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A Mathematical Study of Accessibility and Cohesion Degree in a High-Speed Rail Station Connected to an Urban Bus Transport Network

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Abstract: In the last twenty years, the implementation of High-Speed Rail (HSR) has been one of the major strategies for territorial structuring used by various countries. This model has enhanced the development of countries such as France, Spain, Germany and Japan. At present, the United States and China are also starting to implement this model. Nevertheless, the lack of social and economic profitability of several networks is being increasingly analysed. Many networks located in particular regions serve populations that are not large enough to recover the initial investment. For this reason, it is necessary to evaluate the population served by this transport mode, beyond the number of users. In this sense, it is essential to identify the deficiencies and potentials of implementing a network linked to other secondary networks in a specific territory which can compensate for the so-called tunnel effect. This article proposes to apply a mathematical approach based on graph theory to measure the Degree Accessibility Node (DAN) in a constrained Geographic Information System (GIS) model. Hence, it would be possible to compare regions, especially medium-sized cities, where the implementation of HSR could represent a qualitative leap due to incorporation into large transport networks. The DAN function uses static and dynamic studies to evaluate the level of connection of stations to secondary transport networks—local public transport in this case. Thus, the impact of high-speed trains could be spread to greater territorial and population ranges. Four cases have been studied, two in Germany (one of them, Fulda, is analysed in depth throughout this article) and two in Spain. These two countries were selected since HSR was implemented in the same relative period of time, in comparison with other European countries. Results show relevant differences, suggesting a review of inappropriate policies of transport integration in a city that could weaken the expansion of the positive effects of HSR integration.

Keywords: High-speed rail; dynamic indexes; accessibility; centrality; graph theory

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1 Introduction

Nowadays, the expansion of High-Speed Rail (HSR) networks together with interconnected stops is raising the challenge of integrating the high-speed railway stations within urban dynamics [1]. The newly refurbished or built stations that are to host HSR are conceived as an instrument to decrease travel times dramatically. Consequently, HSR creates great expectations in terms of socio-economic and territorial revitalisation [2]. This fact leads to certain changes in the city, which are enhanced by public administrations in the framework of the pursuit of urban renewal [2].

The present study aims to analyse and compare the accessibility levels of different stations through static and dynamic mathematical procedures. It is worth noting that these methodologies have been traditionally applied in other fields such as road networks [3, 4]. The different local mobility policies analysed in the different cases are derived from the study of railway stations integration at a local level, depending on their location within the public transport network. In this sense, the problems that are normally related to the practical integration of the node do not only show the importance of planning and implementing strategies, but also the necessity of joining urban development and the appropriate implementation of this infrastructure.
Moreover, the connection of stations with other secondary networks contributes to spreading the positive effects of HSR to a greater population and territorial range [5]. Consequently, situations where the station is located in a place lacking possible connections, omitting the need to link the station to other secondary networks, contribute to increasing the unfavorable effects of a transport mode that crosses large distances without stopping in the aforementioned places [6]. For this reason, it is necessary to study and analyse the HSR station together with the different secondary networks in terms of cohesion and accessibility of the different nodes within the network. In this work, the mathematical approach is to compute the Degree Accessibility Node (DAN) for the study of four European networks belonging to the same time period and whose history is long enough to be studied. The DAN function evaluates the accessibility of high-speed railway stations through two different methodologies: on the one hand, a static study to obtain the indexes describing the node’s level of cohesion compared to the rest of the network, and on the other hand, a dynamic study that measures the connection of the stop evaluated compared to other stops of the transport network, an urban bus transport network in this case.

Finally, it is worth noting that the networks under study have to be comparable in order to provide conclusions about the development of the cities. Thus, different features were considered when the cities of Ulm-Neu Ulm (conurbation) and Fulda in Germany and Lleida and Toledo in Spain where selected among a wide range of possibilities. They are all medium-sized cities with comparable populations that range between 50 000 and 200 000 inhabitants, with the train station located in the inner city. In these networks, the implications of HSR are more objective since they are not affected by the impact of other transport modes such as air transport, and the implementation of HSR has a dramatic impact on their accessibility level globally [7]. Furthermore, the cities share structural similarities, where HSR has been implemented for more than twenty years and HSR was commissioned at a similar time—1991 in Germany and 1992 in Spain. Indeed, these factors are necessary in order for transport system dissimilarities to become observable. Moreover, these networks have a historical urban planning tradition administratively implemented in the fields of physical and spatial planning, conceived as a link between economic and social planning. In the cases under study, the countries have over a century of experience in urban planning, which is a mandatory requirement for the comparison, according to the European administration. This fact automatically rejects the possibility of considering other European countries such as Portugal or Italy within the analysis. Finally, Germany and Spain share similarities in terms of territorial organisation into states—16 federal states in the case of Germany, 17 autonomous communities in the case of Spain. This also enables comparison between the different administrative frameworks which make decisions about city development.

1.1 The integration of high-speed rail in Europe: a unique objective in a mixed picture

The implantation of HSR in Europe was considered as an attempt to promote railway as a transport system and articulate the territories of the European Union (EU). The initial idea was to offer a competitive alternative to air travel. Instead of that, it has become a new concept that emphasises the improvement of connectivity between territories and cities. That continental network may contribute to the territorial structuring process necessary to achieve a more uniform and cohesive EU territory [8].

The main benefits of HSR are related to its high speed, large capacity for passengers, punctuality, security, high efficiency and comfort. For these reasons, HSR plays a key role in the transport system between different European regions. One of the most direct effects of its implementation is the improvement of accessibility [9, 10]. HSR represents a key element of territorial structuring, enabling the development of new relationships between remote countries, considerably shortening the distance between them [11]. It promotes mobility and interaction in economic and social aspects [12]. In this sense, different studies have reported variations of accessibility after the implementation of HSR at the European level [13–15].

Nevertheless, an evaluation prior to the multianual renewal of the programme established in the Trans-European Transport Network for the period between 2007–2013 was used to elaborate the report requested by the European Commission. This report emphasised the necessity of continuing with the cohesion plan, establishing a cost-benefit ratio of €1.6 for each euro invested. It also warned that deficiencies in the execution of projects would imply a loss of efficiency in terms of execution and financing. In addition to the budgetary constraints, there are two big areas where the evaluation identified deficiencies in the preparation and implementation of the project. These deficiencies are along two critical lines:
The immaturity of the projects, lacking technical details, attention to the environment, public consultation and appropriate risk assessment.

The poor technical-administrative coordination and the lack of experience of the beneficiaries with requirements established in EU-funded programmes.

1.2 Evaluating accessibility by modelling the real object: the local public transport network

The implementation of HSR and the investment made in Europe throughout the last twenty years have made it possible to link a large number of cities [6]. HSR’s competitiveness is based on the transport marketplace, service quality and access time to the main centres of activity [16]. Nevertheless, due to the features of the mode (it needs to travel long distances to reach the required speed [17]), if the access node to the territory—the station—is not duly linked to other secondary networks, its benefits will exclusively reach the surroundings thereof, exacerbating the effects commonly known as the tunnel effect [18] which consists of communicating those cores served by HSR only while it creates serious imbalances in those locations that are crossed but not served by it. This argument makes it necessary to evaluate the level of connection of stations with other secondary transport networks. In this case, the DAN function makes it possible to evaluate the accessibility level of the station’s node by applying it to the local public bus network, because only a smooth and efficient connection to the city and any potential cores lends continuity to the positive effects of a transport mode based on travel speed. Otherwise, the time saved thanks to the high speed reached between those cities served by HSR is diluted when adding the time necessary to gain access to the end of the route. In this case, the mode’s potential users will no longer think the generally higher cost of HSR compared to other transport modes is justified, thus making HSR less competitive.

On the other hand, in those countries with lower population density, the development of these secondary networks is even more necessary since they are essential to serve a greater population range and to make network implementation and maintenance costs sustainable. Finally, guaranteeing accessibility through public transport is increasingly important due to the fluctuating price of fuel [19] and due to congestion problems [20]. To evaluate accessibility, we usually find studies that are developed from the field of transport, economy or social policy planning [21]. Many of them introduce the accessibility conferred to the territory depending on the connection offered by public transport networks: in this sense, we would like to emphasise how Burns and Inglis [22] evaluate access cost to commercial areas depending on the transport means chosen; Yigitcanlar et al. [23] suggest a LUP-TAI model (Land Use & Public Transport Accessibility Index), comparing access to different points depending on whether the route is on foot or by bus, and Silva [24] develops the SAL system (Structural Accessibility Layer) which measures the population affected by accessibility using private, public and non-motorised transport.

The DAN function implies the evaluation of accessibility of a station node in comparison with the local public transport network through modelling of the real object on the basis of graph theory. By using a method that has already been used by other network scholars such as Garrison [3], each point can perfectly represent a city, a station, a stop or a computer belonging to a network (or any set linked to objects), and the lines that connect them can represent roads, railways or wires (namely, the physical element that links said objects). A graph with these features, integrated by nodes and lines, enable the modelling of a reality that can be a network of roads, of transport made up of different lines, the Internet or industrial processes. Based on the foregoing, Kansky [25] was able to study transport networks by abstracting them through mathematical graph theory [26].

In our case, the graph is made up of trajectories that link any pair of nodes linked by any local transport network, evaluating two perspectives: the static one, applied by authors such as Seguí [27] to motorised road transport to compare the cohesion level of the different nodes (populations in his case), and the dynamic one, to study the accessibility of the different modes more specifically. Nevertheless, despite the fact that the latter has traditionally been applied to the study of private road transport combined with the static perspective, its prior application to a public transport network, as detailed below, has not been found.

We next outline the evaluation process of these parameters in more details through application to one of the four cases: Fulda, and deal with the results obtained in Ulm-Neu Ulm, Toledo and Lleida subsequently. We close with the required comparative valuation under final reflection in the form of conclusions, possible approaches in the future and improvement of the situation under study.
2 Methodology

To find out the accessibility and cohesion degree in a transport network, it is crucial to calculate some features that provide an objective point of view about the existing level of interconnectivity and accessibility among the different bus stops. To achieve this objective, the study of corresponding cartography together with the application of different georeferencing techniques, is essential in order to build up the network model. Furthermore, it is necessary to map the network into a set of vertexes and edges which represent the connectivity and distance between the different bus stops within the model. It is important to remark that in this study, the network is analyzed from two different points of view: static and dynamic. On the one hand, the numbers of vertexes and edges are computed in the static study to quantify the cohesion degree in the network. On the other hand, graph theory is used to quantify other aspects like connectivity and costs among different nodes in the dynamic study. In the subsequent sections both methodologies are described.

2.1 Static network study

Firstly, the generated graph was studied from a static point of view. This allowed us to analyze the direct communication between the whole set of points of interest without taking into consideration the distances between the diverse vertexes. This perspective reports the cohesion degree within the model where the connectivity between vertexes grows as the number of direct interconnections increases, i.e., the more direct connections between vertexes in the model, the higher the cohesion degree. Therefore, considering the graph under study where \( v \) and \( e \) correspond to the number of vertexes and the total number of edges respectively, the maximum connection degree was computed as shown in Eq. (1).

\[
C_d = \frac{v(v - 1)}{2}
\]

(1)

Moreover, the cyclomatic number \( (\mu) \) which represents the number of circuits within the network was computed as shown in Eq. (3). In this regard, the higher the \( \mu \), the higher the cohesion level, with its minimum value equal to zero and the maximum equal to \( (v - 1)(v - 2)/2 \).

\[
\mu = e - (v - 1)
\]

(3)

In addition, the corrected cyclomatic numbers (Kansky’s indexes) were also computed. First, Kansky’s \( \alpha \) index was calculated by computing the relationship between the number of existing circuits and the maximum possible circuits within the graph. This index is expressed mathematically as shown in Eq. (4).

\[
\alpha = \frac{2\mu}{(v - 1)(v - 2)}
\]

(4)

Second, Kansky’s \( \beta \) index is defined as the rate between the number of edges and the number of vertexes, such that the higher the number of edges, the higher the connectivity among vertexes. The possible values for this index range from zero to infinity based on the assumption of the number of connections between vertexes increasing faster than the number of vertexes when a network changes its structure and becomes more cohesive. Its mathematical expression and maximum value for a given number of vertexes can be observed in Eq. (5) and Eq. (6) respectively.

\[
\beta = \frac{e}{v}
\]

(5)

\[
\max(\beta) = \frac{v - 1}{2}
\]

(6)

Third, Kansky’s \( \gamma \) index formulates the relationship between the maximum number of edges in the graph and the existing ones. This index ranges from zero to unity, such that \( \gamma \) increases when the model cohesion grows. It is also frequently expressed as a percentage. This index can be mathematically defined as shown in Eq. (7).

\[
\gamma = \frac{2e}{v(v - 1)}
\]

(7)

Finally, the Zagozdzon \( (G_p) \) index was computed, which indicates the existing development degree in the graph by reporting the missing number of edges per vertex needed to fill the graph completely. Its formula is shown in Eq. (8).

\[
G_P = \frac{v^2 - v - e}{v}
\]

(8)

2.2 Dynamic network study

The other major aspect of this work lies in the analysis of the graph from a dynamic point of view. In this regard, this
methodology allows enhancement of estimations regarding the accessibility and cohesion of the different vertexes in the graph (in relation to the static study). The main reason for this lies in the fact that dynamic study of the graph considers not only the number of vertexes and edges \((u, e)\), but also the cost value for each direct connection between the vertex or node \(i\) and vertex or node \(j\) within the graph. Considering that accessibility is defined as the facility of access from any point within the network to another, it seems appropriate to use the distance \((d)\), measured in meters, to quantify the cost between two connected vertexes. Thus, the cost matrix for each connected vertex on the graph was calculated as shown in Eq. (9).

\[
C(i, j) = \begin{cases} d(i, j) & \text{if } (i, j), \text{ are linked} \\ 0 & \text{if } i = j \\ \infty & \text{if } (i, j) \text{ are not linked} \end{cases} \quad (9)
\]

Moreover, the assessment of whole dynamic markers are based on computing the shortest path of the vertexes in the graph, i.e., finding a path between two nodes in a graph such that the sum of the costs of the edges forming the path is minimized. To resolve the shortest path problem, there exist different algorithms (Dijkstra, Bellman-Ford, \(A^*\), Floyd-Warshall, etc.) which provide the minimum cost matrix defined by \(\Theta \in \mathbb{R}^{n \times n}\), where each number defined in the matrix by the pair of indexes \((i, j)\) contains the shortest path between the node origin \(i\) and the node destiny \(j\).

Given the network proportions regarding the number of vertexes involved, the Floyd-Warshall algorithm was considered optimal to calculate \(\Theta\) because of its simplicity and easy programming. This algorithm uses the matrix \(C\) computed previously as its input argument and also uses it to initialize the output matrix \(\Theta\) such that \(\Theta = C\). Next, in each interaction, the matrix \(\Theta\) is recalculated so it contains costs of paths among all pairs of nodes \((i, j)\), using progressively enlarging sets of intermediate nodes defined by the variable \(k\). Thus, the matrix \(\Theta\) is rewritten in the first iteration, containing cost values among all the pairs of nodes, using exactly only one intermediate predefined node \((k = 1)\). In the second iteration, the matrix \(\Theta\) is redefined again with the resulting costs using two predefined intermediate nodes \((k = 2)\). Finally, the algorithm finishes when the number of intermediate predefined nodes is equal to the total number of vertexes in the graph \((k = \nu)\). This procedure can be summarized though the recursive equation shown in Eq. (10).

\[
\Theta_{i,j}^k := \min(\Theta_{i,j}^{k-1}, \Theta_{i,k}^{k-1} + \Theta_{k,j}^{k-1}) \quad (10)
\]

Once the matrix \(\Theta\) is computed, several dynamic features found in the literature can be used to estimate the accessibility and cohesion level in the network. In this regard, the graph’s diameter corresponds to the longest distance from one vertex to another anywhere in the shortest path matrix. Moreover, the Shimbel index \((\Psi)\), or the index of actual accessibility, is defined as the summation of costs between each vertex and the rest of the vertexes within the shortest path matrix, and can be expressed mathematically as shown in Eq. (11). This index is useful for finding out the core node in the network such that the lowest \(\Psi\) values correspond with the most accessible vertexes and the highest \(\Psi\) values correspond with the least accessible nodes.

\[
\Psi_i = \sum_{j=1}^{\nu} \Theta_{i,j}, \quad i = 1, \ldots, \nu \quad (11)
\]

Given that the \(\Psi\) calculation depends on the number of vertexes \(\nu\) in the network, comparison with other networks is hardly possible. Consequently, the Shimbel index is usually normalized for the number of nodes in the network, resulting in the relative accessibility index defined mathematically as shown in Eq. (12). It is important to remark that the relative accessibility closest to zero corresponds to the core node.

\[
\Omega_i = \frac{\Psi_i - \min(\Psi)}{\max(\Psi) - \min(\Psi)} \times 100 \quad (12)
\]

Finally, to detect nodes with a low and high accessibility, mean and standard deviation are usually applied to these indexes, such that indexes below the mean minus once or twice the standard deviation have a high accessibility and those above the mean plus once or twice the standard deviation are considered as poorly accessible.

In order to calculate both static and dynamic indexes, mathematical software (Matlab) was used for programming the DAN function. Given the dimension of the transport networks (249 rows and 249 columns), automation of this process is required. A fragment of the DAN function is shown in Listing 1.

Listing 1: A fragment of DAN function programmed with Matlab

```matlab
%Global variables;
C = Cost_Matrix();
max1 = 0;
maxn = 0;
node1 = 0;
node2 = 0;
noden1 = 0;
table_acceskm = zeros(length(C), 6);
for i = 1:length(C)
    for j = 1:length(C)
        if C(i, j) == inf
            C(i, j) = 0; % filtering ...
```
A Mathematical Study of Accessibility and Cohesion Degree

end
else max diameter in the network
if C(i, j) > max1
max1 = C(i, j);
nodo1 = i;
nodo2 = j;
end
% max diameter for each node
if C(i, j) > maxn
maxn = C(i, j);
end
end
%Accessibility indexes
table_acceskm(i, 1) = i;
% vertexes
table_acceskm(i, 2) = \sum(C(i,:));
% Shimbel index
table_acceskm(i, 3) = maxn;
% Diameter
table_acceskm(i, 4) = \sum(C(i,:)) / N;
% Average length
maxn=0;
nodon1=0;
end

% Relative accessibility (Omega %100)
M = \max(table_acceskm(:,2));
% max Shimbel index;
m = \min(table_acceskm(:,2));
% min Shimbel index;

for i = 1:length(C)
	table_acceskm(i, 5) = ...
	((table_acceskm(i, 2) - m) / ...
	(M - m)) * 100;
end

3 Static and dynamic analysis of a German city: Fulda

In order to apply the methodology described in Section 2 to different networks, cities under study have to be structurally analogous. This prerequisite makes possible extrapolation of the results and comparison between the different locations. In this regard, four singular cases of intervention in urban projects around European stations in medium-size cities with no more than half a million population were chosen for this analysis. More concretely, Fulda (Germany), Ulm-Neu Ulm (Germany), Lleida (Spain) and Toledo (Spain) were included in the study, given that these cities share structural similarities and adopted very different solutions in the integration of HSR within the urban area. Given that the effects provoked by the integration of HSR is more considerable in medium-sized locations, some authors have reported the suitability of these cities as urban impact indicators [28].

To analyze the cohesion and accessibility of the main attractions through the urban transport network and its relationship with the HSR, it is necessary to carry out a cartographic representation of the local public transport network at an urban level with respect to the surroundings of the HSR stations. This process was accomplished by using a Geographic Information System (GIS), which allowed us to build up the graph of Fulda, consisting of vertexes and edges formed by the public transport network and its interconnectivity, respectively. Moreover, local agents were interviewed to locate the city’s main attractions as can be appreciated in Fig. 1. Thus, the objective is to embed points corresponding to bus stops into the graph to perform a further study of accessibility and centrality regarding the HSR station and the city’s main attractions, as well as the assessment of the relationship with elements of the mobility policy such as public car parks, taxi ranks or other stations. In the case under study, the HSR station is located at city center and a complex bus stop network, taxi rank and public car parks can be found within a 1-kilometer radius (see Fig. 2), thus reinforcing the hypothesis of their relationship.

3.1 Generating the connection matrix

In the case of Fulda, the public transport network is composed exclusively of the public bus network, consisting of nine urban bus lines and 312 bus stops which cover a length of 88.67 km within an area of 60.01 km². Eight out of nine bus lines have a bus stop at the HSR station. Only bus line number 4 has no match with this station. Given that many of the bus stops coincide, the total number of vertexes considered in the graph was 249, such that two coincident nodes were labeled with the same identification. Moreover, the number of total interconnections between them was 292. It is noteworthy that distances were calculated as the minimum path in meters necessary to go from i to j if they were directly connected, 0 in cases where

Static indexes are useful to assess the cohesion degree of different networks, as well as to track the network development degree through time. Therefore, static features defined in Section 2 were applied over the whole graph of Fulda and results are shown in Table 2. The geographical global area under study consists of 249 vertexes and 292 edges, which represents only 0.95% of the total number of possible direct connections among the network, defined by the maximum connection degree index (30876). Consequently, results obtained through Prihar’s index indicated a low cohesion degree with a value of 105.74, the maximum and minimum cohesion levels being 1 and 124.5 respectively. Similarly, the $\mu$ index reported a low number of pos-
Figure 2: Elements of mobility policy in Fulda. P: Public car park; Tx: Taxi rank.

Table 1: Connection matrix.

<table>
<thead>
<tr>
<th>i / j</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>162.10</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
</tr>
<tr>
<td>2</td>
<td>162.10</td>
<td>0</td>
<td>272.60</td>
<td>∞</td>
<td>∞</td>
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<td>∞</td>
<td>∞</td>
</tr>
<tr>
<td>3</td>
<td>∞</td>
<td>∞</td>
<td>0</td>
<td>263.50</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
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</tr>
<tr>
<td>4</td>
<td>∞</td>
<td>∞</td>
<td>263.50</td>
<td>0</td>
<td>179.90</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
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<tr>
<td>5</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
<td>179.90</td>
<td>0</td>
<td>392.70</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
</tr>
<tr>
<td>6</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
<td>0</td>
<td>∞</td>
<td>351.05</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
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<td>∞</td>
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<td>180.70</td>
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<td>8</td>
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<td>∞</td>
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<td>∞</td>
<td>∞</td>
<td>180.70</td>
<td>0</td>
<td>185.10</td>
<td>∞</td>
</tr>
<tr>
<td>9</td>
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<td>∞</td>
<td>∞</td>
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<td>∞</td>
<td>185.10</td>
<td>0</td>
<td>176.20</td>
</tr>
<tr>
<td>10</td>
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<td>∞</td>
<td>∞</td>
<td>∞</td>
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<td>∞</td>
<td>176.20</td>
<td>0</td>
</tr>
</tbody>
</table>
Table 2: Static indexes applied to Fulda’s graph.

<table>
<thead>
<tr>
<th>Index</th>
<th>Global</th>
<th>Area1</th>
<th>Area2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of vertexes</td>
<td>249</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>Number of edges</td>
<td>292</td>
<td>35</td>
<td>27</td>
</tr>
<tr>
<td>Maximum connection degree</td>
<td>30876</td>
<td>300</td>
<td>300</td>
</tr>
<tr>
<td>Prihar’s index / interval</td>
<td>105.74 / [1,124.5]</td>
<td>8.57 / [1, 14.5]</td>
<td>11.11 / [1, 14.5]</td>
</tr>
<tr>
<td>μ index / interval</td>
<td>44 / [0, 30628]</td>
<td>11 / [0, 276]</td>
<td>3 / [0, 276]</td>
</tr>
<tr>
<td>Kansky’s α index</td>
<td>0.14%</td>
<td>3.98%</td>
<td>1.08%</td>
</tr>
<tr>
<td>Kansky’s β index / interval</td>
<td>1.17 / [0, 124]</td>
<td>1.4 / [0, 14]</td>
<td>1.08 / [0, 14]</td>
</tr>
<tr>
<td>Kansky’s γ index / interval</td>
<td>0.0095 / [0, 1]</td>
<td>0.12 / [0, 1]</td>
<td>0.09 / [0, 1]</td>
</tr>
<tr>
<td>$G_p$ index</td>
<td>122.83</td>
<td>10.60</td>
<td>10.92</td>
</tr>
</tbody>
</table>

Although a low cohesion degree was observed considering the entire network, the study of the cohesion degree from different areas within the network is more interesting from a city’s development point of view. Therefore, two geographical areas corresponding with the surroundings of the HSR station, and a peripheral industrial area were assessed. To facilitate this comparison, each area was defined by a cluster formed from the nearest 25 nodes around for each zone, as can be appreciated in Fig. 4. It is important to remark that the connection matrix was recalculated for each area under study, and only direct paths among the involved nodes were considered. Then, the number of edges was taken into account and static markers were applied over both areas. The results of this comparison are shown in Table 2, Columns 2 and 3. Thus, while the number of direct connections in Area 1 represents 11.67% of its specific network, Area 2 reported only 9% of the possible direct connections within its specific network. Consequently, Prihar’s index showed better cohesion degree for Area 1 than Area 2, achieving values of 8.57 against 11.11, where 14.5 represents the worst possible cohesion. Similarly, the number of possible circuits within the graph was higher in Area 1 than in Area 2. Regarding Kansky’s indexes, the three parameters showed the same trend, showing that Area 1 has better cohesion than Area 2. More concretely, Kansky’s α index showed that the number of completed circuits within Area 1 was more than three times higher than in Area 2. Also, Kansky’s β index reported a better relationship between number of edges and vertexes in Area 1 than Area 2. Furthermore, regarding the existing and maximum number of edges in the network, Area 1 presented much better cohesion, achieving 12% while Area 2 presented a 9% cohesion. Finally, the graph’s development index ($G_p$) reported that Area 1 needs a slightly lesser number of edges than Area 2 in order to complete the graph.

3.3 Dynamic study of the Fulda network: accessibility and centrality

To carry out the dynamic study regarding Fulda’s graph, the Floyd algorithm was applied over the connection matrix and the first ten nodes are shown in Table 3. Observing the results, it is noteworthy that all the nodes are connected between them, i.e., it is possible to reach any node from any other within the network. Afterwards, the Shimbel index ($Ψ$) and the normalized Shimbel index ($Ω$) were computed, together with the network diameter and average length. The results for the most interesting nodes, given their location and accessibility, can be observed in Table 4. Thus, the highest accessibility was for node 92, which corresponds to the historic quarter surroundings near the Council area. In contrast, the less accessible nodes can be found outside the urban core, next to the industrial zone. Regarding the HSR station, it is the fourth most accessible node, with $Ω$ equal to 1.89, thus achieving a high accessibility degree as shown in Table 4.
Figure 3: Representation of the Fulda’s network of local public buses by means of a graph.

Table 3: Minimum paths computed throughout Floyd algorithm.

<table>
<thead>
<tr>
<th>i / j</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>162.10</td>
<td>434.70</td>
<td>698.20</td>
<td>878.10</td>
<td>1270.80</td>
<td>1621.85</td>
<td>1802.55</td>
<td>1987.65</td>
<td>2163.85</td>
</tr>
<tr>
<td>2</td>
<td>162.10</td>
<td>0</td>
<td>272.60</td>
<td>536.10</td>
<td>716</td>
<td>1108.70</td>
<td>1459.75</td>
<td>1640.45</td>
<td>1825.55</td>
<td>2001.75</td>
</tr>
<tr>
<td>3</td>
<td>712.43</td>
<td>550.33</td>
<td>0</td>
<td>263.50</td>
<td>443.40</td>
<td>836.10</td>
<td>1187.15</td>
<td>1367.85</td>
<td>1552.95</td>
<td>1729.15</td>
</tr>
<tr>
<td>4</td>
<td>975.93</td>
<td>813.83</td>
<td>263.50</td>
<td>0</td>
<td>179.90</td>
<td>572.60</td>
<td>923.65</td>
<td>1104.35</td>
<td>1289.45</td>
<td>1465.65</td>
</tr>
<tr>
<td>5</td>
<td>1155.83</td>
<td>993.73</td>
<td>443.40</td>
<td>179.90</td>
<td>0</td>
<td>392.70</td>
<td>743.75</td>
<td>924.45</td>
<td>1109.55</td>
<td>1285.75</td>
</tr>
<tr>
<td>6</td>
<td>4319.38</td>
<td>4157.28</td>
<td>3606.95</td>
<td>3343.50</td>
<td>3163.55</td>
<td>0</td>
<td>351.05</td>
<td>531.75</td>
<td>716.85</td>
<td>893.05</td>
</tr>
<tr>
<td>7</td>
<td>3968.33</td>
<td>3806.23</td>
<td>3255.90</td>
<td>2992.40</td>
<td>2812.50</td>
<td>3205.20</td>
<td>0</td>
<td>180.70</td>
<td>365.80</td>
<td>542.00</td>
</tr>
<tr>
<td>8</td>
<td>3787.63</td>
<td>3625.53</td>
<td>3075.20</td>
<td>2811.70</td>
<td>2631.80</td>
<td>3024.50</td>
<td>180.70</td>
<td>0</td>
<td>185.10</td>
<td>361.30</td>
</tr>
<tr>
<td>9</td>
<td>3602.53</td>
<td>3440.43</td>
<td>2890.10</td>
<td>2626.60</td>
<td>2446.70</td>
<td>2839.40</td>
<td>365.80</td>
<td>185.10</td>
<td>0</td>
<td>176.20</td>
</tr>
<tr>
<td>10</td>
<td>3433.63</td>
<td>3271.53</td>
<td>2721.20</td>
<td>2457.70</td>
<td>2277.80</td>
<td>2670.50</td>
<td>542</td>
<td>361.30</td>
<td>176.20</td>
<td>0</td>
</tr>
</tbody>
</table>
Among the most accessible bus stops can be found those belonging to the historic Baroque center and public services (nodes 12, 90, 92, and 94), the HSR central station (node 11), the university (nodes 53, 123 and 124), the administrative center (node 50 and 52), the hospital (node 47) and the congressional palace (node 185). It is noteworthy that these points of interest were identified by the local agents as the most interesting locations within the city. Indeed, considering locations with high accessibility as those nodes with $\Omega$ below the mean minus once the standard deviation, i.e., those vertexes with $\Omega \leq 14.95$, all the aforementioned nodes lie in that range. On the contrary, if nodes with $\Omega$ above the mean plus once the standard deviation are considered as nodes with low accessibility, i.e., vertexes with $\Omega \geq 59.52$, we find locations corresponding with nodes away from the urban center, usually located in industrial areas. Thus, nodes 57, 58, 72, 73 and 135 are the most marginal vertexes in terms of accessibility. In this regard, the two most relevant industrial nodes (node 77 and 84) achieved $\Omega$ values equal to 62.52 and 37.85 respectively, showing a low accessibility within the network.

3.4 Comparison of Fulda with others European cities

In Table 5 can be observed a dynamic comparison between German and Spanish cities regarding their HSR stations and characteristics. More concretely, the Spanish stations are located in Toledo and Lleida, and the German stations are located in Fulda and Ulm-Neu Ulm. These stations were chosen after a preliminary study considering their urban characteristics and population, where all of them are categorized as medium-sized cities with not more that half a million in population.
Table 4: Dynamic indexes applied to Fulda’s graph.

<table>
<thead>
<tr>
<th>Location</th>
<th>Node Id</th>
<th>Diameter (m)</th>
<th>Average length (m)</th>
<th>Ψ</th>
<th>Ω</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hospital</td>
<td>5</td>
<td>7618.45</td>
<td>3930.22</td>
<td>978624.17</td>
<td>27.20</td>
</tr>
<tr>
<td>Historic Quarter</td>
<td>10</td>
<td>6615.60</td>
<td>3077.66</td>
<td>766338.49</td>
<td>7.62</td>
</tr>
<tr>
<td>HSR Central Station</td>
<td>11</td>
<td>6249.30</td>
<td>2828.37</td>
<td>704264.79</td>
<td>1.89</td>
</tr>
<tr>
<td>Historic Quarter</td>
<td>12</td>
<td>5801.10</td>
<td>2811.70</td>
<td>715248.67</td>
<td>2.90</td>
</tr>
<tr>
<td>Hospital</td>
<td>47</td>
<td>7149.50</td>
<td>3327.20</td>
<td>828473.23</td>
<td>13.35</td>
</tr>
<tr>
<td>Public Services</td>
<td>50</td>
<td>6363.90</td>
<td>2915.40</td>
<td>725934.87</td>
<td>3.89</td>
</tr>
<tr>
<td>Public Services</td>
<td>52</td>
<td>5964.80</td>
<td>2915.40</td>
<td>725934.87</td>
<td>3.89</td>
</tr>
<tr>
<td>University</td>
<td>53</td>
<td>5755.50</td>
<td>2872.48</td>
<td>715248.67</td>
<td>2.90</td>
</tr>
<tr>
<td>Industry Area</td>
<td>57</td>
<td>10443.17</td>
<td>6988.37</td>
<td>1740104.21</td>
<td>97.44</td>
</tr>
<tr>
<td>Industry Area</td>
<td>58</td>
<td>10206.57</td>
<td>6769.01</td>
<td>1685482.48</td>
<td>92.40</td>
</tr>
<tr>
<td>Industry Area</td>
<td>72</td>
<td>10727.90</td>
<td>7099.76</td>
<td>1767840.87</td>
<td>100</td>
</tr>
<tr>
<td>Industry Area</td>
<td>73</td>
<td>10402.10</td>
<td>6776.58</td>
<td>1687368.27</td>
<td>92.58</td>
</tr>
<tr>
<td>Industry Area</td>
<td>77</td>
<td>9056.70</td>
<td>5468.14</td>
<td>1357103.17</td>
<td>62.52</td>
</tr>
<tr>
<td>Industry Area</td>
<td>84</td>
<td>7871.80</td>
<td>4393.70</td>
<td>1094031.87</td>
<td>37.85</td>
</tr>
<tr>
<td>Historic Quarter</td>
<td>90</td>
<td>5993.50</td>
<td>2946.31</td>
<td>733630.31</td>
<td>4.60</td>
</tr>
<tr>
<td>Historic Quarter</td>
<td>91</td>
<td>6393.82</td>
<td>3040.90</td>
<td>757185.31</td>
<td>6.77</td>
</tr>
<tr>
<td>Town Hall</td>
<td>92</td>
<td>5545.80</td>
<td>2746.03</td>
<td>683761.31</td>
<td>0.00</td>
</tr>
<tr>
<td>Public Services</td>
<td>94</td>
<td>5713.60</td>
<td>2783.58</td>
<td>693111.41</td>
<td>0.86</td>
</tr>
<tr>
<td>University</td>
<td>123</td>
<td>5811.70</td>
<td>2878.11</td>
<td>716648.21</td>
<td>3.03</td>
</tr>
<tr>
<td>University</td>
<td>124</td>
<td>5808.10</td>
<td>2835.69</td>
<td>706086.51</td>
<td>2.06</td>
</tr>
<tr>
<td>Industry Area</td>
<td>135</td>
<td>10308.00</td>
<td>6796.69</td>
<td>1692376.31</td>
<td>93.04</td>
</tr>
<tr>
<td>Congress Palace</td>
<td>185</td>
<td>6728.20</td>
<td>3163.50</td>
<td>787712.21</td>
<td>9.59</td>
</tr>
</tbody>
</table>

Table 5: Dynamic comparative study among different networks from HSR station point of view.

<table>
<thead>
<tr>
<th></th>
<th>Fulda</th>
<th>Ulm-Neu Ulm</th>
<th>Toledo</th>
<th>Lleida</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population</td>
<td>64177</td>
<td>177986</td>
<td>83108</td>
<td>138416</td>
</tr>
<tr>
<td>Density of population (inh/Km²)</td>
<td>617</td>
<td>894</td>
<td>359</td>
<td>654</td>
</tr>
<tr>
<td>Network length (Km)</td>
<td>88.67</td>
<td>76.38</td>
<td>53.37</td>
<td>106.88</td>
</tr>
<tr>
<td>Number of vertexes</td>
<td>249</td>
<td>177</td>
<td>73</td>
<td>226</td>
</tr>
<tr>
<td>Number of lines</td>
<td>9</td>
<td>13</td>
<td>12</td>
<td>18</td>
</tr>
<tr>
<td>Ψ</td>
<td>704264.79</td>
<td>538681.94</td>
<td>270644.15</td>
<td>798273.90</td>
</tr>
<tr>
<td>Ω</td>
<td>1.89</td>
<td>0.00</td>
<td>5.04</td>
<td>2.56</td>
</tr>
<tr>
<td>Hierarchical position (HSR station)</td>
<td>4th</td>
<td>1st</td>
<td>8th</td>
<td>10th</td>
</tr>
</tbody>
</table>

Some topological aspects were studied in order to check the comparability and reliability of this study. In this regard, some differences were found: while Fulda and Toledo have a quite similar number of inhabitants, Lleida and Ulm-Neu Ulm are relatively higher in population. Another difference, in terms of the bus lines configuration, lies in the fact that Lleida has the highest number of bus lines (18 lines) among all the cases under study, and consequently the highest network length. However, the aforementioned line characteristics do not affect proportionally to the number of vertexes within the network. Indeed, the number of vertexes is quite similar among all the cities (249 in Fulda, 177 in Ulm-Neu Ulm, and 226 in Lleida), with Toledo being the city presenting the lowest density of vertexes (73) with reference to its population.

Regarding relative accessibility, it is noteworthy that while the Fulda HSR station ranks in the fourth position out of 249 nodes, and the HSR station in Ulm-Neu Ulm ranks the first position out of 177, the Toledo HSR station and Lleida HSR station rank the eighth position out of
Table 6: Static comparison study among different networks: Fulda, Ulm-Neu Ulm, Toledo and Lleida.

<table>
<thead>
<tr>
<th>Index</th>
<th>Fulda</th>
<th>Ulm-Neu Ulm</th>
<th>Toledo</th>
<th>Lleida</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of vertexes</td>
<td>249</td>
<td>177</td>
<td>73</td>
<td>226</td>
</tr>
<tr>
<td>Number of edges</td>
<td>292</td>
<td>196</td>
<td>89</td>
<td>271</td>
</tr>
<tr>
<td>Maximum connection degree</td>
<td>30876</td>
<td>15576</td>
<td>2628</td>
<td>25425</td>
</tr>
<tr>
<td>Prihar's index / interval</td>
<td>105.74 / [1,124.5]</td>
<td>79.47 / [1, 88.5]</td>
<td>29.52 / [1, 36.5]</td>
<td>93.82 / [1, 113]</td>
</tr>
<tr>
<td>µ’s index / interval</td>
<td>44 / [0, 30628]</td>
<td>20 / [0, 15400]</td>
<td>17 / [0, 2556]</td>
<td>46 / [0, 25200]</td>
</tr>
<tr>
<td>Kansky’s α index</td>
<td>0.14%</td>
<td>0.13%</td>
<td>0.67%</td>
<td>0.18%</td>
</tr>
<tr>
<td>Kansky’s β index / interval</td>
<td>1.17 / [0, 124]</td>
<td>1.11 / [0, 88]</td>
<td>1.22 / [0, 36]</td>
<td>1.20 / [0, 112.5]</td>
</tr>
<tr>
<td>Kansky’s γ index / interval</td>
<td>0.0095 / [0, 1]</td>
<td>0.0126 / [0, 1]</td>
<td>0.0339 / [0, 1]</td>
<td>0.0107 / [0, 1]</td>
</tr>
<tr>
<td>$G_p$ index</td>
<td>122.83</td>
<td>86.89</td>
<td>34.78</td>
<td>111.30</td>
</tr>
</tbody>
</table>

73 and tenth position out 226, respectively. This fact reinforces the hypothesis that degree of connectivity and development of the surroundings of the HSR station are better for the German than for the Spanish stations. This difference is exacerbated when contrasting the different population densities. Thus the Ulm-Neu Ulm station which has the best connection among all the networks, serves a population of 177986 inhabitants with a density of 894 inhabitant per km$^2$. In the most adverse case (Toledo), whose station ranks in the eight position out of 73, there are 83108 inhabitants with a low density population of 359 inhabitants per km$^2$. Certainly, the Toledo station rank in relation to its secondary transport network limits accessibility for potential customers and fails to guarantee the sustainability and profitability of the HSR network.

With respect to static indexes (see Table 6), all the networks have a similar number of vertexes except Toledo, which presents one-third the number of nodes compared to the rest of the networks. Nevertheless, there exist differences mainly in two markers: maximum degree of connection and the Zagodzdzon index which increase proportionally as the number of vertexes in the network grow. Thus, Toledo network has the lowest maximum degree connection and Zagodzdzon index among all the networks studied. Furthermore all the public bus transport networks show a cohesion degree pretty low within their respective intervals, as reported by the rest of the static indexes used in this study. Therefore, both German and Spanish stations have a low cohesion degree and all of them need a high level of development in order to improve connection and cohesion. However, it is important to remark that the networks are hardly comparable given their differences regarding the number of vertexes.

4 Conclusions

4.1 Conclusions regarding the static assessment

In the case of the static study of the network, applied to Fulda first, it was found that the cohesion level reported by all indexes showed the same low cohesion. The result is logical, since they measure the relation between the number of vertexes in the graph and the potential direct interconnections between them. The more stops are added to the network, the more potential connections appear.

As described in Section 3.2, the static method, which has been widely applied when assessing cohesion, does in fact provide incomplete information in the case of the global study of the network. This method, which is useful when analysing the connection of town by roads that link each one of them with the others, is less reliable when analysing networks such as those of local buses, since they have a large number of nodes (identified by stops in this case). Given that it has a greater number of nodes distributed in the same area, instead of contributing to something that would in principle be deemed to be a network with a higher level of cohesion, the static study penalises this fact, since the figures resulting therefrom to calculate the number of connections needed to link all the stops directly are not compatible with the spatial configuration inherent to transport networks that are like those we are analysing. This way, the results of those networks with a greater number of stops as Fulda (249), Ulm-Neu Ulm (177) or Lleida (226) are comparatively negative in comparison with other networks with a lower number of stops as...
Two ideas are noteworthy:

The deficiencies of direct extrapolation in a method traditionally utilized in road networks should be taken into account, since the number of nodes is much lower in the case of road networks and the connections have a radial trend rather than linear, which does not happen in the case of public transport networks. For this reason, applying static studies to two different places in the same city seemed appropriate: one in the city center, Area 1, and a more outlying zone, Area 2. Thus, upon extrapolation based on these two areas, the method becomes interesting again. In the case of Fulda, the comparative analysis of two areas belonging to the same city and served by the same network, the first one having a more central location and the second one having a more outlying location, shows results differing from the level of cohesion of those nodes belonging to the area with better location and consistently linked to a greater number of nodes, in comparison to the area whose nodes coincide with a lower number of lines, which is more valid when diagnosing those areas with greater cohesion deficit within the networks serving them.

4.2 Conclusions regarding the dynamic assessment

The severity and the scope of the conclusions seems to be logical, making it necessary to take into account any national, regional and local policies that may, in fact, prepare and execute what has been planned under the right conditions. Analyses like that of this research show very different trajectories in the policies affecting the location of HSR stations and their integration in the local mobility. Two ideas are noteworthy:

- The difference in the accessibility level of the stop that corresponds to the HSR station has very favourable values in the German cases, and not so much in the Spanish cases, being especially unfavourable in Toledo case. The necessary territorial spread from the station guaranteed by a suitable link to secondary transport networks is as commonly accepted theoretically as variable in its practical application.
- Different connection levels between the HSR station and those locations that, from the study of the city that has been envisaged by interviewee local agents, are deemed to be poles of generation of activity within local dynamics. In this regard, Fulda presents good levels of accessibility to locations identified as main attractors. This good station accessibility and aforementioned attractors guarantee access of travellers throughout the HSR.

The implementation of the new rail line to the urban area is an opportunity to increase the dynamism of an area that generally is of great value, but which also faces problems that are a result of the scar caused by rail in the 19th century. The scenario chosen when analysing the four cases, with their railway stations located in the centre thereof, allows city halls to execute projects integrating the new transport mode with the other transport systems. That way, efficient use of the new accessibility conditions is guaranteed. The analysis of the systematic studies of the cases in different European countries shows common patterns related to the recipient of the actions. In the first place, it can be noted that there is a direct relationship between those inherent factors of cities and their location within the network and the potential impact of stations in them. Although the aforesaid location factors condition the strategies adopted by the cities, city halls can execute others enabling them to optimise the implementation of HSR in their cities.

Regarding the dynamic analysis of those networks analysed, and in sight of the fact that a favourable accessibility value corresponds to a good location in comparison to the rest of the nodes that make up the network, the value obtained in the case of the high-speed railway station is important because it can guarantee a good relationship between the same and the rest of the local environment, which will in time assure good links between the high-speed network and other secondary networks, as it has commonly been claimed. There is a significant different between the values obtained in the four analyses. In the German cases, that link is fundamentally important. Thanks to this fact, the rank is 4th (out of 249) as the best connected node in the case of Fulda or 1st (out of 177) in the case of Ulm-Neu Ulm. Nevertheless, regarding the Spanish cases and particularly the case of Toledo, ranking 8th (out of 73), the station’s accessibility decreases significantly, making its necessary relationship with the local secondary network worse. Finally, Lleida ranking 10th (out of 226) shows a slightly better result than Toledo regarding the relative accessibility, but worse than the German networks.

It thus seems that, in the near future, an in-depth review is necessary that guarantees multiple objectives for new HSR lines or for those already built, in order to serve a wider range of traffic, which will result in a reasonable level of use. In that case, it would be necessary to review the existing design criteria to meet new demands as
far as possible, reducing in time the need for investment, which is especially important in the current scenario of budgetary constraints.

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