Efficiency of second harmonic generation for partially coherent light in anisotropic crystals

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Abstract: This paper analyzes the second harmonic generation (SHG) efficiency of light with partial temporal coherence due to depolarization effects in birefringent media. It discusses relations between SHG efficiency fading, light source spectrum, crystal birefringence, and phase matching conditions. The efficiency of SHG pumped by the partially coherent light beam that may depolarize light in nonlinear birefringent crystal is also analyzed. The basic theory of SHG with its modification for partially coherent light with depolarization and some numerical calculations of the SHG process are described.

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1. Introduction

Second Harmonic Generation (SHG) is an optical nonlinear phenomenon relying on frequency doubling of the light during its propagation in anisotropic crystals. The efficiency of SHG depends on phase matching between pumping and doubled frequency light beams. Phase matching can be realized in birefringent media, where ordinary and extraordinary waves have different phase velocities. In the first type of phase matching both fundamental and second harmonic waves are perpendicularly polarized. In phase matching of the second type, the fundamental wave includes both ordinary and extraordinary components. Since propagation of the partially coherent light in such media may cause light depolarization [1], the degree of polarization of the fundamental wave becomes an important issue.

2. Depolarization of partially coherent light in anisotropic crystals

In general, the partially coherent light becomes depolarized during propagation through birefringent media. The depolarization effect depends on: the coherency of the light, which is characterized by a source spectrum bandwidth (Δν); birefringence, which is defined in the uniaxial crystal as Δn = |n_e – n_o| where n_o, n_e are ordinary and extraordinary refractive indices respectively; and the azimuth of the light beam versus the fast and slow axes of the birefringent medium [1]. Both components of the linearly polarized light coupled into the crystal may differ-
When calculating amplitudes, the phasor of the complex degree of mutual coherence between orthogonal polarizations $E_x$ and $E_y$ can be ignored and its absolute value will be used in further analysis:

$$|\gamma(\tau)| = |\gamma_0|$$  \hspace{1cm} (4)

The degree of coherence can be connected with the spectrum of the light source [2]:

$$|\gamma_l(\tau)| = \exp\left(-\pi\Delta\nu\tau\right),$$  \hspace{1cm} (5)

for the Lorentzian type of light sources, and

$$|\gamma_C(\tau)| = \exp\left[-\left(\frac{\pi\Delta\nu\tau}{2\sqrt{\ln 2}}\right)^2\right],$$  \hspace{1cm} (6)

for the Gaussian type of light sources.

The bandwidth of the spectrum $\Delta \nu$ can be expressed as wavelength dependent:

$$\Delta \nu = \frac{c}{\lambda} - \frac{c}{\lambda + \Delta \lambda} \approx \frac{\Delta \lambda}{\lambda^2} c.$$  \hspace{1cm} (7)

where $\lambda$ and $\Delta \lambda$ are the central wavelength and bandwidth respectively. If the propagation distance is equal to $z$, then the propagation time is equal to $\tau = \frac{\Delta \lambda z}{c}$, and thus:

$$|\gamma_l(z)| = \exp\left[-\frac{\Delta \nu \pi \Delta n \tau}{c}\right].$$  \hspace{1cm} (8)

$$|\gamma_C(z)| = \exp\left[-\left(\frac{\Delta \nu \pi \Delta n}{2\sqrt{\ln 2}}\right)^2\right].$$  \hspace{1cm} (9)

When a laser beam propagates in an uniaxial crystal along its optical axis $c$ (Fig. 2) it sees the ordinary refractive index $n_o$, and this is exactly the same situation as beam propagation in an isotropic medium. However, when the beam is perpendicular to the optical axis the difference between both ordinary and extraordinary refractive indices reaches its maximum value. The ordinary refractive index is constant for a given wavelength, while the extraordinary refractive index is a function of an angle $\theta$ between the direction of the beam and the optical axis (Fig. 2).

Hence we obtain the refractive index for the extraordinary wave [3]:

$$n_\varepsilon(\theta) = \frac{n_o n_e}{\sqrt{(n_e^2 - n_o^2) \cos^2 \theta + n_o^2}},$$  \hspace{1cm} (10)

and the birefringence present in equations (8-9) can be written in the form:

$$\Delta n = \frac{n_o n_e}{\sqrt{(n_e^2 - n_o^2) \cos^2 \theta + n_o^2}} - n_o.$$  \hspace{1cm} (11)

It means that the DOP fading depends on the beam propagation direction (θ angle) in the uniaxial crystal [4].
3. Second Harmonic Generation

Second harmonic generation (SHG) is a special case of the three-wave mixing (TWM) non-linear process that can be described by a set of differential equations [5, 6]:

\[
\begin{align*}
\frac{dE_{x}^{(\omega)}}{dz} &= -i \frac{\omega}{2\varepsilon_0 n_0} \chi^{(2)}_{\text{ef}} E_{x}^{(2\omega)} E_{y}^{(\omega)} \exp(i\Delta k z) \\
\frac{dE_{y}^{(\omega)}}{dz} &= -i \frac{\omega}{2\varepsilon_0 n_0} \chi^{(2)}_{\text{ef}} E_{x}^{(2\omega)} E_{y}^{(\omega)} \exp(i\Delta k z) \\
\frac{dE_{z}^{(2\omega)}}{dz} &= -i \frac{\omega}{\varepsilon_0 n_0 (2\omega)} \chi^{(2)}_{\text{ef}} E_{x}^{(2\omega)} E_{y}^{(\omega)} \exp(-i\Delta k z)
\end{align*}
\]

(12)

where \(\omega\) is the pumping frequency, \(n_o\) and \(n_y\) are the ordinary and extraordinary refractive indices, \(c\) is the speed of light, \(\varepsilon_0\) is the dielectric constant, \(\chi^{(2)}_{\text{ef}}\) is an effective second-order nonlinear susceptibility, \(E_{x}^{(\omega)}\) and \(E_{y}^{(\omega)}\) are orthogonal components of the pumping electric field amplitude, and \(E_{z}^{(2\omega)}\) is the amplitude of the second harmonic electrical field.

This configuration of the pumping waves, called II SHG type, is schematically shown in Fig. 3.

The most effective SHG process occurs when a phase matching condition (\(\Delta k = 0\)) is fulfilled. We can match the phase by coupling the light along the birefringence axis and by selecting a special type of anisotropic crystal in which:

\[
n_o(2\omega) = \frac{n_o(\omega) + n_y(\omega)}{2}
\]

(13)

The phase matching condition can also be fulfilled in all crystals at the angle \(\theta\) of the input light beam that satisfies the following equation:

\[
n_o(\omega, \theta) = \frac{n_o(\omega) n_y(\omega)}{\sqrt{[\n_o(\omega)]^2 - [n_o(\omega)]^2 \cos^2 \theta + [n_o(\omega)]^2}}
\]

(14)

which leads to:

\[
\theta = \cos^{-1} \sqrt{\frac{[n_o(\omega)]^2 - [n_o(\omega)]^2 \left[\frac{[n_o(\omega)]^2}{[2n_o(2\omega) - n_o(\omega)]^2} - [n_o(\omega)]^2\right]}{[2n_o(2\omega) - n_o(\omega)]^2 \left[\frac{[n_o(\omega)]^2}{[2n_o(2\omega) - n_o(\omega)]^2} - [n_o(\omega)]^2\right]}}.
\]

(15)

The method of acquiring the phase matching condition can be illustrated on the refractive indices ellipsoid, as shown in Fig. 4.

4. SHG for partially coherent light

For partially coherent light, the absolute value of the degree of coherence \(|\gamma|\) should be introduced into the equations (12):

\[
\begin{align*}
\frac{dE_{x}^{(\omega)}}{dz} &= -i \frac{\omega}{2\varepsilon_0 n_0} \chi^{(2)}_{\text{ef}} E_{x}^{(2\omega)} E_{y}^{(\omega)} |\gamma| \exp(i\Delta k z) \\
\frac{dE_{y}^{(\omega)}}{dz} &= -i \frac{\omega}{2\varepsilon_0 n_0} \chi^{(2)}_{\text{ef}} E_{x}^{(2\omega)} E_{y}^{(\omega)} |\gamma| \exp(i\Delta k z) \\
\frac{dE_{z}^{(2\omega)}}{dz} &= -i \frac{\omega}{\varepsilon_0 n_0 (2\omega)} \chi^{(2)}_{\text{ef}} E_{x}^{(2\omega)} E_{y}^{(\omega)} |\gamma| \exp(-i\Delta k z)
\end{align*}
\]

(16)
When the light propagating in a birefringent medium becomes depolarized we need to use a distance-dependent complex degree of coherence (that also depends on the spectrum emitted by the light source), in the form given in (8)-(9) with the birefringence: \( \Delta n = |n_x(\omega) - n_y(\omega)| \).

Fig. 5 presents SHG efficiency numerical results calculated as a function of the light propagation distance for different bandwidths of the Lorentzian-type light source, and in phase matching conditions for the input optical power density of \( 10^{13} \) W/m\(^2\). Such values of optical power can be used e.g. in high-power frequency doublers based on currently grown KTP crystals, which can handle power densities up to \( 1 \times 10^{14} \) W/m\(^2\) in 10 ns pulses for pumping wavelengths of 1064 nm without suffering optical damage [7].

Fig. 6 presents SHG efficiency numerical results calculated as a function of the input optical power for different distances inside the birefringent crystal, and in phase matching conditions for bandwidth \( \Delta \lambda = 0.5 \) nm of the Lorentzian spectrum. It is evident that the SHG efficiency drops with wider source bandwidths. The efficiency of SHG with partially coherent light can reach the same saturation level as SHG with monochromatic light, but requires more input optical power.

Fig. 7 presents SHG efficiency numerical results calculated as a function of distance for different values of phase mismatch, for the input optical power density of \( 10^{13} \) W/m\(^2\) and for the bandwidth \( \Delta \lambda = 0.5 \) nm of the Lorentzian spectrum.

In the case of phase mismatch \( \Delta k \neq 0 \) generation of the second harmonic wave is less effective for a non-coherent source, similar to the phase matching configuration. There are periodical changes of the second harmonic signal typical for this case, as a result of the reversal differential frequency generation process. However, for a non-coherent source, these oscillations decay and the effect is especially visible for a larger mismatch \( \Delta k \), for which the analytical solution of the equations (16) can be obtained.

For large values of the phase mismatch \( \Delta k \gg 0 \) SHG
Numerical results of SHG compared to analytical approximates results of SHG efficiency numerical calculations. By putting $\gamma$ as given in (8) for the Lorentzian source, the last equation of (16) can be written as follows:

$$\frac{dE_x^{(2\omega)}}{dz} = -i \frac{\omega}{\varepsilon_0 n_x(2\omega)} \gamma^{(3)} E_x^{(\omega)} E_y^{(\omega)} \exp(-\phi z - i\Delta k z),$$

where:

$$\phi = \Delta \nu \pi |n_x(\omega) - n_x(\omega)|,$$

and finally we can obtain an analytical solution of the second harmonic amplitude:

$$E_x^{(2\omega)} = i \frac{\omega}{\varepsilon_0 n_x(2\omega)} \gamma^{(3)} \frac{E_x^{(\omega)} E_y^{(\omega)} \exp(-i\Delta k z - \phi z) - 1}{\phi + i\Delta k}$$

Fig. 8 presents results of SHG efficiency numerical calculations with the partially polarized light compared with the analytical approximation. Calculations were carried out for the input optical power density of $10^{13}$ W/m² and for the bandwidth $\Delta \lambda = 0.5$ nm of the Lorentzian spectrum.

The large $\Delta k$ approximation fits to the numerical results only if $\Delta k \gg 0$. For a smaller value of the phase mismatch the analytical solution is no longer valid. The decay of the oscillations of SHG is caused by fading of the DOP (Fig. 8). When the phase difference between the orthogonal components $E_x^{(\omega)}$ and $E_y^{(\omega)}$ becomes undetermined, both SHG and differential frequency generation decrease. As a result, the value of the SHG signal saturates at a certain average level.

All calculations were carried out for refractive indices and nonlinearity coefficients of the KTP crystal ($n_x(\omega) = 1.7400$, $n_x(\omega) = 1.8304$, $n_x(2\omega) = 1.7787$, $\gamma^{(3)} / \varepsilon_0 = 2.4$ pm/V, $\lambda = 1064$ nm).

5. Conclusions

We have demonstrated that the absolute value of the degree of coherence, which for a polarization azimuth of $\alpha = 45^\circ$ is equal to the degree of polarization (DOP = $|\gamma|$), has influence on the II type SHG efficiency.

Real light sources with finite spectrum bandwidths generate light with partial temporal coherence that is responsible for depolarizing effects, and depolarization of light emitted by the Lorentzian sources is stronger than depolarization of light emitted by the Gaussian sources in the same birefringent medium. It has been proven, that SHG is less effective with larger spectrum bandwidths.

The SHG process induced with the partially polarized light can become saturated at a certain interaction distance, but it requires more input optical power than SHG pumped by a monochromatic source with high degree of coherence.

It has been shown that in view of phase mismatch, second harmonic intensity decays periodically with distance, and as a result the second harmonic signal at large distances reaches a saturated constant value.

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