_s \mathbf{z}_s^j u(i-j),
\]

where \( \mathbf{H}_s \mathbf{z}_s^j \) is the \( (j+1) \)th element of \( \mathbf{H}_s \)

Applying the \( z \)-transformation to the residual (11) yields the following digital filter structure:

\[
r_s(z) = \sum_{j=0}^{M-1} h_s(j)y(z) - \sum_{j=0}^{M-1} \mathbf{H}_s z^{-j} u(z),
\]

where \( h_s(j) \) and \( -\mathbf{H}_s z^{-j} \) become filter coefficients. It is noted that the digital filter structure (12) is a well-known moving average process whose functional relation between inputs \( y(z) \), \( u(z) \) and output \( r_s(z) \) is nonrecursive. The block diagram of the proposed finite memory residual generation filter (8) for a hardware implementation can be represented as Fig.1.

![Block diagram representation of the finite memory residual generation filter.](image)

5. EXTENSIVE COMPUTER SIMULATIONS

Extensive computer simulations using MATLAB software package are performed for a DC motor system with sensor faults to illustrate the fault detection capability of the proposed finite memory residual generation filter.

A. Discretized DC motor system

First, it is necessary to discretize the continuous-time state model of the DC motor system in order to facilitate the design in discrete-time state space model. The continuous-time state space model of the DC motor system can be represented by
The continuous-time state space model (13) can be discretized as follows:

\[
\begin{align*}
x(i+1) &= \Phi x(i) + Du(i), \\
y(i) &= Cx(i),
\end{align*}
\]

where

\[
\Phi = e^{\Psi T}, \quad D = \left[ \int_0^T e^{\Psi \tau} d\tau \right] \Xi = \left[ e^{\Psi T} - I \right] T^{-1} \Xi
\]

The performance of the proposed finite memory residual generation filter is evaluated and compared with a Kalman filtering-based residual generation filter with infinite memory structure through computer simulation in [4]-[9]. In these numerical examples, the window length is taken as \( M = 10 \) System noises are generated with covariances \( Q_w = 0.01^2 \) and \( Q_v = diag(0.02^2, 0.02^2) \). Dealing with different noise levels requires that three cases of measurement noises are generated: high noise level with \( R = diag(0.4^2, 1^2) \), medium noise level with \( R = diag(0.2^2, 1^2) \), and low noise level with \( R = diag(0.1^2, 1^2) \).

A fault is modeled for two scenarios as shown in Fig.2. The fault is modeled as an incipient soft bias-type fault in the first scenario, while the fault is modeled as an abrupt bias-type fault in the second; the second scenario might be more feasible than the first. The proposed finite memory residual generation filter is compared with the Kalman filtering-based approach with infinite memory structure for both scenarios.

Fig.3, and Fig.4, show plots for the residuals of both residual generation filters. As shown in Fig.3, the proposed finite memory residual generation filter can be compared to the Kalman filtering-based approach for the first scenario with incipient soft bias-type fault. In contrast, as shown in Fig.4, meaningful results are given for the second scenario with abrupt bias-type fault. For all cases of noise levels, the tracking of the proposed finite memory residual generation filter is much faster than that of the Kalman filtering-based approach when the abrupt bias-type fault occurs and disappears. One possible explanation for this is the finite convergence time and fast tracking ability of the proposed finite memory residual generation filter. However, in the case of high noise level, the Kalman filtering-based approach shows better noise suppression capabilities than the proposed finite memory residual generation filter. As mentioned in Section 3, the noise suppression of the proposed residual generation filter might be closely related to the window length \( M \). The proposed residual generation filter...
filter can gain better noise suppression as the window length grows, which means that the Kalman filtering-based residual generation filter with infinite memory structure can have good noise suppression ability.

Although the proposed finite memory residual generation filter can have greater noise suppression as the window length $M$ increases, the tracking speed worsens in proportion with the window length $M$. This can be observed from simulations results according to diverse window lengths, as shown in Fig.5. These simulation results illustrate the proposed finite memory residual generation filter's compromise between noise suppression and the tracking speed of residuals.

6. CONCLUDING REMARKS AND FUTURE WORKS

This paper has proposed a residual generation filter with finite memory structure for sensor fault detection. The proposed finite memory residual generation filter provides the residuals via real-time filtering of fault vectors using only the most recent finite measurements and inputs on the window. It has been shown that the residual given by the proposed residual generation filter provides the exact fault for noise-free systems. The proposed residual generation filter has been specified to the digital filter structure for its amenability to hardware implementation. The capability of the proposed residual generation filter has been verified through numerical examples for the discretized DC motor system with sensor faults. In particular, simulation results have been shown to be meaningful for abrupt bias-type fault according to diverse noise levels and window lengths.

Although a guideline for the choice of window length $M$ has been described, this could still be somewhat non-systematic. Therefore, a more systematic approach of determining window length should be researched in future work. In addition, noise covariance matrices $Q$ and $R$ might be useful design parameters under the assumption that information about them is unknown. Hence, another future work might examine how to tune noise covariance matrices to improve residual performance such as via noise suppression and tracking abilities.

Fig.3. Residuals for the first scenario.

![Fig.3](image)

a) High noise level: $R = \text{diag}(0.4^2, 1^2)$

![Fig.3](image)

b) Medium noise level: $R = \text{diag}(0.2^2, 1^2)$

![Fig.3](image)

c) Low noise level: $R = \text{diag}(0.1^2, 1^2)$

Fig.4. Residuals according to diverse noise levels for the second scenario.
Results according to diverse window lengths

-0.2
0
0.2
0.4
0.6

100 200 300 400 500

M=5
M=10
M=20

-0.2
0
0.2
0.4
0.6

60 80 100 120 140 160 180 200 220 240

Time

M=5
M=10
M=20

Fig.5. Residuals according to diverse window length for the second scenario.

REFERENCES


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