Research on carbonate reservoir interwell connectivity based on a modified diffusivity filter model

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Abstract: According to the solution of dual-porosity model, a diffusivity filter model of carbonate reservoir was established, which can effectively illustrate the injection signal attenuation and lag characteristic. The interwell dynamic connectivity inversion model combines a multivariate linear regression (MLR) analysis with a correction coefficient to eliminate the effect of fluctuating bottom-hole pressure (BHP). The modified MLR model was validated by synthetic field with fluctuating BHP. The method was applied to Tahe oilfield which showed that the inversion result was reliable. The interwell dynamic connectivity coefficients could reflect the real interwell connectivity of reservoir. The method is easy to use and proved to be effective in field applications.

Keywords: Carbonate reservoir; filter coefficient; interwell connectivity; bottom-hole pressure fluctuations

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1 Introduction

Carbonate reservoir has various storage types that, are mainly cavities and fractures. It is difficult to describe the interwell connectivity due to its serious heterogeneity [1–3]. The evaluation of interwell connectivity is an important part of reservoir analysis, especially in carbonate reservoirs [4]. The interwell connectivity provides us a tool to divide fracture-vug units, identify location for new wells, and manage waterflood performance.

Both the injection and production rates are the most available data in any waterflooding reservoir. Numerous studies to investigate interwell connectivity using these data have been carried out. Heffer et al. [5] and Refunjol [6] used Spearman analysis to determine flow trends in a reservoir. Albertoni and Lake [7] used multivariate linear regression (MLR) analysis of the flow rates in a waterflooding reservoir to estimate interwell connectivity. They also introduced diffusivity filters to account for the delay where an injector signal reaches a producer. Yousef et al. [8] and Zhao et al. [9] introduced a capacitance model, which evaluated the connectivity by combining producer bottom-hole pressure (BHP) and flow rates.

There are many methods to analyze the interwell connectivity in a carbonate reservoir, such as pressure system analysis, interference test and similar interference test, fluid property contrast, water injection response, tracer agent, etc [10, 11]. Some researchers combined the dynamic and static data, including the palaeogeomorphology, formation structure, fracture, tracer and well interference, to estimate the interwell connectivity [12, 13]. However, the required parameters in these methods are difficult to obtain and the corresponding methods are expensive. In addition, some methods need to shut-in wells, which will affect the normal production.

In this paper, a diffusivity filter model of carbonate reservoir is established based on the solution of an infinite reservoir with the double-porosity model proposed by Warren and Root [14]. In previous studies by Albertoni and Lake [7], MLR analysis was used to determine the interwell connectivity coefficients according to the injection and production data. We also propose a modified MLR model to account for the case of BHP fluctuating. The developed technique is applied to a synthetic field firstly and then applied to a real carbonate reservoir in Tahe oilfield.
2 Diffusivity filters of carbonate reservoir

2.1 Diffusivity filters development

According to the research results of reservoir studies, the fractured carbonate reservoir could be described by a double-porosity system: matrix system and fracture system (see Figure 1). The fracture system is the principle channel for fluid flow, and the matrix system is the major storage space. The fluid flow and pressure diffusion in such reservoirs are not like those in single porosity reservoirs [15]. In the double-porosity system, the fractures have higher permeability than the matrix. The fluids travel from the high permeability fractures system to the wellbore and the fracture pressure begins to decline. The hydrocarbons contained within the matrix begin to flow from matrix to nearby fractures.

Barenblatt et al. [16] suggested that the flow of a single phase slightly compressible fluid in a double-porosity medium could be expressed by the following equations:

\[
\frac{k_f}{\mu} \left( \frac{\partial^2 p_f}{\partial r^2} + \frac{1}{r} \frac{\partial p_f}{\partial r} \right) + \frac{\alpha}{\mu} (p_m - p_f) = \phi_f C_f \frac{\partial p_f}{\partial t} \\
\frac{\alpha}{\mu} (p_m - p_f) + \phi_mC_m \frac{\partial p_m}{\partial t} = 0
\]  
(1)

where \( p \) and \( \mu \) are pressure and viscosity of fluid, respectively; \( k \), \( \phi \), \( C \) are permeability, porosity and total compressibility, respectively; \( \alpha \) is a geometric factor; and subscripts \( f \) and \( m \) denote the fracture system and matrix system, respectively.

Warren and Root [14] took the effects of porosity of fracture system and wellbore diameter into consideration and obtained an approximate solution for an infinite reservoir. According to the solution, the pressure change caused by a change of injection rate can be expressed as:

\[
\Delta p = p_i - p_f (r_w, t) \\
= -\frac{q \mu}{4\pi k_f h} \left[ \ln \frac{r_w^2}{r_f^2} + Ei(-at) - Ei(-a\omega t) + 0.809 \right] \\
= C_1 \left[ \ln \frac{n_f}{r_w^2} + Ei(-at) - Ei(-a\omega t) + 0.809 \right]
\]  
(2)

where \( C_1 = -\frac{q \mu}{4\pi k_f h} \), \( \eta = \frac{k_f}{\mu(C_f/\phi_f+C_m/\phi_m)} \), \( \omega = \frac{C_f \phi_f}{r_f^2 C_m \phi_m} \), \( a = \frac{\lambda \theta}{\omega(1-\alpha)} \), \( \lambda \) is the pressure conductivity coefficient.

Using the linear model of fluid productivity index and the superposition principle, the change in production rate caused by a unit injection impulse can be expressed as:

\[
\Delta q = \begin{cases} 
C_2 \left[ \ln \frac{n_f}{r_w^2} + Ei(-at) - Ei(-a\omega t) + 0.809 \right] & \text{if } t \leq 1 \\
C_2 \left[ \ln \frac{n_f}{r_w^2} + Ei(-at) - Ei(-a\omega t) - \ln \frac{n_f}{r_w^2} \right] \\
- Ei(-a(t-1)) + Ei(-a\omega(t-1)) & \text{if } t > 1
\end{cases}
\]  
(3)

where \( \Delta q \) is production rate change; \( C_2 = C_1 J_f; J_f \) is productivity index; \( d \) is the distance between injection well and production well.

According to the definition of diffusivity filter proposed by Albertoni and Lake [7], the carbonate reservoir discrete diffusivity filter coefficients can be expressed as:

\[
a^{(n)}(r_w) = \frac{\sum_{t=0}^{t=n} \sum_{t=n+1} \Delta q dt}{\sum_{t=0}^{t=n} \sum_{t=n+1} \Delta q dt} = (n = 1, 2, \ldots, N)
\]  
(4)

where \( a^{(n)} \) is the diffusivity filter coefficient; \( N \) is the total number of filter coefficients. The normalization of the coefficients determines that the \( a^{(n)} \) are less or equal to one, and the sum of all the coefficients is equal to one.

2.2 Validation of the diffusivity filters

Using two numerical simulation reservoir models to validate the diffusivity filters: one is a carbonate reservoir described by double-porosity model and the other is a sand reservoir described by single porosity model. Both are undersaturated oil reservoirs and the oil-water mobility ratio is equal to one. There is only one injector and one producer in the models. The water is injected only in the first month (one injection impulse) and the simulation extends for 12 months. Then the production rates are normalized to 0 and 1. If there was no dissipation in the reservoir, an impulse
in the injector would be instantaneously produced in the producer. If there is some dissipation in the reservoir, the production response will remain for some months (see Figure 2).

From Figure 2, we know that the injection signal attenuation and time lag characteristics in carbonate reservoir are significantly different from those in the sand reservoir. There are lots of natural fractures in the carbonate reservoir and the permeability is larger than the sand reservoirs, so there is less dissipation in the carbonate reservoir. The production peak occurs in the first month and the impulse dissipation effect will remain for several months. There is a larger dissipation in the sand reservoir because of its lower permeability which makes the production peak much lower and occurs one month later than the impulse.

Figure 3 shows the diffusivity filter coefficients generated by Eq. 4 in the carbonate reservoir and sand reservoir. Comparing Figure 3 with Figure 2, the diffusivity filter coefficients are the same as the normalized production rate of the numerical simulation model. Therefore, the carbonate reservoir diffusivity filters can accurately describe the time lag and signal attenuation between injector and producer in the carbonate reservoir.

$$i_{ij}(t) = \sum_{n=0}^{N_i} \alpha_{ij}^{(n)}i_{ij}(t-n) \quad (5)$$

where $i_{ij}$ is effective injection rate; $i_{ij}$ is actual injection rate; and subscripts $i$ and $j$ denote injector index and producer index, respectively.

### 3.2 Interwell dynamic connectivity model

The most widely used interwell dynamic connectivity model is MLR model. In this model, the estimated production rate of a producer is given by:

$$\hat{q}_j(t) = \beta_{0j} + \sum_{i=1}^{I} \beta_{ij}(i_{ij}(t)) \quad i = 1, 2, 3, \ldots, I \quad j = 1, 2, 3, \ldots, J \quad (6)$$

Where $\hat{q}_j(t)$ is the estimated production rate of producer $j$ at time $t$; $\beta_{0j}$ is an additive constant term, when the waterflooding is balanced, $\beta_{0j} = 0$; $I$ is the total number of injectors; $J$ is the total number of producers; $\beta_{ij}$ is the connectivity coefficient between injector $i$ and producer $j$, the larger $\beta_{ij}$, the greater the connectivity.

We assume that the producer BHP is constant and the change in the production rate is exclusively caused by changes in the injection rates. Therefore, MLR model states that at any time the total production rate at well $j$ is a linear combination of the rates of every injector plus a constant term.

The MLR model assumes that within the period of time selected for analysis, the producer BHP must be constant, no new wells and no new completions. In a real field, it is
difficult to keep producer BHP constant. When the operation conditions of producer do not change, producer BHP is approximately constant.

Changing producer BHP would also affect production rates and then influence the injector-producer connectivity determination. By changing BHP of any producer, the production rates of the rest wells will be changed. When the producer BHP changes, we just shift the expected production rates by a constant rate [17]. Therefore, the modified MLR model can be rewritten as:

\[ \hat{q}_j(t) = \beta_{0j} + \sum_{i=1}^{I} \beta_{ij} \bar{c}_i(t) + q_{BHP}(s) \]  

(7)

Where

\[ q_{BHP}(s) = \sum_{k=1}^{K} J \left[ p_{wfj}(T_{s-1}) - p_{wfk}(T_s) \right] \]  

(8)

is the correction coefficient; \( s \) is the time interval for BHP changes; \( T_s \) are the segmentation times; \( p_{wf} \) is BHP of the producer; \( J \) is the liquid productivity index.

Albertoni and Lake [7] present the solution of a MLR problem. The minimization of the errors leads to the following set of \( I \) linear equations:

\[
\begin{pmatrix}
\sigma_{11}^2 & \sigma_{12}^2 & \cdots & \sigma_{1I}^2 \\
\sigma_{21}^2 & \sigma_{22}^2 & \cdots & \sigma_{2I}^2 \\
\vdots & \vdots & \ddots & \vdots \\
\sigma_{I1}^2 & \sigma_{I2}^2 & \cdots & \sigma_{II}^2
\end{pmatrix}
\begin{pmatrix}
\beta_{1j} \\
\beta_{2j} \\
\vdots \\
\beta_{Ij}
\end{pmatrix}
=
\begin{pmatrix}
\sigma_{1j}^2 \\
\sigma_{2j}^2 \\
\vdots \\
\sigma_{Ij}^2
\end{pmatrix}
\]  

(9)

which can be solved by standard means. The constant term is given by:

\[ \beta_{0j} = \bar{p}_j - \sum_{i=1}^{I} \beta_{ij} \bar{p}_i \]  

(10)

where \( \sigma_{ij}^2 \) and \( \sigma_{ij}^2 \) are the injector-producer and injector-producer covariances, respectively.

### 3.3 Model validation

In order to verify the accuracy of the modified model, we use numerical simulation to generate a set of production rates from a given set of injection rates. There are five injectors and four producers in a homogeneous double-porosity reservoir. The numerical simulation is run for 100 months under waterflooding and BHP of the producer P1 changes four times during the production period (Figure 4). Minimize the sum of squared error between actual and predicted production rates of all the producers which are calculated by the Eq. 7. Figure 5 shows a comparison between the production rate using MLR model, modified MLR model and the production rate observed in the numerical simulation. The coefficient of determination between MLR model production and numerical simulation production is \( R^2 = 0.28 \), while the modified MLR model improves the coefficient of determination to \( R^2 = 0.99 \) as shown in the Figure 5. Comparing the estimated \( \beta \), the modified MLR model provides the most accurate estimation of the connectivity coefficient \( \beta \) (Figure 6). Therefore, the modified MLR model is able to predict the production rate accurately and it determines the true connectivity coefficients.
According to the actual production data of the oilfield, the correction coefficient is often difficult to solve due to the lack of bottom-hole pressure data. However, we can use the time of producer production rate, which is changing dramatically as the time of BHP is fluctuating, which can be automatically identified by the computer algorithm, so an initial correction coefficient value can be given. Then the genetic algorithm is used to optimize the solution to minimize the error between the predicted production rate and actual production rate. After the time of BHP fluctuating is identified, the fluctuation of the production rate caused by the change of the production well’s own working system should also be excluded.

4 Field application

The modified technique was applied to the S48 unit in Tahe oilfield. The S48 unit is one of the main cave systems put into development firstly in Tahe oilfield. The storage space is composed of a large number of caves, fractures and pores which leads to a strong heterogeneity. The S48 unit started water waterflooding development since May 10, 2005 and the waterflood was balanced (the field injection rate is approximately equal to the total production rate). Due to the large number of fractures and vugs, the injection signal propagation is very fast in the reservoir, so the daily injection rate and daily production rate are selected as input signals. Meanwhile, the wellbore pressure fluctuation and shut-in situation happens frequently caused by water flooding method and many adjustment measures. Therefore, the modified MLR model which considers the fluctuation of bottom-hole pressure is applied.

The three-month production dynamic data was selected to study the interwell dynamic connectivity of the reservoir. The predicted production rate and the actual production rate curve of the block are shown in Figure 7, and the coefficient of determination between them is 88.7%. The average error of all production wells is 6.89%, which meets the requirement of field engineering. The S48 unit interwell dynamic connectivity diagram is plotted by using dynamic connectivity coefficient in Figure 8, where the reverse arrow points from the injector to the producer. Thus, the displacement direction is demonstrated intuitively in the reservoir. The injection rate is proportional to the dynamic connectivity coefficient of the well.

As shown in Figure 8, due to the strong heterogeneity of reservoir, dynamic connectivity between injectors and producers in the different directions are significantly different. The larger connectivity coefficients occur in the NW-SE trending, clearly showing the preferential permeability orientation. The main cave of unit S48, located in a NW-SE trending inherent karst highland, was formed by the first-order underground river [18]. Therefore, the injectors and producers in this direction are well-connected, which is consistent with the interwell connectivity distribution. Yang [19] has found TK410-TK425-S48-T401-TK412-T402 well groups are connected using the pressure transient analysis method before the unit had started water injection development. Many other researchers [20] also obtained the similar conclusions using other methods such as the pressure gradient method and the interwell energy interference analysis method.
After the waterflooding development, based on the injection and production performance data, interwell connectivity can be determined using our new method. The results show that TK412 is connected dynamically with well T402 and well T401, and TK425 is connected with well TK410 and well S48, which are consistent with the previous analysis. TK411 well has been injected with tracer during water injection. The tracer interpretation result demonstrates that the connectivity level of the wells S48, T401, TK467 and TK411 is the first order while the connectivity level of TK476, TK428CH and TK411 is the third order. The conclusion is consistent with the inversion result of this paper.

5 Conclusions

(1) The inject impulse dissipation in the carbonate reservoir is different from that in the sand reservoir. Due to fractures and vugs, the time lag and signal attenuation in the carbonate reservoir is smaller than that in the sand reservoir. Based on the typical pressure solution of double-porosity reservoirs, diffusivity filter coefficient model has been built to model the propagation characteristics of injection signals in carbonate reservoirs, which can describe the attenuation and time lag of the injected signal accurately.

(2) The MLR model with fluctuating BHPs gives more reliable evaluations of interwell connectivity between injectors and producers, which greatly increases the amount of data available in practical application and improves the accuracy of inversion results.

(3) The inversion results of our method are consistent with the interwell connectivity of the reservoir. It can also characterize the connectivity between injectors and producers quantitatively as well as provide guidance for later injection. Our method is simple and convenient to apply. In addition, it doesn't influence the normal production of oil field.

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References


