Cent. Eur. J. Phys. • 12(11) • 2014 • 785-791
DOI: 10.2478/s11534-014-0513-4

VERSITA

Central European Journal of **Physics**

Investigation of isospin forbidden $0^+ \rightarrow 0^+$ Fermi beta decays with Pyatov method

Research Article

Abdullah Engin Çalık[1] *

1 *Department of Physics, Faculty of Arts and Sciences, Dumlupınar University, 43100 Kütahya, Turkey*

Received 3 January 2014; accepted 21 June 2014

Abstract: In the present work, the matrix elements, isospin impurities and $\log ft$ values of the isospin forbidden $0^+ \rightarrow 0^+$ beta decays have been investigated. The calculated results have been compared with available experimental and another theoretical data. The isotopic invariance of the Hamiltonian has been restored by Pyatov method. Within the quasi-particle random phase approximation (QRPA), the computations have been performed both in presence and absence of the pairing interactions.

PACS (2008): 21.60.Cs, 23.40.Hc, 23.40.-s

Keywords: isospin forbidden beta decay • isobar analog state • $\log ft$ value

© *Versita sp. z o.o.*

1. Introduction

Superallowed beta decays are pure Fermi type transitions and are seen between two components of the same isospin multiplet *i.e.*, in between isobar analogue states. Isospin forbidden beta decays, however, occur between members of two different isospin multiplets [1]. Isospin forbidden transitions occur for both $0^+ \rightarrow 0^+$ and $J^\pi \rightarrow J^\pi (J \neq 0)$ states. In $0^+ \rightarrow 0^+$, $\triangle T = \pm 1$, the transition is pure Fermi decay and in $J^\pi \rightarrow J^\pi (J \neq 0)$, $\triangle T = \pm 1$, the transition is isospin-allowed Gammow Teller or isospin forbidden Fermi decay [2].

For the case of isospin forbidden $0^+ \rightarrow 0^+$, $\triangle T = \pm 1$ Fermi beta decay, if there are no charge dependent effects, the Fermi matrix elements should be zero. If the matrix element is not zero, the isospin is not a good quantum number. Hence, the matrix element is proportional to the magnitude of the isospin impurities in states [1, 2].
We have considered a $0^+ \rightarrow 0^+$ β^- decay, in which the initial state is denoted by $\mid i \rangle = \mid J = 0^+, T, T_0 \rangle$ and the final state is denoded by $\mid f \rangle = \mid J = 0^+, T', T_0 - 1 \rangle$, respectively. The Fermi matrix element for the related decay has the form;

$$M_V = \langle f \mid \widehat{T}_- \mid i \rangle, \tag{1}$$

where, J and T are angular momentum and isospin quantum numbers, respectively. T_0 is the third component of T. For the $\widehat{T}_\pm$ raising (lowering) isospin operator according to the angular momentum is written as;

$$\widehat{T}_\pm \mid T, T_0 \rangle = \sqrt{(T \mp T_0)(T \pm T_0 + 1)} \mid T, T_0 \pm 1 \rangle. \tag{2}$$

*E-mail: aengin.calik@dpu.edu.tr

Unauthenticated
Download Date | 9/26/18 5:43 AM

According to the Eq. (1) and (2) the matrix element is obtained as,

$$M_V = \langle T', T_0 - 1 \mid \widehat{T}_- \mid T, T_0 \rangle = 0. \tag{3}$$

As a consequence of Eq. (3), the transition cannot occur. However, when the analysis are experimentally reported, beta transition is detected between these states, as the initial and final states are not pure isospin states. The analogue state of the initial state has admixed into the final state of the transition. The analog state of the initial state is

$$\mid T, T_0 - 1 \rangle = \widehat{T}_- \mid T, T_0 \rangle. \tag{4}$$

New situation of the final state is denoted as:

$$\mid f \rangle = \mid T', T_0 - 1 \rangle + \alpha \mid T, T_0 - 1 \rangle, \tag{5}$$

where, α is the admixture amplitude.
From Eq. (5), for this new situation, the β^- Fermi matrix element is given as,

$$\begin{aligned} M_V &= (\langle T', T_0 - 1 \mid + \alpha \langle T, T_0 - 1 \mid) \widehat{T}_- \mid T, T_0 \rangle \\ &= \alpha \sqrt{(T + T_0)(T - T_0 + 1)}. \end{aligned} \tag{6}$$

Above equation shows that the beta transition has also occurred theoretically.
Up to date, the isospin forbidden Fermi beta decays have been studied in many scientific studies. The general formalism, experimental data and theoretical calculations were expressed by Blin-Stoyle [1]. Isospin forbidden beta decay of ^{28}Mg was experimentally investigated by Dickey *et al.* [2]. They deduced a charge-dependent isospin matrix element. The effective non-conserving interaction [3], charge symmetry breaking nucleon-nucleon interaction [4] and, effects of the charge-symmetry-breaking and charge-independence-breaking terms of nuclear force [5] in 1s0d-shell were studied by Nakamura *et al.* The $\log ft$ values of 514 nuclei were calculated in Ref. [5]. The up-down quark mass difference was evaluated in 1s0d shell nuclei and matrix elements were computed in Ref. [6]. The Fermi matrix elements in the beta decay of ^{234}Np were investigated both theoretically and experimentally in Ref. [7]. The isospin impurities were analyzed and, admixture amplitudes were computed for 26 different isospin forbidden beta decays by Bertsch and Mekjian [8]. An effective one body spheroidal Coulomb potential was used by Yap and Saw, to calculate the matrix elements and admixture amplitudes for the isospin forbidden beta decays of ^{8}Li and ^{134}Cs [9]. The relatively large Fermi matrix element of isospin forbidden beta decay of ^{57}Ni [10] and, isospin forbidden positron decay of ^{46}V [11] were studied by the same authors. Isospin forbidden transitions to the low-lying states in ^{26}Al with distored wave Born approximation (DWBA) were investigated by Yasue *et al.* [12].

In the present study, the isospin symmetry breaking has been restored by Pyatov method [13]. This method has been used to achieve self-consistency between residual interaction and shell-model potential. The isotopic invariance of nuclear forces and self-consistency conditions make the theory free of any adjustable parameters [14]. The matrix elements, isospin impurities and $\log ft$ values of the isospin forbidden $0^+ \to 0^+$ Fermi beta decays have been investigated based on Pyatov's method.

Pyatov method was used in several studies [14–23]. In our previous studies, the Cabbibo-Kobayashi-Maskawa (CKM) matrix unitarity [24, 25], the ft values of superallowed fermi beta decays [26] and isospin admixtures and isospin structure of isobar analog resonance states of superallowed beta decays [27] were investigated without and with pairing interactions, by using this method.

The paper is organized as follows: In Sec. 2, the details of Pyatov method and in Sec. 3 the Fermi matrix elements are given. The $\log ft$ values of the isospin forbidden $0^+ \to 0^+$ Fermi beta decays are then presented in Sec. 4. Finally, the computed results and conclusions are presented in Sec. 5 and Sec. 6, respectively.

2. Method

Here, we have only given main formalism of the Pyatov method. The details of method can be found in Refs [13–16].

In a closed system; if $\left[\widehat{H}, \widehat{T}\right] \neq 0$, there is a symmetry breaking due to the nuclear model, where $\widehat{H}$ and $\widehat{T}$ are Hamiltonian and isospin operator, respectively. The isospin symmetry violated with Coulomb interaction is natural. Hence, $\left[\widehat{H} - V_C, \widehat{T}\right]$ commutation is supposed to be zero, while it is not zero. Because the shell model potential includes isovector term and, the effect of isospin symmetry breaking caused by isovector term should be eliminated using a method. According to Pyatov's restoration method, the breaking symmetry of model Hamiltonian is restored by adding a proper residual force to the Hamiltonian. The residual interaction $\widehat{h}$ should satisfy the following condition:

$$\left[\widehat{H}_{sqp} + \widehat{h} - V_C, \widehat{T}^{\rho}\right] = 0, \tag{7}$$

Unauthenticated
Download Date | 9/26/18 5:43 AM

where, the effective interaction term $\widehat{h}$ is defined by Pyatov [13, 14] as:

$$\widehat{h}=\sum_{\rho=\pm}\frac{1}{4\gamma_\rho}\left[\widehat{H}_{sqp}-V_C,\widehat{T}^\rho\right]^\dagger\left[\widehat{H}_{sqp}-V_C,\widehat{T}^\rho\right] \quad (8)$$

where, γ_ρ is an average of double commutator in the ground state

$$\gamma_\rho\equiv\frac{\rho}{2}\langle 0\mid\left[\left[\widehat{H}_{sqp}-V_C,\widehat{T}^\rho\right],\widehat{T}^\rho\right]\mid 0\rangle. \quad (9)$$

The form of the effective interaction in Eq. (8) allows to treat the Coulomb mixing effects in a self-consistent way. Here, in the second quantization representation, single-quasiparticle Hamiltonian is

$$\widehat{H}_{sqp}=\sum_{j,\tau}\varepsilon_{j_\tau}\alpha^\dagger_{j_\tau}\alpha_{j_\tau}(\tau=n,\ p) \quad (10)$$

where, ε_j is the single quasiparticle energy of the nucleons and $\alpha^\dagger_{j_\tau}(\alpha_{j_\tau})$ is the quasiparticle creation (annihilation) operator.
The Coulomb potential is

$$V_C=\sum_i^A v_c(i)\left(\frac{1}{2}-t_z(i)\right)\ ,\ t_z(i)=\begin{cases}\frac{1}{2} & \text{for neutrons,}\\ -\frac{1}{2} & \text{for protons.}\end{cases} \quad (11)$$

The isospin operators, $\widehat{T}^\rho$, are defined as:

$$\widehat{T}^\rho=\frac{1}{2}\left(\widehat{T}_+ +\rho\widehat{T}_-\right)=\begin{cases}\widehat{T}_x & \rho=+1\\ i\widehat{T}_y & \rho=-1\end{cases}\quad ,\quad \widehat{T}_\pm=\sum_{k=1}\widehat{t}^k_\pm, \quad (12)$$

where, $\widehat{t}^k_+(\widehat{t}^k_-)$ are raising (lowering) isospin operators.
The isobaric 0^+ excitations in odd-odd nuclei generated from the correlated ground state of the parent even-even nuclei by the charge-exchange forces have been considered. The eigenstates of the single quasi-particle Hamiltonian, $\widehat{H}_{sqp}$, have been used as a basis. The basis set of the particle-hole operators is defined as

$$\widehat{A}^\dagger_{np}\equiv\frac{1}{\sqrt{2j_p+1}}\sum_{m_p,m_n}(-1)^{j_n-m_n}\alpha^\dagger_{j_pm_p}\alpha^\dagger_{j_n-m_n},$$

$$\widehat{A}_{np}\equiv\frac{1}{\sqrt{2j_p+1}}\sum_{m_p,m_n}(-1)^{j_p-m_p}\alpha_{j_p-m_p}\alpha_{j_nm_n}. \quad (13)$$

The bosonic commutations of these operators are given by following relations,

$$\left[\widehat{A}_{n_1p_1},\widehat{A}^\dagger_{n_2p_2}\right]=\delta_{n_1n_2}\delta_{p_1p_2}\ ,\ \left[\widehat{A}_{n_1p_1},\widehat{A}_{n_2p_2}\right]=0\ ,$$
$$\left[\widehat{A}^\dagger_{n_1p_1},\widehat{A}^\dagger_{n_2p_2}\right]=0.$$

The form of $\widehat{h}$ and γ in quasi-particle space can be written as

$$\widehat{h}=\sum_\rho\frac{1}{4\gamma_\rho}\Big(E^\rho_{n_1p_1}(j)\,E^\rho_{n_2p_2}(j)\left(\widehat{A}_{n_1p_1}-\rho\widehat{A}^\dagger_{n_1p_1}\right)\left(\widehat{A}^\dagger_{n_2p_2}-\rho\widehat{A}_{n_2p_2}\right)\Big) \quad (14)$$

and

$$\gamma_\rho=-\frac{1}{2}\sum_{np}E^\rho_{np}(j)\left(\overline{b}_{np}+\rho b_{np}\right), \quad (15)$$

respectively, with

$$\begin{aligned}E^\rho_{np}(j)&\equiv\frac{1}{2}\left(\varepsilon_{np}\left(\overline{b}_{np}+\rho b_{np}\right)-\left(\rho V_{np}-\overline{V}_{np}\right)\right),\\ b_{np}&\equiv U_pV_n\langle j_p\ \ j_n\rangle\ \ ,\ \ V_{np}\equiv U_pV_n\langle j_p\ \ v_c\ \ j_n\rangle,\\ \overline{b}_{np}&\equiv U_nV_p\langle j_p\ \ j_n\rangle\ \ ,\ \ \overline{V}_{np}\equiv U_nV_p\langle j_p\ \ v_c\ \ j_n\rangle.\end{aligned}$$

V's and U's are the occupation and unoccupation amplitudes obtained in BCS calculations [28]. In QRPA, the collective 0^+ states considered as one phonon excitations are given as

$$\widehat{Q}^\dagger_i\mid 0\rangle=\sum_{np}\left(\psi^i_{np}\widehat{A}^\dagger_{np}-\varphi^i_{np}\widehat{A}_{np}\right)\mid 0\rangle. \quad (16)$$

where, ψ^i_{np}, φ^i_{np} and $\widehat{Q}^\dagger_i$ are real amplitudes and phonon creation operator, in turn. The $\mid 0\rangle$ is the phonon vacuum which corresponds to the ground state of the even-even nucleus,

$$\widehat{Q}_i\mid 0\rangle=0. \quad (17)$$

The following orthonormalization condition for the amplitudes is obtained as,

$$\left[\widehat{Q}_i,\widehat{Q}^\dagger_j\right]=\sum_{np}\left(\left(\psi^i_{np}\right)^2-\left(\varphi^i_{np}\right)^2\right)\delta_{ij} \quad (18)$$

The eigenvalues and eigenfunctions of the restored Hamiltonian can be obtained by solving the equation of motion in QRPA,

$$\left[\widehat{H},\widehat{Q}^\dagger_i\right]\mid 0\rangle=\omega_i\widehat{Q}^\dagger_i\mid 0\rangle. \quad (19)$$

Herein, ω_i's are energies of the isobaric 0^+ states. Employing the conventional procedure of QRPA, the dispersion equation for the excitation energy of the isobaric 0^+ states is obtained as

$$\left(1-\sum_{np}\frac{\left(E^+_{np}\right)^2}{\gamma_+}\frac{\varepsilon_{np}}{\left(\omega_i^2-\varepsilon_{np}^2\right)}\right)\left(1-\sum_{np}\frac{\left(E^-_{np}\right)^2}{\gamma_-}\frac{\varepsilon_{np}}{\left(\omega_i^2-\varepsilon_{np}^2\right)}\right)$$
$$-\left(\sum_{np}\frac{E^+_{np}E^-_{np}}{\gamma_-}\frac{\omega_i^2}{\left(\omega_i^2-\varepsilon_{np}^2\right)}\right)^2=0,$$

Unauthenticated
Download Date | 9/26/18 5:43 AM

with $\varepsilon_{np} \equiv \left(\varepsilon_n + \varepsilon_p\right)$.
The amplitude can be analytically expressed in the following form:

$$\psi_{np}^i = \frac{1}{\sqrt{Z(\omega_i)}} \frac{1}{(\omega_i - \varepsilon_{np})} \left(\frac{E_{np}^+}{2\gamma_+} + \frac{E_{np}^-}{2\gamma_-} L(\omega_i) \right)$$

$$\varphi_{np}^i = \frac{1}{\sqrt{Z(\omega_i)}} \frac{1}{(\omega_i + \varepsilon_{np})} \left(\frac{E_{np}^+}{2\gamma_+} - \frac{E_{np}^-}{2\gamma_-} L(\omega_i) \right), \quad (20)$$

with

$$L(\omega_i) \equiv \frac{1 - \sum_{np} \frac{(E_{np}^+)^2}{\gamma_+} \frac{\varepsilon_{np}}{(\omega_i^2 - \varepsilon_{np}^2)}}{\sum_{np} \frac{E_{np}^+ E_{np}^-}{\gamma_-} \frac{\omega_i}{(\omega_i^2 - \varepsilon_{np}^2)}}$$

and,

$$Z(\omega_i) \equiv \sum_{np} \left(\left(\frac{1}{(\omega_i - \varepsilon_{np})} \left(\frac{E_{np}^+}{2\gamma_+} + \frac{E_{np}^-}{2\gamma_-} L(\omega_i) \right) \right)^2 - \left(\frac{1}{(\omega_i + \varepsilon_{np})} \left(\frac{E_{np}^+}{2\gamma_+} - \frac{E_{np}^-}{2\gamma_-} L(\omega_i) \right) \right)^2 \right).$$

3. Fermi matrix elements

The Fermi transition matrix elements between the isobaric 0^+ states of the neighbour nuclei are defined as:
a) for the transitions (N,Z) $\to$ (N-1,Z+1)

$$M_{\beta^-}^i = \langle 0 \,|\, \left[\widehat{Q}_i, \widehat{T}_- \right] | \, 0 \rangle = \sum_{np} \left(b_{np} \psi_{np}^i + \overline{b}_{np} \varphi_{np}^i \right), \quad (21)$$

b) for the transitions (N, Z)$\to$ (N+1, Z-1)

$$M_{\beta^+}^i = \langle 0 \,|\, \left[\widehat{Q}_i, \widehat{T}_+ \right] | \, 0 \rangle = \sum_{np} \left(\overline{b}_{np} \psi_{np}^i + b_{np} \varphi_{np}^i \right). \quad (22)$$

It is possible to show that the aforementioned transitions obey the Fermi sum rule,

$$\sum_i \left(|M_{\beta^-}^i|^2 - |M_{\beta^+}^i|^2 \right) = \sum_{np} \left(b_{np}^2 + \overline{b}_{np}^2 \right) = N - Z = 2T_0. \quad (23)$$

4. $\log ft$ values

In a beta transition, ft value is given by

$$ft = \frac{K}{G_V^2 |M_V|^2 + G_A^2 |M_A|^2} \quad (24)$$

where, $K/(\hbar c)^6 = 2\pi^3 \hbar \ln 2/(m_e c^2)^5 = (8120.271 \pm 0.012) \times 10^{-10}$ GeV^{-4}s, G_V and G_A are vector and axial coupling constant, respectively. M_V and M_A are Fermi and Gammow-Teller type matrix elements. The ft product is a degree of forbiddenness of beta decays. t is the half-life of nucleus and, an integral f is the statistical rate function depends on the energy and structure of the transition.
In the present work, for the $J^\pi = 0^+ \to 0^+$, isospin forbidden transitions depend only on Fermi matrix element M_V. The Gammow-Teller matrix elements are defined as $M_A = 0$ and, Eq. (24) becomes:

$$ft = \frac{K}{G_V^2 |M_V|^2}. \quad (25)$$

As seen above, the experimental ft value is related to the vector coupling constant G_V. In electroweak theory, the relationship between the Fermi and vector coupling constant is expressed as

$$G_V = G_F V_{ud}. \quad (26)$$

The Fermi coupling constant G_F is derived from the muon beta decay and its numerical value is [29]:

$$\frac{G_F}{(\hbar c)^3} = (1.16637 \pm 0.00001) \times 10^{-5} \text{GeV}^{-2}.$$

The V_{ud} is an element of CKM mixing matrix which represents the up-down quark mixing. The V_{ud} value is adopted from Ref. [30] as

$$V_{ud} = 0.97418 \pm 0.00026.$$

From the Eq. (25) and Eq. (26), the ft value is found as

$$ft = \frac{6289.55 \pm 2.38}{|M_V|^2}. \quad (27)$$

Unauthenticated
Download Date | 9/26/18 5:43 AM

5. Results and discussions

In this section, the numerical computations for matrix elements, $\log ft$ values and admixtures amplitutes for the isospin forbidden $0^+ \rightarrow 0^+$ transitions have been performed by considering the pairing correlations between nucleons and, including the effective Fermi interaction term in a self-consistent way.

In the computations, the Woods-Saxon potential with the Chepurnov parametrization has been used [28]. The pairing correlation function has been chosen as $C_n = C_p \approx 12/\sqrt{A}$ for open shell nuclei. The computations have been performed for eleven well known isospin forbidden Fermi beta transitions.

In Table 1, the first column represents the isospin forbidden $0^+ \rightarrow 0^+$ Fermi beta transitions between the isobaric analog states. In the second and third columns, the nuclear matrix elements calculations based on the Pyatov method have been tabulated in presence and absence of the pairing interactions, respectively. The matrix elements are calculated from Eq. (21) and Eq. (22). As seen in the Table 1, the matrix elements values with pairing interactions are smaller than without pairing ones. This is an expected result. When the pairing interactions are taken into account, the numbers of transition states increase and, the value of the matrix elements decreases as a result of the strength of the beta transition re-distribution between these new states. In the fourth column the experimental results and, in fifth and sixth columns the theoretical results are shown. For ^{64}Ga, ^{66}Ga and ^{234}Np, when comparing experimental data with theoretical values, our results are better than another. Also this is clearly seen in Table 2.

In Table 1, the seventh and eighth columns represent the admixture amplitudes without and with pairing interactions defined in Eq. (6), respectively. Since T_0 took maximum value of the T , Eq. (6) has became [1, 2]

$$M_V = \alpha\sqrt{2T}. \tag{28}$$

Moreover, the admixture amplitudes are calculated by the expression [1, 2],

$$\alpha = \frac{\langle | V_{CD} | \rangle}{\triangle E}. \tag{29}$$

V_{CD} and $\triangle E$ represent all charge dependent terms in the nuclear Hamiltonian and the magnitude of the energy separation between the final state and the analogue state of the initial state, respectively.

Calculations have been performed by Eq. (28). Since α connected to the M_V as linear, the values of admixture amplitudes with pairing interactions have been smaller than the values of without pairing. In tenth and eleventh column, the admixture amplitude was calculated using Eq. (29). In Refs [4] and [6] V_{CD} was calculated as 46.8 keV and 39.7 keV, respectively. $\triangle E$ = 5.992 MeV was adopted from Ref.[5]. Ref. [2] is the experimental result. Especially for ^{28}Mg, when the our values are compared with the Refs [2], [4] and [6], it is seen that present results are more closer to the experimental one. In the calculations without pairing interactions, the isovector and Coulomb potential are caused to isospin breaking. But, in with pairing calculations, there is also a pairing potential. Hence, the effect of the other terms caused to isospin breaking is decreased by pairing potential. The calculations with pairing interactions are closer to the other theoretical results in the last column.

The calculated $\log ft$ values of isospin forbidden $0^+ \rightarrow 0^+$ Fermi beta transitions are given in Table 2 and Figure 1. For these $\log ft$ values, there are quite small and negligible error bars such as ± 0.00016. Calculations have been performed using the Eq. (27). In Table 2, the experimental results are given in the ninth column for all nuclei, and the relative deviations (RD) of $\left(\left| \log ft^{cal} - \log ft^{\text{expt}} \right| / \log ft^{\text{expt}} \right) \times 100\%$ are shown in the last two columns. As seen from table, both without and with pairing calculations are in good agrement with the experimental results. Relative deviations of calculations with pairing are smaller than the without pairing ones. The calculations with pairing are more closer to the experimental results. Especially, when nucleon numbers of the nuclide increase, this compatibility is further increased. The reason of this situation is that the pairing potential is considered in the calculations with pairing interactions. When the nucleon numbers increase, the effect of isovector potential decreases. But effect of pairing potential is more effective. The relative deviations of with pairing calculations decrease with increasing mass number. For instance; the calculated result for ^{170}Lu is equal to the experimental value. Furthermore, for ^{64}Ga, ^{66}Ga and ^{234}Np our results are more consistent to the experimental ones than the theoretical results given in seventh and eight columns. This theoretical results were obtained from the values in the fifth and sixth columns of the Table 1 used in Eq. (27).

In Figure 1, the calculated $\log ft$ values with and without paring interactions and experimental results are plotted, for comparison. The $\log ft$ values of isospin forbidden $0^+ \rightarrow 0^+$ Fermi beta transitions are given in the same order as in Table 2 *i.e.*, the x-axis numbers correspond to the S. No. in Table 2. As can be seen in figure, the plot without pairing is lower than with pairing and experimental ones. Especially, in ^{54}Co, ^{66}Ga and ^{78}Rb this difference is more pronounced. The calculations with pairing in harmony with the experimental results is clearly seen from the figure.

Unauthenticated
Download Date | 9/26/18 5:43 AM

Table 1. Fermi matrix elements and admixture amplitudes of the isospin forbidden $0^+ \rightarrow 0^+$ Fermi beta decay. In the second and third columns, the nuclear matrix elements calculations based on the Pyatov method are tabulated in presence and absence of the pairing interactions, respectively. In the fourth column the experimental results and, in fifth and sixth columns the theoretical results are shown. The seventh and eighth columns represent the admixture amplitudes without and with pairing interactions found in this study. The remaining columns show the results of the other studies.

Nuclide	$\|M_V\| \times 10^3$					$\alpha \times 10^3$					
	without pairing	with pairing	[1]	[7]	[9]	without pairing	with pairing	[2]	[4]	[6]	[8]
^{28}Mg	8.869	7.230	–	–	–	4.435	3.615	3.400±0.9	7.81	6.63	–
^{42}Sc	37.376	27.141	–	–	–	26.429	19.192	–	–	–	–
^{54}Co	38.470	13.336	–	–	–	27.202	9.431	–	–	–	–
^{60}Mn	39.623	34.498	–	–	–	14.009	12.197	–	–	–	–
^{64}Ga	43.243	38.174	–	–	71.2	21.622	19.087	–	–	–	19.500
^{66}Ga	29.832	11.984	–	–	33.5	12.179	4.893	–	–	–	3.500
^{70}Se	55.767	47.710	–	–	–	39.433	33.737	–	–	–	–
^{78}Rb	19.235	10.218	–	–	–	7.853	4.172	–	–	–	–
^{156}Eu	3.105	1.506	1.02±0.05	1.5	–	0.587	0.285	–	–	–	0.190
^{170}Lu	1.304	1.112	1.03±0.15	1.0	–	0.238	0.203	–	–	–	0.170
^{234}Np	7.090	3.250	4.2±1.6	25.0	–	1.003	0.459	–	–	–	0.600

Table 2. $\log ft$ values of the isospin forbidden $0^+ \rightarrow 0^+$ Fermi beta transitions (All units are seconds). The calculated $\log ft$ values based on Pyatov method are given in third and fourth columns. The fifth - eighth columns represent the results of other studies and the experimental $\log ft$ data are given in ninth column. Relative deviations (RD) of $\left(\left|\log ft^{cal} - \log ft^{\text{expt}}\right| / \log ft^{\text{expt}}\right) \times 100\%$ are shown in the last two columns.

S.No	Nuclide	$\log ft$							RD	
		without pairing	with pairing	[2]	[5]	[7]	[9]	Experiment [31]	without pairing	with pairing
1	^{28}Mg	7.90	8.08	8.13	7.60	–	–	7.96±0.1	0.75	1.51
2	^{42}Sc	6.65	6.93	–	–	–	–	6.78±0.1	1.92	2.21
3	^{54}Co	6.63	7.55	–	–	–	–	7.40±0.1	10.41	2.03
4	^{60}Mn	6.60	6.72	–	–	–	–	6.70±0.1	1.49	0.30
5	^{64}Ga	6.53	6.64	–	–	–	6.09	6.57±0.1	0.61	1.07
6	^{66}Ga	6.85	7.64	–	–	–	6.75	7.89±0.1	13.18	3.17
7	^{70}Se	6.31	6.44	–	–	–	–	6.40±0.1	1.41	0.63
8	^{78}Rb	7.23	7.78	–	–	–	–	7.93±0.1	8.83	1.89
9	^{156}Eu	8.81	9.44	–	–	9.45	–	9.83±0.1	10.38	3.97
10	^{170}Lu	9.57	9.71	–	–	9.80	–	9.71±0.1	1.44	0.00
11	^{234}Np	8.10	8.77	–	–	7.00	–	8.54±0.1	5.15	2.69

6. Conclusions

The isotopic symmetry has been broken by both Coulomb forces and the isovector term in the nuclear shell model Hamiltonian. The isovector terms effect is not natural. Also, it is necessary to compensate its effect in the wave functions and matrix elements. This point has not been emphasized in another similar studies. In the present work, the isospin breaking due to the isovector part of the shell model potential has been separated and, its effect is eliminated by Pyatov's restoration method. After the restoration, the model is free of any adjustable parameters.

The matrix elements, admixture amplitudes and $\log ft$ values of isospin forbidden $0^+ \rightarrow 0^+$ Fermi beta decays are herein reported for the first time. When we compare our results to the existing literature data existed, they are slightly better than others. The effect of pairing correlations between nucleons on the admixture amplitudes has been dominantly seen in the isospin forbidden $0^+ \rightarrow 0^+$ beta transitions. The calculations of admixture amplitudes with pairing interactions of the transitions are closer to the other studies in the literature. As seen from the calculated

Unauthenticated
Download Date | 9/26/18 5:43 AM

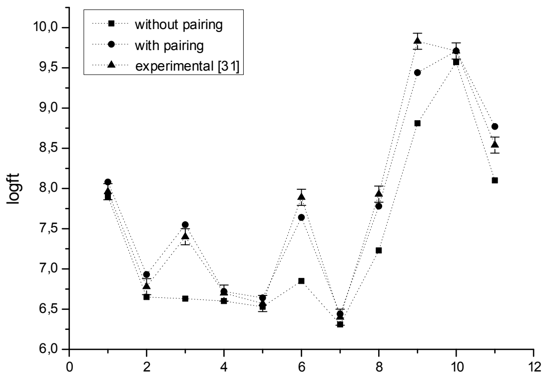

Figure 1. The calculated and experimental $\log ft$ values of isospin forbidden $0^+ \to 0^+$ Fermi beta transitions (the x-axis numbers correspond to the S. No. in Table 2). For the calculated $\log ft$ values, there are quite small and negligible error bars such as ± 0.00016.

values of $\log ft$, both without and with pairing calculations are in good agreement with experimental results. This harmony due to the effect of the pairing potential is seen well in the calculations with pairing. Especially, for ^{28}Mg in the admixture amplitude, for ^{64}Ga, ^{66}Ga and ^{234}Np in the $\log ft$ values, the presented results are excellent agreement with all the experimental findings when comparing another theoretical data.

Acknowledgements

The author thanks to Prof. Dr. Cevat Selam and Prof. Dr. Murat Gerçeklioğlu from Mus Alparslan and Ege University, for their contributions, respectively. Also, the author is gratefully acknowledges the assistance of Assoc. Prof. Dr. Cemal Parlak and Assist. Prof. Dr. Hüseyin Ertik from Dumlupınar and Akdeniz University, respectively, in drafting and editing the manuscript. The author would like to thank the editor and anonymous reviewers for their useful comments to improve the manuscript.

References

[1] R. J. Blin-Stoyle, Isospin in Nuclear Physics, Edited by D H Wilkinson (North-Holland, Amsterdam, 1969)
[2] P. A. Dickey, J. E. Bussoletti, E. G. Adelberger, Nucl. Phys. A 303, 442 (1978)
[3] S. Nakamura, K. Muto, T. Oda, Phys. Lett. B 297, 14 (1992)
[4] S. Nakamura, K. Muto, T. Oda, Phys. Lett. B 311, 15 (1993)
[5] S. Nakamura, K. Muto, T. Oda, Nucl. Phys. A 575, 1 (1994)
[6] S. Nakamura, K. Muto, M. Oka, S. Takeuchi, T. Oda, Phys. Rev. Lett. 76, 881 (1996)
[7] P. G. Hansen, H. L. Nielsen, K. Wilsky, J. G. Cuninghame, Phys. Lett. B 24, 95 (1967)
[8] G. F. Bertsch, A. Mekjian, Ann. Rev. Nucl. Sci. 22, 25 (1972)
[9] E. L. Saw, C. T. Yap, Il Nuovo Cimento 39, 643 (1977)
[10] C. T. Yap and E. L. Saw, Acta Phys. Pol. B 16, 75 (1985)
[11] C. T. Yap and E. L. Saw, Phys. Rev. C 38, 2332 (1988)
[12] M. Yasue et al., Phys. Rev. C 43, 510 (1991)
[13] N. I. Pyatov, D. I. Salamov, Nukleonika 22, 127 (1977)
[14] N. I. Pyatov et al., Sov. J. Nucl. Phys. 29, 10 (1979)
[15] T. Babacan et al., J. Phys. G 30, 759 (2004)
[16] A. Küçükbursa, D. I. Salamov, T. Babacan, H. A. Aygor, Pramana 63, 947 (2004)
[17] T. Babacan, D. I. Salamov, A. Küçükbursa, Phys. Rev. C 71, 037303 (2005)
[18] D. I. Salamov et al., Pramana 66, 1105 (2006)
[19] D. I. Salamov, S. Ünlü, and N. Çakmak, Pramana 69, 369 (2007)
[20] T. Babacan, D. I. Salamov, A. Küçükbursa, Nucl. Phys. A 788, 279 (2007)
[21] N. Çakmak, S. Ünlü, C. Selam, Phys. Atom. Nucl. 75, 8 (2012)
[22] N. Çakmak, S. Ünlü, C. Selam, Pramana 75, 649 (2010)
[23] N. Çakmak, S. Ünlü, K. Manisa, C. Selam, Pramana 74, 541 (2010)
[24] A. E. Çalık, M. Gerçeklioğlu, D.I. Salamov, Z. Naturforsch. 64 A, 865 (2009)
[25] A. E. Çalık, M. Gerçeklioğlu, C. Selam, Phys. Atom. Nucl. 76, 549 (2013)
[26] A. E. Çalık, M. Gerçeklioğlu, D. I. Salamov, Sci. China- Phys. Mech. Astron. 56, 718 (2013)
[27] A. E. Çalık, M. Gerçeklioğlu, D. I. Salamov, Pramana 79, 417 (2012)
[28] V. G. Soloviev, Theory of Complex Nuclei, (Pergamon, New York, 1976)
[29] J. C. Hardy, I. S. Towner, Phys. Rev. C 79, 055502 (2009)
[30] I. S. Towner, J. C. Hardy, Phys. Rev. C 77, 025501 (2008)
[31] B. Singh, J. L. Rodrigues, S. S. M. Wong, J. K. Tuli, Nucl. Data Sheets 84, 487 (1998)

Unauthenticated
Download Date | 9/26/18 5:43 AM