Research Article

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Bohr against Bell: complementarity versus nonlocality

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Abstract: In this note we compare the views of Bohr (known as the Copenhagen interpretation of quantum mechanics) with the views of Bell: complementarity versus nonlocality.

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1 Introduction

Success of the recent experiments to violate the Bell inequality [1] aimed to close all basic loopholes¹ [2–4] give the impression that the last chapter of the long “EPR-Bell story” has been finally completed, see [5, 6] for assertions of this point of view (but cf. [7, 8]). My personal conviction is that no experiment (independent of its excellence) can be used as the deciding argument in the debates related to meaning of the phrase “violation of Bell’s inequality”, see [8]. Therefore, although this special issue has some orientation toward the analysis of loopholes, this note is devoted to foundational aspects. At the same time it is clear that further analyses of loopholes and experimental data are needed, cf. [9–11].

The main aim of this note is not to confront local realism, the quantum theory and experiment (as it is typically done), but to analyze the additional impact of Bell’s argument on the foundations of quantum theory. I compare the positions of Bohr and Bell, complementarity versus nonlocality. To appreciate the following, we must not forget that (at least originally) Bell was very sympathetic to the ideas of Bohm about the emergence of quantum mechanics from a nonlocal realistic theory - Bohmian mechanics.² (Some prominent Bohmians are even angry that the “Bell-project” (including experiments) did not lead to the world-wide recognition of Bohmian mechanics as the only correct interpretation of quantum mechanics.) Nonlocality is an “additional element” which Bell consistently advertised for the quantum foundations. My position is that this additional element is entirely unnecessary and that the consequences of the experimental violations of Bell’s inequality are zero when compared to the experimental confirmation of interference in quantum systems! Of course, this is true only from the viewpoint of quantum foundations. The Bell-type experiments have a great technological value, since they demonstrated that correlations between quantum systems can survive at very long distances [12].

2 Bohr’s complementarity

There is general feeling that the writings of Niels Bohr [13] are difficult to understand. Therefore few researchers read Bohr: the young generation of quantum physicist has typically never read Bohr (nor von Neumann). In fact, however, Bohr’s position is very clear:

– (B1): An output of any observable is composed of contributions from a system under measurement and the measurement device.
– (B2): Therefore the whole experimental arrangement (context) has to be taken into account.

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1 See the web-links at https://lnu.se/en/staff/andrei.khrennikov for videos of the presentations of all main actors of these experiments.
2 This is the good place to make the following remark about the views of Bohm and de Broglie. In the foundational literature, Bohmian mechanics is considered as an improvement of de Broglie’s double solution model. And this is correct, but only from a mathematical viewpoint. From the view of interpretation, the theories of Bohm and de Broglie differ crucially. In contrast to Bohm, de Broglie did not consider the double solution model as nonlocal. He emphasized that the apparent nonlocality is of a purely mathematical origin. Such a mathematical nonlocality does not contradict the possibility to create a subquantum local realistic model, see [8]
(B3): There is no reason to expect that all experimental contexts can be combined. Therefore there is no reason to expect that all observables can be measured jointly. This is the essence of the principle of complementarity. Hence, there can exist incompatible observables. Their existence is proved by interference experiments.

Comments:

(CB2): Here context is understood generally as a complex of experimental physical conditions. So, it is not reduced to a measurement of a compatible observable, as it is common in modern discussions about quantum contextuality. The Bohr viewpoint on context was elaborated in my previous work, see, e.g., monograph [16].

(CB3): The principle of complementarity is so natural that one should be surprised if it were not valid. The main reason for being surprised by quantum complementarity is that the preceding probabilistic model, classical statistical mechanics, was formulated without the reference to this principle. Here the observable and intrinsic features of systems were identified. We remark that the principle of complementarity was better: assimilated by Bohr from psychology, where the existence of the complementarity of cognitive contexts was widely accepted [17]. Moreover, it seems that classical statistical mechanics is the only model of natural science and humanities that can neglect this principle.

Niels Bohr elaborated on these views in the process of the analysis of Heisenberg’s uncertainty principle and the two slit experiment. The latter was considered as a set of three experiments, corresponding to three different experimental contexts: $C_{12}$ - both slits are open, $C_1$ - only the upper slit is open, $C_2$ only the lower slit is open. Interference experiments for a variety of quantum systems provide the experimental confirmation of Bohr’s principle of complementarity.

We now present the direct citation of Bohr (from his discussions with Einstein [18]) - just to show that Bohr is readable:

"The new progress in atomic physics was commented upon from various sides at the International Physical Congress held in September 1927, at Como in commemoration of Volta. In a lecture on that occasion, I advocated a point of view conveniently termed “complementarity”...

This crucial point, which was to become a main theme of the discussions reported in the following, implies the impossibility of any sharp separation between the behaviour of atomic objects and the interaction with the measuring instruments which serve to define the conditions under which the phenomena appear. In fact, the individuality of the typical quantum effects finds its proper expression in the circumstance that any attempt of subdividing the phenomena will demand a change in the experimental arrangement introducing new possibilities of interaction between objects and measuring instruments which in principle cannot be controlled. Consequently, evidence obtained under different experimental conditions cannot be comprehended within a single picture, but must be regarded as complementary in the sense that only the totality of the phenomena exhausts the possible information about the objects. Under these circumstances an essential element of ambiguity is involved in ascribing conventional physical attributes to atomic objects, as is at once evident in the dilemma regarding the corpuscular and wave properties of electrons and photons, where we have to do with contrasting pictures, each referring to an essential aspect of empirical evidence.

We comment that often Bohr’s principle of complementarity is reduced to (B3), existence of complementary experimental contexts. However, as we can see from Bohr’s writing, (B1) and (B2) are the fundamental counterparts of this principle.

We now briefly discuss the EPR-Bohr debate. Many authors point to lack of clarity of Bohr’s reply [19] to the EPR paper [20]. However, the reply can be summarized in a very simple way: it is impossible to measure in one experiment incompatible observables even in the EPR-setting; therefore the EPR definition of realism is metaphysical - the end of the debate (from Bohr’s viewpoint).

3 Probabilistic representation of Bohr’s complementarity

A weak side of Bohr’s position is that he did not use the formal probabilistic framework for the principle of complementarity. However, this can be explained by a purely historical reason: the rigorous mathematical formalization of classical probability theory was finished by A. N. Kolmogorov only in 1933 [21].

The probabilistic representation of the principle of complementarity is given in the framework of contextual probability theory elaborated by the author [16]. Here each experimental context $C$ generates its own probability space, a triple $\mathcal{P}_C = (\Omega_C, \mathcal{F}_C, P_C)$, where $\Omega_C$ is a set of elementary events, $\mathcal{F}_C$ is a system of subsets of $\Omega_C$ representing events (mathematically formalized as so-called
of a random variable, gives the possibility to define the probability distribution of observables are represented by random variables, maps \( \xi : \Omega_C \to \mathbb{R} \) such that for any interval \( A \subset \mathbb{R} \) its inverse image \( \xi^{-1}(A) = \{ \omega \in \Omega_C : \xi(\omega) \in A \} \subset \mathcal{F} \). The latter condition gives the possibility to define the probability distribution of a random variable, \( p_\xi(A) = P_C(\omega \in \xi^{-1}(A)) \). If a few observables are compatible, i.e., they can be measured under the common experimental context \( C \), then they can be represented as random variables on the same probability space \( \mathcal{P}_C \). In this (and only this) case their joint probability distribution is well defined:

\[
p_{\xi_1, \ldots, \xi_n}(A_1, \ldots, A_n) = P_C(\omega \in A_1 : \xi_1(\omega) \in A_1, \ldots, \xi_n(\omega) \in A_n).
\] (3.1)

Although the consistent representation of contextual probability theory was finalized only recently [16], the founders of probability theory were well aware of the context-dependence of probabilistic representations. In Kolmogorov’s book [21] presenting the axiomatics of the modern probability theory, it was explicitly pointed out that each complex of experimental conditions (in our terminology - context) generates its own probability space, see section 2 [21]. Unfortunately, Kolmogorov did not label probability spaces by the context index. And it appears that this very important interpretational issue was completely missed by readers of his book. Later he published a paper (but in Russian) explaining the contextuality of probability, see [22]. Here all probabilities were labeled by corresponding contexts, he used notation \( P(A|C) \). He stressed that in general this is not the usual classical conditional probability given by the Bayes formula. Unfortunately, this paper is practically unknown (I got to know about it from the former students of Kolmogorov, Bulinskii and Shiryaev, in a private conversation). Moreover, already in the 19th century Boole (who presented one of the first rigorous mathematical models of probability [23], [24]) pointed to the existence of incompatible observables and studied conditions for the existence of the common probability representation for a group of observables. He was the first who derived the inequality which nowadays is known as the Bell inequality. Later in the early 1960s Soviet probabilist Vorob’ev investigated this problem in great detail [25]. He derived all inequalities which then were rediscovered by physicists. However, the main stream of probability theory ignored this important problem and the interest to Boole-Vorob’ev type inequalities was generated only after the works of Bell who derived independently one of such inequalities. We stress once again that in mathematics such studies were considered as studies about compatibility of observables and the possibility to represent them in a common probability space.

4 Does the Copenhagenist need nonlocality?

The Bell argument is based on the violation of some inequalities for joint probabilities or correlations [16, 22, 27, 28], which can be derived only under the assumption that there exists the common (Kolmogorov) probability space \( \mathcal{P} \) in which all observables involved in such inequalities can be jointly represented as random variables. In physical terms these observables should be compatible. A follower of the Copenhagen interpretation sharing the views of Niels Bohr would not be able to derive the Bell inequality for observables considered in Bell’s argument. Thus he would say that Bell’s argument contradicts the Copenhagen interpretation of quantum mechanics.

We want to stress that one who accepts the Copenhagen interpretation would not need nonlocality to explain a violation of Bell’s inequality for quantum observables. For him, this is the direct consequence of the principle of complementarity. For Copenhagenist, an experimental violation of Bell’s inequality is just another confirmation of the principle of complementarity, so to say a complement to the interference experiment. (In fact, the situation is even simpler: experiments to violate Bell’s inequality can be reformulated in the form of interference experiments [29]).

Of course, someone who rejects the Copenhagen interpretation can be excited by Bell’s attempt to go beyond quantum theory. But even for such a person, the initial guess would be that the principle of complementarity might be violated. At least its violation sounds not as mystical as nonlocality.

5 Are novel quantum technologies based on spooky action at a distance?

There is the very common opinion that the present quantum technological revolution is based on “new quantum mechanics” and that quantum nonlocality is the most important foundational component of this revolution. I am not sure that this opinion is justified. It seems that com-
plementarity (interference) is the most basic foundational condition of successful development of quantum technologies. Consider the project on quantum random generators. “Irreducible quantum randomness” (what ever it means) was invented by von Neumann and it is totally based on complementarity, see (B1), section 2. Quantum cryptography is a consequence of complementarity, see (B3), section 2.

Now we move to the most exciting feature of quantum systems - entanglement. However, by itself an entangled state $\psi \in H_1 \otimes H_2$ represented as superposition of the form

$$\psi = \sum_{ij} c_{ij} |i\rangle |j\rangle, \quad (5.1)$$

for the two fixed bases $|i\rangle_1$ and $|j\rangle_2$ (in corresponding Hilbert state spaces) does not exhibit nonclassical features. Nonclassicality arises from the possibility to operate with superpositions with respect to “incompatible bases”, i.e., complementarity of the observables of the form $A_k \otimes B_k$, $k = 1, 2, \ldots$. Of course, the conventionally oriented fellow would emphasize that nonseparability is not a consequence of complementarity. And this is correct. But, nonseparability is just the quantum mechanical representation of correlations between degrees of freedom of two systems. If $\psi$ is separable, $\psi = \psi_1 \otimes \psi_2$, then mathematical expectations are factorizable:

$$\langle A \otimes B \rangle = \langle A \rangle \langle B \rangle \quad (5.2)$$

(there are no correlations at the level of mathematical expectations of the observables). If $\psi$ is entangled, then we can find two such observables $A$ and $B$ that

$$\langle A \otimes B \rangle' = \langle A \rangle \langle B \rangle. \quad (5.3)$$

In quantum information theory loudly emphasizes the entangled states are more powerful information resource than separable states. But who would be surprised that correlations carry the additional information resource? See paper [30] on entanglement for classical Brownian motions for further discussion.

Now we turn to the most complicated issue, quantum computing. I again claim that complementarity, i.e., the existence and the possibility to process in parallel complementary representations of information, is the crucial issue in justification of quantum computing. Entanglement provides an additional (and very important) computational resource as carrying correlations which are produced in the processes of preparation and the dynamical performance of quantum algorithms. The latter can be based either on the unitary Schrödinger dynamics or dynamics described by theory of open quantum systems, e.g., in its approximate form based on Gorini-Kossakowski-Sudarshan-Lindblad equation. It would be interesting to present this heuristic statement in the formal mathematical framework.

Since we started to speak about the state dynamics, this is a good place to point to one of the main distinguishing contributions of quantum information theory: emphasizing the possibility of preserving complementarity and correlations encoded in entangled state during the state dynamics. Of course, it was important to find the right quantum variables with such features. In fact, it was done by Bohm who reformulated the EPR-argument by using electron’s spin projections, instead of the position and momentum observables. This approach was automatically extended to photon’s polarization projections. The following experimental research to demonstrate preserving of correlations and complementarity (culminated in [2–4]) played the determining role in development of quantum information theory and modern quantum technologies.

Thus “new quantum mechanics” based on the quantum information theory foundationally is the same as “old quantum mechanics” of Bohr and Heisenberg. Finally, we remark that all basic foundational features of entangled states can be found already in the book of von Neumann (although he did not use the term “entanglement”). Von Neumann treated entanglement in the framework of Bohr’s complementarity, see [31] for English edition.

6 Concluding remarks

One cannot sit on two chairs. Either you follow the conventional quantum theory and the incompatibility interpretation of observables represented by non-commuting operators in this theory or you involve the so-called “quantum nonlocality”.

Even those who reject Bohr’s views and wish to go beyond quantum theory, should rather question the principle of complementarity and the existence of incompatible observables.

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5 Although nowadays this project seems to be not more actual from the viewpoint of real applications, it is, nevertheless, very interesting from the foundational viewpoint.

6 By the modern terminology: preventing decoherence.

7 I repeat once again that this note is concentrated on the foundational issues. “New quantum mechanics” contains huge body of technical results.
Those who are interested in quantum nonlocality must find a possibility to separate the issues of nonlocality and incompatibility. If this mystical action at a distance cannot be uncoupled from complementarity then, by Occam's Razor argument, nonlocality should be rejected as unnecessary addition to the theory.

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References


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