Modelling axial vibration in windings of power transformers

Abstract: This paper describes the method of homogenization of material properties applied to windings used in power transformers. Exemplary results of natural modes of vibrations obtained by means of finite elements method are also included.

Keywords: power transformer; winding; vibration; finite elements

1 Introduction

The windings in power transformers have a complex and composite structure which is mechanically highly anisotropic and relatively weak, especially in high voltage units where the amount of insulation material is substantial. The presence of insulation, whose rigidity is about two hundred times less than for copper, is most important when deformations in axial direction are considered. The proper estimation of these phenomena is important for the analysis of the winding resistance against operational short-circuit [1] and also for the winding vibration at load [2–6]. In either case arises the question of introducing an equivalent material for the winding area because the exact representation of complex geometry and material heterogeneity would require unacceptable computational effort. The significance of winding vibration in power transformers is clearly visible for units with powers of 100 MVA or greater for which the load noise dominates that of magnetostriction origin.

2 Winding structure in power transformers

The typical structure of low and high voltage windings in large power transformers consists of a series of coils axially separated by a set of spacers uniformly distributed along the winding circumference and forming the radial cooling ducts filled with oil. The coils are usually wired with continuously transposed cable (CTC). The above system includes a set of repetitive portions with the same geometry, material structure and electromagnetic excitation. Each portion consists of sectors of a single coil accompanied with four quarters of spacers. The outlook of a sector together with a cross-section of CTC is displayed in Figure 1. The distribution of mechanical and electromagnetic stress acting on this elementary section is quite complex and it depends on the section’s position in the winding. The all further analysis will concern the excitations and constraints in the axial direction only.

3 Principle of homogenization

The aim of this investigation is to find the equivalent properties of a homogeneous material having the same outer dimensions \((d_0, \alpha_0, h_0)\) as the sector in Figure 1 and simultaneously, reacting in the same way under external exci-
tation. The equation governing this analysis is the virtual work principle

$$\iiint_{S(V)} \sigma_{ij} n_j u_i \, dS = \iiint_{V(S)} \sigma_{ij} \epsilon_{ij} \, dV$$ \hfill (1)

where $\sigma_{ij}$, $\epsilon_{ij}$ are stress and strain and $u_i$ is a virtual displacement. The condition of static equilibrium requires the null value of displacement along the part of outer boundary. If the homogenous volume, having a constant cross-section $S_h$ normal to $0z$ axis, is subjected to the unidirectional load created by axial displacement of the boundary surface $u_{z0}$ the equation (1) converts into

$$p_h S_h u_{z0} = E_z \left( \frac{\partial u_z}{\partial z} \right)^2 S_h h_0$$ \hfill (2)

where $p_h$ denotes the pressure on the displaced surface. The strain varies linearly in this case and therefore, we obtain Hooke’s law

$$p_h = E_z \frac{u_{z0}}{h_0}$$ \hfill (3)

In other words, the homogenization of some volume of interest means that having the same displacement and elastic energy we can approximate the integral equation

$$F_i = Ku_i$$ \hfill (4)

introducing the artificial stiffness $K$ linking the components of force $F_i$ and displacement $u_i$ which are mean quantities for the given volume.

4 Numerical model of winding sector

The transformer windings are significantly pre-stressed in the axial direction. It means that CTC filaments in the area directly under spacers may be assumed to be tightly connected by the friction forces, but outside they can move more or less independently. To designate the equivalent Young modulus in the axial direction a numerical model consisting of two CTC filaments, together with four quarters of a spacer, was developed. Its outlook is shown in Figure 2.

The model is subjected to virtual, axial symmetric displacements $u_{zm}$ (the same as in the homogenous case) and constrained in the circumferential direction. Results of the calculations are presented in Figure 3.

Observing fields shown in Figure 3 we see that the energy of the deformations $\sigma_{zz} \epsilon_{zz}$ is stored in the spacers only, besides, in a uniform manner. It is worth noting that shear stresses are almost absent on the outer surface of the model. The choice of the equivalent homogenous material is made assuming the same axial outer displacement and total energy stored as in a real case. The above remarks lead to the following equation resulting from (1) and (2)

$$E_s \left( \frac{u_{z0}}{h_s} \right)^2 V_s = E_z \left( \frac{u_{z0}}{h_0} \right)^2 V_h$$ \hfill (5)

where $V_s$ and $h_s$ are the volume and height of the spacer and $V_h$ and $h_0$ refer to the homogenous material. After simple manipulations we get an expression describing the relation between the values of axial Young modulus $E_z$ of the equivalent homogenous structure and of the spacer’s material $E_s$

$$E_z = E_s \frac{h_0}{h_s} \frac{a_0 - a_1}{a_0}$$ \hfill (6)

Geometric dimensions are presented in Figure 1. Inserting their values for 120 MVA units we have for HV and LV windings produce the following results: $E_{zHV} = 1.47 E_s$ and $E_{zLV} = 1.58 E_s$.

Quite often CTC are hardened by their epoxy filler during the drying process of transformer windings. In such a case we cannot assume that CTC filaments directly contact...
themselves and we must take into account the amount of relatively soft resin in between. A finite elements model showing the sector of an exemplary CTC is presented in Figure 4. The number of filaments inside is even not odd in order to simplify the analytic expressions. Bearing in mind that the radius of transformer coils is much larger than their thickness, the Cartesian system is used but preserving the notation of axes like in cylindrical coordinates.

The virtual displacement was applied along the 0r axis for points belonging to external planes normal to that direction. The remaining outer boundary was left free, creating the one-dimensional load case. Distributions of dominating strain and stress components are presented in Figure 5. The strain has a non-zero value in the areas of epoxy filler only, therefore, the elastic energy is just stored there as well. Observing the profile of displacement $u_r$ shown in Figure 6 along axis $n_r$ presented in Figure 4, we may conclude that each layer of resin contains almost the same amount of elastic energy.

The outer dimensions of the CTC cross-section are $h_r$, $h_z$ and $h_\alpha$ (see Figure 4) and the relative volumes of filler along axes $n_r$ and $n_z$ are $g_r$ and $g_z$. Denoting the virtual displacement value by $u_0$ and Young modulus of filler by $E_f$ we may compute the elastic energy stored in the CTC from

$$W = E_f \frac{u_0^2}{h_z^2} V_{CTC}^2$$  \hspace{1cm} (7)

where $V_{CTC}^a$ denotes the active volume of the filler

$$V_{CTC}^a = g_r h_r h_a (1 - g_z) h_z$$  \hspace{1cm} (8)

The same amount of energy stored in the homogenized equivalent material under the same virtual displacement in the radial direction is given by

$$W = E_{er} \frac{u_0^2}{h_z^2} h_r h_a h_z$$  \hspace{1cm} (9)

what immediately results with the value of equivalent material property in the radial direction is

$$E_{er} = E_f \frac{1 - g_z}{g_r}$$  \hspace{1cm} (10)

The property of CTC in the axial direction is obtained by the simple exchange of subscripts

$$E_{ez} = E_f \frac{1 - g_r}{g_z}$$  \hspace{1cm} (11)

When the winding has radial cooling ducts we must repeat the analysis of axial deformation described earlier but now with two areas having different elastic properties: $E_s$ for spacer and $E_{ez}$ for CTC. Applying the one-dimensional virtual displacement $u_0$ to the outer surfaces of the spacers we must find the unknown displacement $u_1$ on the boundary between spacer and CTC. It results from

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure4}
\caption{Numerical model of CTC}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure5}
\caption{Strain and stress fields inside CTC cross-section (white is null value)}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure6}
\caption{Radial displacement field inside CTC cross-section}
\end{figure}
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stress equilibrium
\[ \sigma_{zz} = E_s \frac{u_0 - u_1}{h_s} = E_{ez} \frac{u_1}{h_z} \]  \hfill (12)

where \( h_s \) is spacer thickness.

Elastic energy stored now has two components \( W_1 \) for spacer, \( W_2 \) for CTC and it is calculated from

\[ W_1 = E_s \left( \frac{u_0 - u_1}{h_s} \right)^2 h_s (a_0 - a_1) \]
\[ W_2 = E_{ez} \left( \frac{u_1}{h_z} \right)^2 h_z (a_0 - a_1) \]  \hfill (13)

The sum of \( W_1 \) and \( W_2 \) must be equal to energy in the homogenous material \( W \) having larger axial and circumferential size

\[ W = E_{eqz} \left( \frac{u_0}{h_z + h_s} \right)^2 h_z (h_z + h_s) a_0 \]  \hfill (14)

The equivalent material property in the axial direction of the winding sector containing the CTC turns and spacer is noted here by \( E_{eqz} \). Equating (12) and (13) and making some simple manipulations we get the final expression

\[ E_{eqz} = \frac{1}{\frac{1}{E_s} + \frac{g_s}{E_{ez}}} \frac{a_0 - a_1}{a_0} \]  \hfill (15)

where \( g_s \) is the relative size of the spacer

\[ g_s = \frac{h_s}{h_z + h_s} \]  \hfill (16)

Equation (14) is an extension of (6) for two elastic materials subjected to the same stress. It is necessary to remember that amount of insulation material inside CTC depends heavily on its type and its stiffness has a non-linear shape against the pre-stress value. The remaining moduli of an orthotropic material, namely \( E_{eqr} \) and \( E_{eq\alpha} \) are calculated in an analogous way. It should be underlined that material having orthotropic properties needs to also specify three shear moduli and three Poisson factors. The shear moduli are obtained by similarly applying the virtual displacement but now in tangent direction. The Poisson factors are calculated from their definition as the ratio of strain along two perpendicular axes.

5 Exemplary results

The winding area in a 120 MVA transformer was modelled using solid and shell elements having orthotropic properties, where windings with axial wedges were constructed within cylindrical coordinates but support plates were modelled under the Cartesian system. All materials used in the winding zone have a significant anisotropy. For example, the equivalent material in a HV winding has the following properties along cylindrical coordinates: \( (E_r, E_\alpha, E_z) = (3.2, 67.8, 0.54) \) GPa and the pressboard is represented in Cartesian system by \( (E_x, E_y, E_z) = (9.1, 6.8, 0.5) \) GPa.
6 Conclusions

The analysis presented above clearly indicates the necessity of representation of transformer windings for vibration investigations by anisotropic materials. The main difficulty here is to get the proper values of these parameters which depend on pre-stress of the winding zone and also on distribution of insulation inside the CTC area. The careful measurements of particular properties of elements of winding structure connected with a mathematical model of the winding are essential for the accuracy of the final results.

References