Energy and momentum losses in the process of neutrino scattering on plasma electrons in the presence of a magnetic field.

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Abstract: The neutrino-electron scattering in a dense degenerate magnetized plasma under the conditions \( \mu^2 > 2eB \gg \mu E \) is investigated. The volume density of the neutrino energy and momentum losses due to this process are calculated. The results we have obtained demonstrate that plasma in the presence of an external magnetic field is more transparent for neutrino than for non-magnetized plasma. It is shown that neutrino scattering under conditions considered does not lead to the neutrino force acting on plasma.

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1 Introduction.

It is known that neutrino physics plays a unique role in astrophysics and cosmology. In particular, these light weakly interacting particles are special for astrophysical phenomena like supernova explosions when a large number of neutrinos are produced in a collapsing stellar core (Raffelt 1996). The compact core with the typical radius \( R \sim 10 \, \text{km} \), the supranuclear density \( \rho \sim 10^{14} \, \text{g/cm}^3 \) and the high temperature \( T \sim 30 \, \text{MeV} \), is opaque for neutrinos.

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While a rather rarified remnant envelope with the typical density $\rho \sim 10^{10} - 10^{12}$ g/cm$^3$ and temperature of the order of few MeV, becomes partially transparent for the neutrino flux.

It should be noted that in investigations of neutrino processes in a dense medium the magnetic field should also be taken into account. We stress that a magnetic field can play the role of an additional component of an active medium, and substantially influence particle properties. This influence becomes especially important in the case when the magnetic field strength reaches the critical, Schwinger value $B_e = m_e^2/e = 4.41 \times 10^{14}$ G.\footnote{We use natural units in which $c = \hbar = 1$, $e > 0$ is the elementary charge.} According to modern astrophysical models, very strong magnetic fields up to $10^{17}$ G could be generated, for example, in a rapidly rotating supernova remnant (Duncan and Thompson 1992; Bisnovatyi-Kogan 1993; Mathews et al. 1997).

In previous studies of neutrino interactions with a dense stellar medium, the main attention was given to the neutrino-nucleon processes. This is due to the fact that the Urca-processes and the neutrino-nucleon scattering defined the major contribution to the energy balance of the collapsing core, and were considered as a main source of neutrino opacity. The neutrino-electron processes were investigated less. However, as it was pointed out in studies by Mezzacappa and Bruenn (1993), taking account of the neutrino-electron scattering in a detailed analysis of the supernova dynamics is physically well justified. In particular, the neutrino-electron processes can contribute significantly to the asymmetry and provide a competition with the neutrino-nucleon processes. For example, in the paper by Kuznetsov and Mikheev (2000), the total set of neutrino-electron processes ($\nu e^\pm \leftrightarrow \nu e^\pm$, $\nu e^-e^+ \leftrightarrow \nu$, $e^\pm \leftrightarrow \nu \bar{\nu} e^\pm$) was investigated in a strong magnetic field limit, when electrons and positrons occupied the lowest Landau level. It was shown that the neutrino force on plasma along the magnetic field turns out to be of the same order and, more importantly, of the same sign as the one caused by the $\beta$-processes (Gvozdev and Ognev 1999).

Thus, the investigations of the neutrino-electron processes under extreme conditions of high density and/or temperature of matter and also of strong magnetic field are the subject of great interest.

In this study we investigate the neutrino-electron processes in a dense magnetized plasma. In contrast to (Kuznetsov and Mikheev 2000) we consider the physical situation when the magnetic field is not so strong, whereas the density of plasma is large. Thus $\mu$, the chemical potential of electrons, is the dominant factor:

$$\mu^2 > 2eB \gg T^2, \quad E^2 \gg m_e^2,$$

(1)

where $T$ is the plasma temperature, $E$ is the typical neutrino energy. Under the conditions (1) plasma electrons occupy the excited Landau levels. At the same time it is assumed that the magnetic field strength being relatively weak, (1), is strong enough, so that the following condition is satisfied:

$$eB \gg \mu E.$$  

(2)
In the present astrophysical context, conditions (1), (2) could be realized, for example, in a supernova envelope, where the electron chemical potential is assumed to be \( \mu \sim 15 \text{ MeV} \), plasma temperature \( T \sim 3 \text{ MeV} \). The magnetic field could be as high as \( 10^{15} - 10^{16} \text{ G} \). Under the conditions considered, the approximation of ultrarelativistic plasma is a good one, so we shall neglect the electron mass wherever this causes no complications.

As it was shown in the paper by Mikheev and Narynskaya (2000), under the conditions (1) and (2) that the total set of neutrino-electron processes reduces to the process of neutrino scattering on plasma electrons. Moreover, both initial and final electrons occupy the same Landau level.

The neutrino-electron scattering in dense magnetized plasma was investigated by Bezchastnov and Haensel (1996). Numerical calculations of the differential cross-section of this process in the limit of a weak magnetic field \( (eB < \mu E) \) were performed. The purpose of our work is to calculate analytically not only the probability of this process, but also the volume density of the neutrino energy and momentum losses under the conditions (1) and (2).

## 2 Neutrino-electron scattering probability.

We start from the effective local Lagrangian of the neutrino-electron interaction in the framework of the Standard Model:

\[
L_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[ \bar{\nu}_{\alpha} (c_v - c_a \gamma_5) e \right] j^\alpha,
\]

where \( j^\alpha = \left[ \bar{\nu}_{\alpha} (1 - \gamma_5) \nu \right] \) is the current of massless left neutrinos, \( c_v = \pm 1/2 + 2 \sin^2 \theta_W \), \( c_a = \pm 1/2 \). Here upper signs correspond to the electron neutrino \( (\nu = \nu_e) \) when both Z and W boson exchange takes part in a process. The lower signs correspond to \( \mu \) and \( \tau \) neutrino \( (\nu = \nu_\mu, \nu_\tau) \), when only the Z boson exchange is included in the Lagrangian (3).

In order to impart a physical meaning to the probability of the neutrino-electron scattering per unit time, it is necessary to integrate not only over the final but also over the initial electron states:

\[
W_{(\nu_e \rightarrow \nu_e)} = \sum_{n=0}^{n_{\text{max}}} \frac{1}{T} \int \sum_{s,s'} |S|^2 \, dn_{e} - dn_{e}' \, \frac{d^3 k}{(2\pi)^3} \, V (1 - f(E')).
\]

Here \( n_{\text{max}} \) corresponds to the maximum possible Landau level number, which is defined as the integer part of the ratio \( \mu^2/2eB \geq 1 \), \( T \) is the total interaction time, \( |S|^2 \) is the square of S-matrix element the process considered, \( V \) is the normalization volume, \( f(E') \) is a distribution function of final neutrinos, \( f(E') = \frac{e^{(E' - \mu_\nu)/T_\nu} + 1}{e^{(E' - \mu_\nu)/T_\nu}} \), \( E' \) is the final neutrino energy, \( \mu_\nu \) and \( T_\nu \) are the effective chemical potential and the spectral temperature of the neutrino gas respectively. In a general case the neutrino spectral temperature \( T_\nu \) can differ from the plasma temperature \( T \) (we do not assume equilibrium between neutrino gas and plasma). The phase-space elements of the initial and final
plasma electrons in the presence of a magnetic field are defined as follows: 
\[dn_e = \frac{dp_y dp_z}{(2\pi)^2} L_y L_z f(\varepsilon_n),\]
\[dn'_e = \frac{dp'_y dp'_z}{(2\pi)^2} L_y L_z (1 - f'(\varepsilon'_n)),\]
where \(p_z\) is the electron momentum along the magnetic field, \(p_y\) is the generalized momentum which defines the position of the center of a Gaussian wave packet along the \(x\) axis, \(x_0 = -p_y/eB\), while \(\varepsilon_n \approx \sqrt{p_z^2 + 2eBn}\) is the energy of an ultrarelativistic plasma electron occupying the \(n\)-th Landau level, \(f(\varepsilon_n)\) is a distribution function of electrons, \(f(\varepsilon_n) = [e^{(\varepsilon_n - \mu)/T} + 1]^{-1}\).

The details of integration over the phase space of particles had been published in our previous paper (Mikheev and Narynskaya 2000). The result of the calculation of the probability (4) can be presented in a relatively simple form:
\[
W(\nu_e \rightarrow \nu_e) = \frac{G_F^2 (c_\nu^2 + c_0^2) eB T^2 E}{4 \pi^3} \sum_{n=0}^{n_{\text{max}}} \frac{1}{z^2} \times \left\{(1 + z^2)(1 + u^2) - 4uz \right\} \int_{-a}^{b} \Phi(\xi) d\xi \\
+ \frac{1}{zr\tau}(z^2 - 1)(z - u) \int_{-a}^{b} \xi \Phi(\xi) d\xi \right\} + (u \rightarrow -u),
\]
where \(z = \sqrt{1 - 2eBn/\mu^2}\), \(\Phi(\xi) = \xi[(e^\xi - 1)(e^{\eta_\nu - r - \xi/\tau} + 1)]^{-1}\), \(a = r\tau z(1 + u)/(1 + z)\) and \(b = r\tau z(1 - u)/(1 - z)\), \(r = E/T_\nu\), \(\tau = T_\nu/T\), \(\eta_\nu = \mu_\nu/T_\nu\), \(u = \cos\theta\), \(\theta\) is the angle between the initial neutrino momentum \(\vec{k}\) and the magnetic field direction. The variable \(\xi\) defines the spectrum of the probability (5) on the final neutrino energy, \(\xi = (E' - E)/T\).

In the limit of a very dense plasma (\(\mu^2 \gg eB\)), when a great number of Landau levels are occupied by plasma electrons, one can transform the summation over \(n\) to an integration over \(z\):
\[
\sum_{n=0}^{[\mu^2/2eB]} F(z) \simeq \frac{\mu^2}{eB} \int_{0}^{1} F(z) z \, dz.
\]

In this case the contribution from the lowest Landau levels turns out to be negligibly small, so the main contribution to the probability arises from the highest Landau levels. In this limit the probability (5) can be rewritten in the following form:
\[
W(\nu_e \rightarrow \nu_e) = \frac{G_F^2 (c_\nu^2 + c_0^2) \mu^2 T^2 E}{4 \pi^3} \int_{0}^{1} \frac{dz}{z} \\
\times \left\{(1 + z^2)(1 + u^2) - 4uz \right\} \int_{-a}^{b} \Phi(\xi) d\xi 
\]
\[\text{we use the gauge } A^\mu = (0, 0, Bx, 0), \text{ the magnetic field is directed along the } z \text{ axis.}\]
\[ + \frac{1}{zr^2} (z^2 - 1)(z - u) \int_{-a}^{b} \xi \Phi(\xi) d\xi \} + (u \to -u). \]

As one can see, the probability (6) does not depend on the value of the magnetic field strength, but is not isotropic. The dependence on the angle \( \theta \) manifests this anisotropy of the neutrino-electron process in the presence of a magnetic field. In the limit of a rare neutrino gas when \( f(E') \ll 1 \), the result has a more simple form:

\[
W_{\nu_e \rightarrow \nu_e} \simeq \frac{G_F^2 (c_v^2 + c_a^2) \mu^2 E^3}{12\pi^3} I(u),
\]

\[
I(u) = \int_{0}^{1} \frac{zdz}{(1+z)^2} (u^4 (3z^2 + 2z + 1) - 12u^2 z + z^2 + 2z + 3).
\]

For purposes of comparison, we present here the probability of the neutrino-electron scattering in the absence of field in the same limit of the rare neutrino gas:

\[
W_{\text{vac}} = \frac{G_F^2 (c_v^2 + c_a^2) \mu^2 E^3}{15\pi^3}.
\]

![Fig. 1](image_url) The relative probability of the neutrino-electron scattering in a magnetized plasma as a function of the angle between the initial neutrino momentum and the magnetic field direction. \( W_{\text{vac}} \) is the probability in a non-magnetized plasma.

The numerical estimate of the ratio of the probabilities (7) and (8) is presented in Fig.1. It is seen that the probability in a magnetized plasma exceeds the vacuum probability in the vicinity of the point \( \theta = \pi/2 \) only.

### 3 Integral neutrino action on a magnetized plasma.

In this section we will calculate the volume density of neutrino energy and momentum losses per unit time in a medium, which could be defined as follows:

\[
(\hat{\varepsilon}, \hat{\mathcal{F}}) = \frac{1}{(2\pi)^3} \int \frac{(q_0, \vec{q}) d^3k}{e^{(E - \mu_\nu)/T_\nu} + 1} dW;
\]
where $q_\alpha$ is the difference between the momenta of the initial and final neutrinos, $q_\alpha = k_\alpha - k'_\alpha$. The zeroth component, $\hat{q}$, determines the neutrino energy loss in unit volume per unit time. In general, a neutrino propagating through plasma can both lose and capture energy. So, we will mean the ”loss” of energy in the algebraic sense.

The vector $\vec{F}$ in Eq.(9) is associated with the volume density of the neutrino momentum loss in unit time, and therefore it defines the neutrino force acting on plasma. Because of the isotropy of plasma in the absence of a magnetic field, one would expect that in the presence of a magnetic field the neutrino force action would be along the magnetic field only. However, as it was shown before, the probability of the neutrino-electron scattering (6) is symmetric with respect to the substitution $u \to -u$ (or $\theta \to \pi - \theta$). This means that the neutrino scattering on excited electrons does not give a contribution to the neutrino force acting on plasma along the magnetic field. Thus, under the conditions (1) and (2), there is no neutrino force on plasma at all. Therefore, this force is caused by a contribution of neutrino interactions with ground Landau level electrons only, and the result obtained by Kuznetsov and Mikheev (2000) has in fact a more general applicability. It may be used even in the limit of dense plasma when chemical potential is considerably greater than the magnetic field strength ($\mu^2 \gg eB$).

For the neutrino energy loss in unit volume per unit time in the limit of a very dense plasma we obtain the following result:

$$\dot{\varepsilon}_B = \frac{G_F^2 (c_v^2 + c_a^2)}{\pi^3} \mu^2 T^4 n_\nu J_B(\tau), \quad (10)$$

$$J_B(\tau) = \frac{\tau^4}{2} \int_0^1 \int_0^\infty \int_0^{\infty} dy \, y^2 \left[ y(1 - z^2) + 4z(1 + z^2) \right] \times \frac{1 - e^{y(1-\tau)}}{1 - e^{-y\tau}} e^{-y(1+\tau)/2z}, \quad (11)$$

where $n_\nu$ is the concentration of initial neutrinos, the parameter $\tau$ has a meaning of a relative neutrino spectral temperature, $\tau = T_\nu / T$. It is interesting to compare this result with the one in a non-magnetized plasma which can be presented in a similar form:

$$\dot{\varepsilon}_{B=0} = \frac{G_F^2 (c_v^2 + c_a^2)}{\pi^3} \mu^2 T^4 n_\nu J_{B=0}(\tau), \quad (12)$$

$$J_{B=0}(\tau) = 4\tau^4 \int_0^1 d\xi \, \xi^2 \frac{e^{\xi(\tau-1)} - 1}{e^{\xi\tau} - 1}. \quad (13)$$

The functions $J_B(\tau)$ and $J_{B=0}(\tau)$ define the dependence of the neutrino energy losses on the relative neutrino spectral temperature in a magnetized plasma and in a plasma without field respectively. In the limit of a sufficiently large neutrino spectral temperature ($e^\tau \gg 1$) they reduce to the power functions:

$$J_B(\tau) \simeq 4.35 \tau^4,$$

$$J_{B=0}(\tau) \simeq 8 \tau^4.$$

The graphs of the functions (11) and (13) are presented in Fig.2. As one would expect, at neutrino spectral temperature smaller than the plasma temperature ($\tau < 1$), the values
of the functions $J_B(\tau)$ and $J_{B=0}(\tau)$ are negative. It implies that a neutrino propagating in a medium captures energy from the plasma. When $T_\nu > T_\text{pl}$, $(\tau > 1)$, the neutrino gives up energy to the plasma. When $\tau = 1$ there is a thermal equilibrium when there is no energy exchange between neutrino and electron-positron plasma. It can be seen that the neutrino energy loss in a magnetized plasma is less than the one in a non-magnetized plasma. Hence, under the conditions (1) and (2) the magnetized plasma becomes more transparent for neutrinos than in the case of plasma without field.

4 Conclusions.

In this paper we have investigated the neutrino-electron scattering in a dense magnetized plasma. We have considered the physical situation when the plasma component is the dominant one of the two components of the active medium. At the same time, a magnetic field was assumed to be not too small ($\mu^2 > 2eB \gg \mu E$). The probability and the volume density of the neutrino energy-momentum losses have been calculated.

It is found that the neutrino scattering on excited electrons does not give a contribution to the neutrino force acting on plasma. This force is caused by the neutrino-electron processes when plasma electrons occupy the lowest Landau level only. Thus the result for this force, obtained in the paper by Kuznetsov and Mikheev (2000), has a more wide area of application. It may be used under a condition $\mu^2 \gg eB$ as well.

It is shown that under the conditions (1) and (2) the combined effect of plasma and strong magnetic field leads to a decrease of the neutrino energy loss in comparison to the one in a pure plasma. Therefore, the complex medium, plasma plus strong magnetic field, is more transparent for neutrinos than non-magnetized plasma.

We believe that the result obtained will be useful for the detailed analysis of astrophysical cataclysms such as supernova explosions.
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References


