Contradictory results, arising from great theories of science and their experimental tests, have been the focus of intense discussions and seeds for progress of past and present scientific research. Perhaps one of the clearest examples for this fact has been presented by discussions of Einstein and Poincaré related to Euclidean geometry [1] and the apparent contradictions to its results by Einstein’s general relativity (which was confirmed by measurements of deflection of starlight by the sun during the 1919 total eclipse).

How could Euclidean geometry be wrong as a mathematical-logical framework? The solution of this conundrum by Einstein and Poincaré is as follows. Any framework like Euclidean Geometry seen as a mathematical framework has as such nothing to do with nature. The axioms can be seen as definitions and, therefore, such a framework cannot contradict the experiments, because it has (in principle) nothing to do with the experiments. As such it also cannot contradict other mathematical-logical frameworks as long as these are only considered as such with axioms that again can be seen as definitions.

A link to experiments needs to be established that then extends the purely logical-mathematical theory to the objects of the physical reality and, therefore, extends it to a physical science. This extension of Euclidean Geometry had been achieved by the introduction of the absolutely rigid body. Only with this additional concept can Euclidean geometry be compared to experiments. If then there exists a discrepancy to experimental results, it is that concept of the rigid body which needs to be rejected, if we do not wish to give up logic.

This special issue discusses a conundrum that arises from the results of quantum theory for certain measurements that are in contradiction to the theoretical framework of John S. Bell and his followers. They derived inequalities (which they often claim are based only on Einstein’s physics, particularly Einstein’s separation principle) that contradict quantum theory and a large number of recent experiments related to quantum theory.

The proposal of Bell and many of his followers to resolve this conundrum turns against the spirit of Einstein’s relativity theory, because it contains the introduction of instantaneous influences faster than the speed of light that occur over arbitrary large distances between so called “entangled” particles. This proposal is currently accepted by a large number of physicists and by almost all science writers.

There exists, however, also a significant number of researchers that have found issues with the work of Bell and his followers. Some of these issues were dubbed “loopholes”. These loopholes in Bell’s reasoning arose in our opinion mainly from the following facts. First, to make an airtight case for the physical validity of his inequality, Bell gave additional directions of how the experiments needed to be performed, directions that had no counterpart, and could not have a counterpart, in his theory. For example, the physically so important time-variable cannot be included into Bell’s formalism in any straightforward way, because all his variables may (and some even must) vary in a totally random way. However, time is not a random variable in any sense of the word. Second, Bell used the concept of probability and joint probability measures in his derivations. The existence of such measures is not given by any logical or mathematical reason but determined by the relation of the mathematical abstractions to the experiments, for example to the fact whether the experiments are performed in pairs, triples or quadruples. For an elaborate discussion of related problems see Khrennikov this
issue and [2]. As a consequence of these and other facts, so dubbed loopholes were discovered in Bell’s reasoning and the burden to close these loopholes was shifted to the experimenters.

This special issue contains contributions that resolve the Bell-conundrum in a variety of ways, which are related to the connections of Einstein’s classical physics and of quantum theory to the experiments. Problems of Bell’s argument and the connections of his specific approach to the experiments are, of course, also central to several contributions. For example, the assumption of triples and quadruples of experiments instead of just pairs that indeed are actually measured is pinpointed as a cause of the conundrum (analogous to the assumption of the absolutely rigid body in the Einstein-Poincare discussions). The guest editors of this special issue are convinced by these contributions that the Bell-conundrum can definitely be resolved without recourse to instantaneous influences at a distance or “spooky influences” as Einstein called them.

1 Einstein-Podolsky-Rosen’s Gedankenexperiment and Bell-type inequalities

The Basis for Bell’s work is the well know Gedankenexperiment of Einstein, Podolsky and Rosen in its modification by Bohm (EPRB). Two atomic or subatomic particles (photons, electrons etc.) are prepared at a source in a correlated state and are sent out in opposite directions to two spatially separated measurement stations, where their polarization is measured by complicated equipment and where by some means it is assured that one deals with a sent-out correlated pair. The experiment is geared to assess the validity of Einstein separability i.e. whether or not the experiments in the two stations are independent of the experimental arrangement and measurement of the other station at the moments of measurement of the pair.

The measurement settings (polarizers etc.) are, therefore, switched quickly in order to exclude the possibility of any information-exchange with the speed of light or slower during the moments of the pair measurement. Light moves in a nanosecond about 30 cm and polarizer settings can be switched easily within 100 nanoseconds or less, which means the two measurement stations may be as close as a few meters, but have actually been implemented at a distance of tens of miles, for example at the islands of Tenerife and La Palma. Recent measurements have even involved a satellite and the Chinese cities of Delingha and Lijiang.

It is now commonly reported or at least implied by science writers that it has been proven, by measurement of a single correlated pair, that the measurement in one station influences instantaneously the outcome in the other station; implying thus signalling much faster than the speed of light. This claim is false. No single pair measurement has ever shown any influence faster than the speed of light. Such a fact would completely destroy Einstein’s theory of relativity and no sane physicist believes such flapdoodle (as Murray Gell-Mann called it).

Kupczynski explains how reported violations of non-signalling in twin-photon beam experiments may be caused by setting dependent post-selection of data necessary to identify correlated detector clicks (see also section Bell game below). Graft shows that the projection postulate suffers from fundamental deficiencies that brings its validity into serious doubt. Its application to EPRB experiments in particular must be invalid, because it requires superluminal transmission of information in contradiction with special relativity. If projection is excluded, the EPR paradox is resolved and quantum nonlocality is a non sequitur.

Bell (in his later years) and his followers did and do indeed assert that influences faster than the speed of light are involved in EPRB experiment. However, they deduce this assertion from the statistics of very many pair-measurements. The basis of their deduction is the statistical violation of an inequality derived by Bell and variations of Bell’s inequality that were proposed by Clauser, Horne, Shimony and Holt as well as Eberhardt and others. These inequalities are often presented, even in textbooks, as if they were only based on the rules of adding and subtracting numbers, or only based on group theory. However, as explained in this special issue, such presentations amount to gross oversimplification. A short explanation of this fact is given next.

Bell-type inequalities are usually derived for 3 or 4 pair measurements each corresponding to a pair of equipment settings and resulting in detector outcomes that are mostly represented by real numbers of the interval \([-1, +1]\). For example we can have possible measurement outcomes \(A, B, C, D\) corresponding to different measurement settings \(a, b, c, d\) with \(-1 \leq A, B, C, D \leq +1\) or more often just with the digital result \(A, B, C, D = \pm 1\). The Clauser-Horne-Shimony-Holt (CHSH) inequality which is a Bell-type inequality frequently addressed in actual experiments is then stated to be:

\[
|AB + AC + DB + DC| \leq 2
\]  

(1.1)
It is very easy to insert all possible values of $A, B, C, D = \pm 1$ into the equality and convince oneself that it is satisfied just by the laws of integer numbers and, with a little more effort, also if the real numbers are used. In many popular presentations the CHSH inequality is, therefore, presented as an obvious fact followed by the remark that some results of quantum theory surprisingly violate this inequality.

Rosinger shows in this issue, with precise mathematical logic, that if the above view were true, the consequences would be dire indeed for the consistency of either mathematics or quantum theory or both.

We have thus a situation that is analogous to the above mentioned Einstein-Poincare discussions and must investigate the connection of both Bell-type inequalities and quantum theory to the elements of physical reality, the data and attempt to find an inaccuracy or weakness in this connection, just as Einstein pinpointed the absolutely rigid body as the problem. An extensive literature on these problematic connections has been accumulated since the appearance of Bell’s seminal paper. Kracklauer reviews early objections to Bell’s approach that are still valid.

The problems of the approach of Bell and his followers are further traced in this issue to several problematic links of Bell-type theories with the actual EPRB experiments: (i) counterfactual reasoning about the experiments, reasoning that would not be permitted in court, (ii) the measurements are assumed in Bell-type work to be performed in triples, quadruples etc., while they are performed in pairs in the actual experiments. Related to this assumption (iii) Bell implies the existence of certain joint probabilities of possible experimental outcomes that cannot be derived from and are inconsistent with the actual experiments much as the absolutely rigid body is inconsistent with the findings of Einstein’s general relativity. Finally (iv) a slightly changed inequality has been presented by Wigner that is believed by many to only involve group theory and Einstein locality (his separation principle). A closer look, however shows that (iv) is actually based on (iii). These points are discussed below and in other contributions of this special issue.

First, a detailed investigation of related books, for example those of Peres and Leggett shows that Eq. (2.1) is based on more than the laws of real numbers and is typically connected to the data of experiments by additional so called counterfactual reasoning (reasoning involving “data” that would have been obtained if the measurements would have been performed in other ways). The problems with counterfactual reasoning have been discussed in the literature and it has been shown that counterfactual reasoning restricts the connection of Bell’s theory to experiments so significantly that it makes the application of that theory to EPRB experiments questionable. It is also unacceptable to abandon the consistency of mathematics and quantum theory just because of counterfactual arguments.

Second, if the possible outcomes $A, B, C, D$ can be simultaneously measured then indeed Eq. (2.1) follows and is valid and still contradicts some of the results of quantum theory. The expression “simultaneously measured” has here nothing to do with Einstein’s definition of simultaneity but simple means that about all data can be ordered in quadruples $A, B, C, D$ or even better in quadruples of the pairs of Eq. (2.1). It is, however, a fact that all actual measurements are just performed in pairs and not simultaneously in triples or quadruples etc.. This fact makes it impossible to prove Eq. (2.1) without additional assumptions and also points to the possibility that quantum theory agrees with the CHSH inequality, if more than one pair of settings is involved in the measurements. This latter fact is discussed in detail by Sica and also by Graft.

Third, there exists another simple reasoning that seems to validate Eq. (2.1). Just assume that the joint probability measures for outcomes $A, B, C, D$ exist, so that we have, for example, a probability measure of 0.1 for the result $A = +1, B = −1, C = −1$ and $D = +1$ and similar for all other possible outcomes. Then again Eq. (2.1) is valid, the $A, B, C, D$ now being Boolean variables or Kolmogorov’s random variables. Ways out of this conundrum are given in paper by Sica. Others ways have also been discussed in the literature.

Finally we do not even need to regard the $A, B, C, D$ as numbers but may regard them as any form of outcome such as up or down, plus or minus and so on. Wigner has shown that one can formulate an inequality just for the equal and not equal outcomes of measured pairs and many have claimed that Wigner’s proof is only based on group theory. This result, however brings us back to Rosinger’s work. Group theory is also the basis of quantum mechanics and how can a group theoretical theorem contradict quantum mechanics without the breakdown of our whole mathematical-physics framework?

It was also shown recently by Hess, De Raedt and Michielsen that Wigner used the assumption of the existence of joint probabilities and with it violated topological-combinatorial factors that are important for the actual experiments [3].

Overall, the contributions in this issue (together with previous publications of some of the contributors) show that the connection of the Bell-type inequalities to actual experiments and also the connections of quantum mechanics to the actual experiments need to be carefully considered and the inclusion of these considerations permit
to remove in one way or the other the conundrum. The
conundrum shows mainly that Bell-type formulations are
based on unwarranted assumptions.

Some may still claim that Eq. (2.1) can also be justi-
ified by the assumption that the velocity of light c in vac-
uum is the limit of all possible velocities, from which Ein-
stein’s separation principle follows. However no connec-
tion of Eq. (2.1) to the Einstein separation principle (Ein-
stein locality) has ever been proven. Bell’s assumption that
his variable λ does not depend on the equipment settings is
simply not a necessary assumption to fulfill the separa-
tion principle, because λ may depend on the setting of the
local equipment.

One asks then, how can a reasonable person still be-
lieve in the relevance of the CHSH inequality and all Bell-
type inequalities for actual EPRB experiments. The answer
to this question is at least threefold.

First none of the Bell-type derivations contain all the
measured data that are necessary for the identification and
count of the pairs. The measurement stations (including
the source of the particle pairs) contain equipment that
tells us which signals belong to correlated pairs and which
signals may not or do not belong to this set. Bell’s the-
ory is, therefore incomplete. Bell and followers do not ac-
knowledge this fact and do not attempt to complete their
theory. Instead they try to take care of deficiencies by im-
posing additional requirements on the experiments. Viola-
tion of these requirements are the so called “loopholes”
and many researchers still believe that they can close these
loopholes or have already closed them.

Second, violations of the inequality can be explained
by faster then light communication between the stations
in a very simple way, actually in the simplest way and
thus are commensurate with Occam’s razor (unfortunately
spooky influences are always the simplest way).

Third, and most important, no one seems to be able
to play the so called Bell game. We discuss this game next
and show in this issue how it can be played without instan-
taneous influences at a distance. This latter possibility also
hints to the fact that the closure of all loopholes is very dif-
cult and, in the opinion of the guest editors and authors
of this issue close to impossible.

2 The Bell Game and how it can be played

The requirement of the followers of Bell that the so called
“Bell game” needs to be played by opponents, who must
master it without instantaneous influences at a distance,
appears at first glance reasonable. The Bell game involves
two players Alice and Bob who work at separated measure-
ment stations, say at Delingha and Lijiang respectively.
They have no communication with each other and know
nothing of the other station. The settings in their stations are
randomly changed between a, d in Delingha and b, c
in Lijiang. When receiving a clue that a correlated particle
has arrived, they need to choose a measurement outcome
+1 or −1. After many such choices, averages (...) are taken
and the CHSH inequality for these averages:

$$\langle AB + AC + DB − DC \rangle ≤ 2 \quad (2.1)$$

must be violated. If you can play this game without know-
ing anything from the other side you have defeated the
CHSH or any other Bell-type inequalities.

The followers of Bell believe that this game cannot be
played and that one does need instantaneous influences
at a distance to play it. This means Alice knows somehow
Bob’s equipment settings and Bob knows Alice’s all at the
moment of measuring a somehow correlated pair.

One of us (K.H.) was told on numerous occasions that
nature can play this game, so why can’t you? The reason
why one cannot play the game, as becomes clear from the
paper by De Raedt, Michielsen and Hess in this special
issue, is that correlations of spatially separated measure-
ments cannot even be conceived if the experimenters in
the stations know nothing from each other. How do Alice
or Bob know that they are dealing with one particle of a
correlated pair if they know nothing about each other, if
they do not even know what is going on on the other side.
We are arriving here at an epistemological problem. How
can we know about a correlation at a distance if we do not
know how the events occur in space and time, how the de-
tectors confirm the measurement of a particle and how the
selection of the particle as part of a correlated pair is made?
How do the settings of the local measurement stations in-
fluence the selection of a particular particle to be part of
a distant pair? And selections must be made, which also
means that the measured set of pairs is reduced from the
larger set of sent out pairs, a fact already discussed by Fine
(see also [4]).

Indeed all actual experiments do include methods of
measurement to determine the pairing. This can be done
by synchronized clock’s and corresponding local time
measurements or by a chosen threshold for the detectors
in both stations and by combinations of these and other
means. This knowledge then opens, as is shown by the
paper of De Raedt, Michielsen and Hess, a door to play
the Bell game without any instantaneous influences at a
distance, without any quantum nonlocality. They dubbed
this door the “photon identification loophole”. This loop-
hole permits to violate Bell type inequalities by computer experiments and to obtain the result predicted by quantum theory for the example of the CHSH inequality and measurements of M. Giustina et al.. These computer experiments use exclusively local (in the respective measurement stations) selection of photons and utilize a mechanism that involves a dependence of the selection on the local measurement settings through considerations involving the detection mechanism; a dependence that appears to be fully commensurate with the experimental arrangements and data.

The paper of Graft investigates a number of well known loopholes related to detector inefficiencies, improper post-selection and other factors and demonstrates convincingly that the loopholes are not closed by the experiments of Hensen et al. who claimed such closure in a recent publication that has received much attention and a report in the New York Times. Kupcynski also discusses the contextuality loophole that, in his opinion, cannot be closed.

### 3 Conclusion

The papers of our special issue demonstrate clearly that the framework of Bell and his followers contains significant weaknesses that make its connection and applicability to the actual EPRB experiments questionable. These weaknesses and corresponding loopholes cannot be closed by just varying the experimental conditions. The demarcation line that Bell-type inequalities represent is extinguished by fundamental deficiencies of Bell-type derivations and by loopholes ranging from post-selection to photon-identification. It appears, therefore, imperative to view EPRB experiments within a broader perspective, a perspective that includes careful investigations of the detailed photon(particle)-identification-process and also of the precision with which the experiments agree with quantum theory.

### References