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Amplification of solitary optical waves in fibers with positive group velocity dispersion

Todor N. Arabadzhiev[1*], Ivan M. Uzunov[2†]

[1] *Institute of Electronics, Sofia 1784, Bulgaria*

[2] *Dept. of Applied Physics, Technical University-Sofia, 1000 Sofia, Bulgaria*

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Abstract: Solitary optical waves with characteristic red frequency shift have been numerically identified in fibers with positive group velocity dispersion in the presence of amplification by semiconductor amplifiers. Influence of the Henry factor, the net gain, gain saturation characteristics, and frequency modulation on the frequency shift of the observed solitary optical waves has been examined. Normalized pulse energy as a function of the gain saturation characteristics has been studied.

Keywords: Solitary optical waves, pulse amplification, semiconductor laser amplifiers
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1 Introduction

It is well known that stationary solitary optical waves (SW) can be generated in dissipative nonlinear media with group velocity dispersion (GVD) and amplification [1–7]. It has been shown that solitary optical waves can propagate in cascaded optical communication systems with in-line semiconductor laser amplifiers (SLA) in spite of the noise generated in SLA [2]. If one neglects the noise, the preservation of solitary wave in such media can be explained on one hand with the presence of strong balance between the group velocity dispersion and nonlinearity, and on the other hand with the presence of

* E-mail: tna@abv.bg
† E-mail: ivan_uzunov@tu-sofia.bg

the delicate balance between loses and nonlinear amplification. The authors have previously described a numerical investigation of the frequency characteristics of the two kind solitary optical waves - averaged and dissipative solitons in dissipative nonlinear media with cascaded amplification by the SLA and negative GVD [7]. These two kind solitary waves demonstrate different dynamics of their averaged frequency. The influence of the positive GVD on the frequency characteristics of the solitary optical waves in dissipative media with amplification by the SLA still remain poorly studied. This is the main goal of this paper. The preliminary results of this paper have been reported [9].

2 Theoretical model

In contrast with the classical Schrödinger solitons, the solitary waves in dissipative media emerge as a complex balance between linear and nonlinear effects, including linear and nonlinear losses and amplification. Therefore, they have qualitatively different dynamics and properties. Fiber transmission system with periodic amplification of optical pulses with SLA and bandpass optical filters is a typical system for consideration of such waves. In difference with the papers [2–7], where the model with averaged amplification (see below) was used in the study of such fiber transmission systems, we apply here a model with the periodical lump amplification by SLA [7].

The propagation of optical waves in a system of optical fiber and periodically inserted SLA can be described by following equations for the slowly varying field envelope $A(Z,T) = A(z,t)\Big/\sqrt{P_0}$ and gain of SLA [1–4]:

$$i\frac{\partial A(Z,T)}{\partial Z} - \frac{1}{2}\frac{\partial^2 A(Z,T)}{\partial T^2} + |A(Z,T)|^2 A(Z,T) = -i\Gamma A(Z,T) + iG(Z)A(Z,T) \quad (1)$$

$$G(Z)A(Z,T) = \sum_{j=1}^{N} \frac{A_{out}(l,T)}{A_{in}(l=0,T)}\delta(Z - jL_a)A(Z,T) \quad (2)$$

$$\frac{dh(T)}{dT} = \frac{(h_0 - h(T))}{\tau_c}t_0 - \frac{P(l=0,T)}{E_S}t_0\left(\exp\left(h(T)\right) - 1\right) \quad (3)$$

$$A_{out}(l,T) = A_{in}(l=0,T)\exp\left((1 - i\alpha)\frac{h(T)}{2}\right) \quad (4)$$

Here $A_{in}(l=0,T)$ and $A_{out}(l,T)$ are input and output in the SLA field envelopes, T=t/t$_0$, Z=z/L$_D$, t_0 is the arbitrary normalization time, and L_D is the dispersion length. $\Gamma = \gamma L_D/2$ is the normalized fiber loss coefficient (On graphics below the fiber loss coefficient is presented in units γ_{dB} [dB/km]) and $L_a = z_a/L_D$ is the normalized distance between amplifiers. The main parameters of the SLA that determine the pulse's shape and frequency chirping are: the small – signal gain $G_0 = \exp(h_0)$ (On graphics below the gain is presented in units G_{dB} [dB]), the carrier lifetime τ_c, the saturation energy E$_S$, and the Henry factor α. Here $h(T) = \int_0^l g(z,T)dz$ is the integrated gain, l is the length of SLA. Saturation energy is defined by $E_S = h\upsilon\sigma_m/\sigma_g$, where σ_m is the mode cross-section, and σ_g is the differential gain. Henry, or linewidth enhancement factor is given with

$\alpha = -\frac{Re\chi_p}{Im\chi_p} = -2k_0\frac{\partial\mu/\partial N}{\partial g/\partial N}$, where χ_p is the additional contribution to the susceptibility related to strength of pumping, μ is the index of refraction and N is the carrier density. The Henry factor accounts for the refractive index changes that accompany the carrier-density variations due to the gain saturation effect, and have typical values in the range 4-6 for InGaAsP amplifiers.

Equations 1 and 2 describe the propagation of the pulse in the optical fiber and periodical amplification with SLA. It is solved by the Split Step Fourier method. Equation 3 describes the dynamics of the integrated gain in the SLA [1]. This equation is solved by the Runge-Kuta method. The equation 4 describes the pulse amplification in SLA. The amplifier noise is neglected.

3 Results

Because of the amplification process, resonant linear waves are generated between the amplifiers. In order to minimize this effect, the distance between amplifiers L_A has to be smaller than the dispersion length L_D. Then the main influence on the wave's shape at high input energy exerts the balance between losses and large gain modulation.

In case, when waves have input energy smaller than the gain saturation energy of SLA, the balance between GVD and nonlinearity is still important. In the positive GVD region, this causes deformation of the waves, as shown in Figure 1.

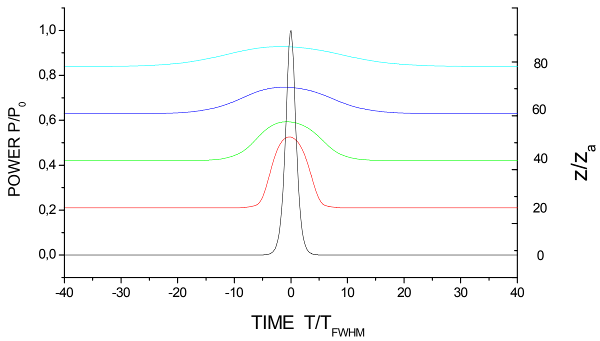

Fig. 1 (color online) GVD>0, A(t,0)=$a_0$sech(T), a_0=2.849, $G_{dB} - \gamma_{dB}$ z_a=0, E/E_S=1/240, α=5, γ_{dB}=0.4db/km, z_a ~31.38 km, L_D=262 km.

In the case of gain saturation, and small excess gain G-γz_a>0, stable waves (SW) can exist in the positive GVD region. Appearance of such SW is demonstrated in Fig. 2.

In Fig. 3, a continuous red frequency shift of the amplified SW can be clearly observed.

This shift is different from the case of negative GVD region where the wave's frequency shifts toward blue side [7]. This phenomenon is due to the interplay of the positive GVD in the fiber and the frequency modulation generated by the process of amplification in the SLA. The spontaneous emission noise of SLA, neglected in this work can influence this frequency shift.

The mean frequency of the spectrum $\langle\omega\rangle = \int_{-\infty}^{\infty} \omega\,|A(\omega)|^2\,d\omega / \int_{-\infty}^{\infty} |A(\omega)|^2\,d\omega$ is pre-

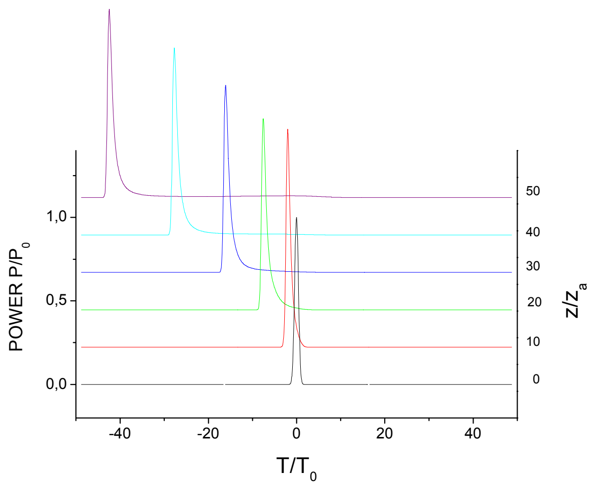

Fig. 2 (color online) A(T,0)=exp(-T^2), t_0=5,6 ps, E/E_s=1/10, γ_{dB}=0.4dB/km, τ_C=200ps, G_{dB}=13.69dB, α=5, z_a=28.76 km, D=-0.121ps/nm.km, L_D=262 km.

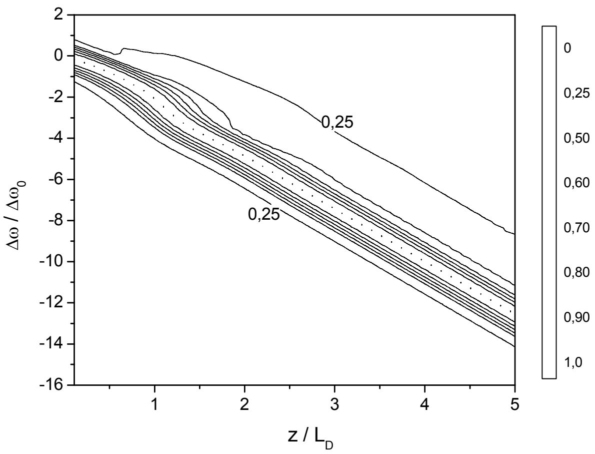

Fig. 3 The red shift of the frequency spectrum of the SW.

sented in the Fig. 4 as a function of the Henry factor α and distance of propagation. The frequency shift can be reduced by reducing α. Fig. 4*b* shows the linear dependence of the frequency shift on α, for a fixed distance of propagation.

As could be expected, the frequency shift is linearly proportional to the Henry factor.

The frequency shift of the SW can be controlled by the values of the net gain and gain saturation parameter E/E_S (Fig. 5) In Fig. 5 the control of frequency and time shifts by means of the value of the net gain for fixed value of the gain saturation parameter E/E_S is demonstrated.

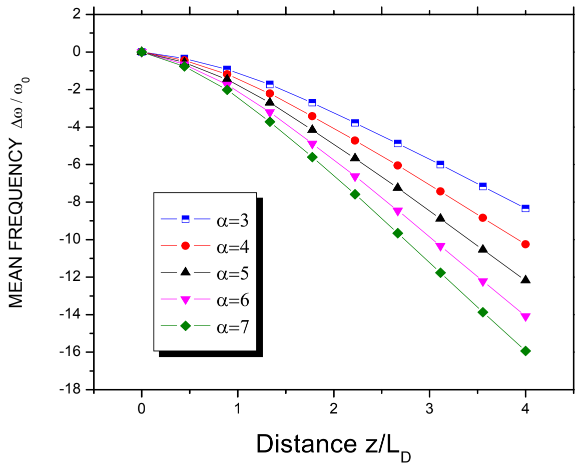

Fig. 4*a* (color online) The mean frequency for different values of the Henry factor.

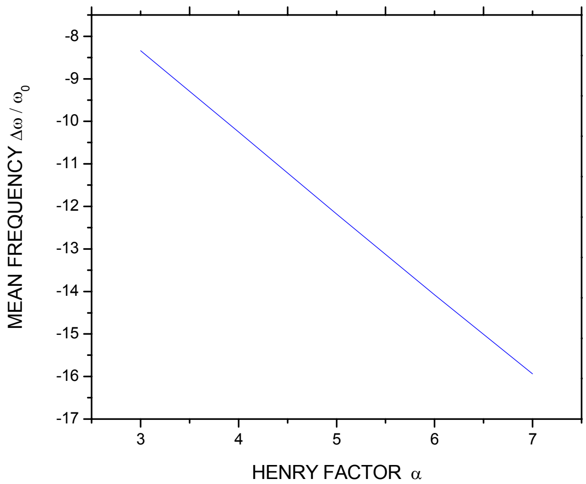

Fig. 4*b* (color online) Dependence of the frequency on the Henry factor for fixed distance z/L_D=4.

When the amplifier distance L_A is much smaller than the dispersion length L_D, the wave's propagation can be described by a model with averaged (distributed) amplification [3, 5, 6, 8]:

$$i\frac{\partial Q}{\partial Z} - \frac{1}{2}\frac{\partial^2 Q}{\partial T^2} + Q\left|Q\right|^2 = i\gamma_1 Q - \gamma_2(\alpha + i)Q\int\limits_{-\infty}^{T}\left|Q\left(T'\right)\right|^2\left[-\frac{\left(T - T'\right)}{\tau_C}t_0\right]dT' \qquad (5)$$

where: $Q = \frac{A(Z,T)}{a(Z)}$, $a_0^2 = \frac{2\gamma z_a}{1-\exp(-2\gamma z_a)}$, $\gamma_1 = \frac{h_0}{2L_a} - \gamma L_D$ is the net gain, and $\gamma_2 = (P_0 t_0 a_0^2)/(2L_a E_S)$. Our numerical results show a good agreement with known results [1–5] received by the model with averaged amplification for comparable values of pulse's and system's parameters.

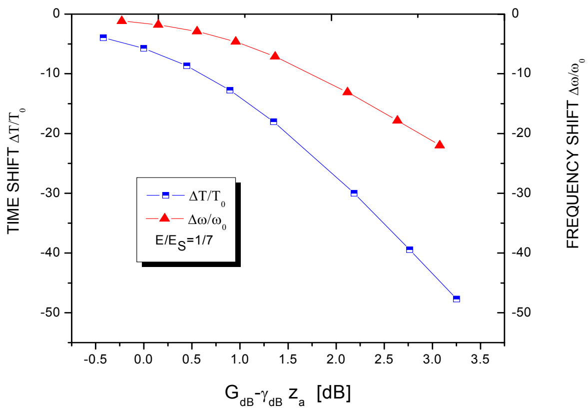

Fig. 5 (color online) Time and frequency shift as function of the net gain ($G_{dB} - \gamma_{dB}\ z_a$) for fixed distance z/L_D=4 and ratio E/E_S=1/7.

Following consideration of dissipative solitons performed in [4], we introduce normalized pulse energy: $W(Z) = \int\limits_{-\infty}^{\infty} |Q(Z,T)|^2\, dT$, which in the case of $\alpha = 0$, satisfies equation:

$$\frac{dW(Z)}{dZ} = 2\gamma_1 W(Z) - \gamma_2 W(Z)^2 \Rightarrow W(Z) = \frac{W(0)\exp(2\gamma_1 Z)}{1 + \frac{\gamma_2}{2\gamma_1} W(0)\left[\exp(2\gamma_1 Z) - 1\right]} \tag{6}$$

The analytically computed stationary value of the normalized pulse energy has been also confirmed by means of numerical simulations and considered as an important characteristic of the dissipative solitons (see Ref. [4]). We obtain $W(\infty) = 2\gamma_1/\gamma_2$. In case when $\gamma_1 = 1$, $\gamma_2 = 1$ we the result of Ref. [4] is obtained $W(\infty) = 2$.

In our lump model, the energy is computed only after amplifiers. Between amplifiers the pulse continuously attenuates. Our results also show the possibility to obtain stationary values of the energy. We have calculated numerically pulse energy of SW as a function of the distance of propagation and ratio E/E_S. Fig. 6 clearly demonstrates the appearance of stationary value of the normalized pulse energy. The asymptotic values of the normalized pulse energy depend on the gain saturation characteristics, as could be expected from the relation $W(\infty) = 2\gamma_1/\gamma_2$.

Further we analyze the influence of the initial pulse chirp on the frequency shift. Initial pulse shape is described by the formula $A(T) = \exp(-(1 + iC)T^2)$, where C is a chirp parameter. Depending on the sign of the chirp parameter, change in the value of the frequency shift is numerically observed. At positive sign the value of the frequency shift decreases, while at negative sign increases (Fig. 7).

The stronger frequency shift in the case of negative values of the chirp parameter is due to the fact that such modulation causes compression of the pulse at its initial stage of propagation and therefore increases the influence of the gain saturation effect.

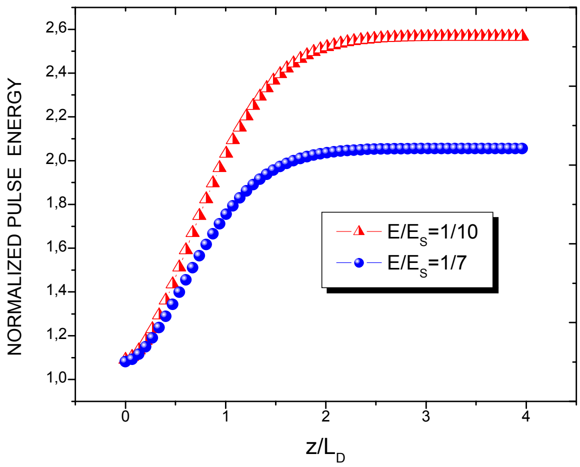

Fig. 6 (color online) Stationary values of the normalized pulse energy of the SW in the lumped model.

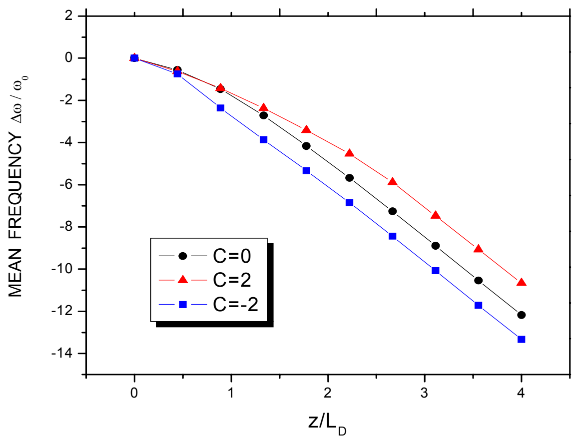

Fig. 7 (color online) The mean frequency for different values of chirp parameter.

4 Conclusions

It was shown that stable optical pulses can exist in regime of the positive group-velocity dispersion in optical fibers in presence of amplification by semiconductor laser amplifiers. The observed stable pulses demonstrate red frequency shift of their mean frequency, contrary to the case of negative group-velocity dispersion [7].

The mean frequency shift was studied as a function of the distance of propagation, Henry factor, net gain, gain saturation parameter E/E_S and chirp parameter C. It was revealed that the value of the mean frequency of the observed optical pulses can be controlled by means of the gain and gain saturation characteristics of the amplifier, as well as with the initial frequency modulation of the pulse. It is shown that the mean frequency is linearly proportional to the Henry factor for a fixed distance of propagation.

Appearance of stationary values of the normalized pulse energy was established. The asymptotic values of the normalized pulse energy depend on the gain saturation characteristics.

The effect of frequency shift of the numerically observed dissipative structures which was analyzed in this paper could be related to the slow-light effects [10].

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