Three-dimensional atom localization via probe absorption in a cascade four-level atomic system

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Abstract: For an atomic system with cascade four-level type, a useful scheme about three-dimensional (3D) atom localization is proposed. In our scheme the atomic system is coherently controlled by using a radio-frequency field to couple with two-folded levels under the condition of the existence of probe absorption. Our results show that detecting precision of 3D atom localization may be obviously improved by properly adjusting the frequency detuning and strength of the radio-frequency driving field. So our scheme could be helpful to realize 3D atom localization with high-efficiency and high-precision. In the field of laser cooling or the atom nano-lithography, our studies provide potential applications.

Keywords: atom localization, probe absorption, coherent manipulation

PACS: 42.50.-p, 42.50.Ct

1 Introduction

With the development of the technology of quantum coherent control, the research about atom localization has developed rapidly, and some researchers pay attention to this physical field. Via manipulation of laser, several pioneering works about atom localization has been proposed, where position of an atom going through a standard light field can be localized by carrying out a phase measurement on a probe field [1–3], and also be localized by using Ramsey interferometry [4], and other optical methods [5, 6]. Subsequently, in the papers of Zubairy and his coworkers, they have given some rather simple schemes in which they restrict the space of an atomic notion via applying the technology of Autler-Townes microscopy, detecting photons of spontaneous emission, and controlling phase and amplitude of the absorption spectra [7–9]. In Zubairy’s schemes, spontaneous emission need to be well controlled. We know it is difficult to effectively control spontaneous radiation in experiment, so some other new schemes [10–16] have been introduced, in which one can obtain atom localization by measuring atomic population of the upperstate or detecting absorption spectra of probe field, and by means of some coherent process, such as coherent population trapping, double-dark resonances, or coherent manipulation of the Raman gain. Besides these works in the field of theory, recently, atom localization in one-dimensional space has been achieved in a proof-of-principle experiment with the help of the technique of electromagnetically induced transparency [17].

Along with deep studies about atom localization, several theoretical schemes about atom localization in two-dimensional space have also been put forward by using two standing-wave light fields whose propagation direction is orthogonal to each other to couple with atomic system. For example, taking a measurement on the atom population of ground state or of upper state, Ivanov’s research group gave a new scheme where they obtained atom localization in two-dimensional space [18]. Later, a number of research papers [19–25] about atom localization in two-dimensional space have been published successively.

For an atom with motion, researchers prefer to limit its motion in three-dimensional space, so how to realize the atom localization in three-dimensional space begins to be concerned. Several schemes [26–28] about atom localization in three-dimensional (3D) space in various systems of atom have been reported. However, 3D atom localization based on different coupling mechanisms via taking a measurement on absorption and gain spectrum of the probe field is not considered. In this paper, we study a four-level atomic system with cascade type and we investigate its three-dimensional atom localization through detecting the absorption and gain spectra of the probe field. In our system, interaction between field and atom is dependent on space coordinates, so we can use the technol-
ogy of detecting probe absorption and gain spectra to determine the position probability distribution of the atoms when they go through standing waves field. There are two channels of excitation in the closed interacting system: one is one-photon excitation and the other is three-photon excitation that depends on phase, and the quantum interference between two channels helps in achieving 3D atom localization. Through measuring absorption of probe field or its gain at a special frequency, we can find atom with 100% probability in three-dimensional space. By contrast with some schemes introduced in references [26–28], our scheme is added by a factor of 4 or 8.

2 The physical model and dynamic evolution

An atomic system with four cascade energy levels is studied and its concrete atomic structures are shown in Figure 1. States $|1\rangle$ is a ground state, $|2\rangle$ is an intermediate state, and the top two states $|3\rangle$ and $|4\rangle$ are two-folded levels. Where we use three standing-wave fields these are orthogonal to each other and whose Rabi frequency $2\Omega (j \sin (kj)(j = x, y, z; k = \omega / c)$ depends on position to couple with the transition between levels $|2\rangle$ and $|3\rangle$, and the total Rabi frequency $2\Omega_0(x, y, z)$ may be expressed as $2\Omega_0(x, y, z) = 2\Omega(x) \sin (kx) + 2\Omega(y) \sin (ky) + 2\Omega(z) \sin (kz)$. A weak probe field with angular frequency $\omega_p$ and Rabi frequency $2\Omega_p$ interacts with the atomic system and excites the transition between levels $|2\rangle$ and $|1\rangle$. A radio-frequency field with angular frequency $\omega_{rf}$ and Larmor frequency $2\Omega_{lf}$ is applied to drive transition between the two-folded levels $|3\rangle$ and $|4\rangle$.

Here we neglect the kinetic part of the atom from the Hamiltonian according to the Raman-Nath approximation, and suppose that the center-of-mass coordinate of the atom almost does not change with time along with the directions of the laser when the intensity of Rabi frequency of the laser field is big enough. Under the condition of rotation wave approximation, the interaction Hamiltonian of this system is given by

$$H_I = (\Delta_{xyz} + \Delta_p) |3\rangle \langle 3| + \Delta_{lf} |4\rangle \langle 4| + \Omega_s(x, y, z) |3\rangle \langle 2| + \Omega_p |2\rangle \langle 1| + \Omega_{lf} |4\rangle \langle 3| + H.c.),$$

where we let $h = 0$, $\Delta_{lf} = \omega_{s3} - \omega_{lf}$, $\Delta_{xyz} = \omega_{s2} - \omega_{xyz}$, and $\Delta_p = \omega_{p21} - \omega_p$ are corresponding to detuning of the radio-frequency field, the standing-wave field and the probe field, respectively, and $\omega_{s3}$, $\omega_{s2}$, and $\omega_{p21}$ are corresponding to transition frequency between levels $|4\rangle$ and $|3\rangle$, between levels $|3\rangle$ and $|2\rangle$, and between levels $|2\rangle$ and $|1\rangle$, respectively.

According to the equation satisfied by the evolution of density matrix elements

$$\frac{d\rho_{mn}}{dt} = \frac{1}{i\hbar} \sum_k (H_{mk} \rho_{kn} - \rho_{mk} H_{kn})$$

where $H_{mn}$ stands for relaxation matrix elements, and it can be expressed as $\langle n | H | m \rangle = \gamma_n \delta_{nm}$, and $\gamma_n$ is the decay rate of level $|n\rangle$. Based on Eqs. (1) and (2), we can obtain the first derivative of density matrix elements with time with:

$$\frac{d\rho_{11}}{dt} = \gamma_2 \rho_{22} - i\Omega_p \rho_{12} + i\Delta_{lf} \rho_{21},$$

$$\frac{d\rho_{22}}{dt} = -\gamma_2 \rho_{22} + \gamma_3 \rho_{33} + \gamma_4 \rho_{44} + i\Omega_p \rho_{12} - i\Delta_{lf} \rho_{21} - i\Omega_4 \rho_{23} + i\Omega_s(x, y, z) \rho_{32},$$

$$\frac{d\rho_{33}}{dt} = -\gamma_3 \rho_{33} + i\Delta_{lf} \rho_{31} - i\Omega_4 \rho_{34},$$

$$\frac{d\rho_{44}}{dt} = -\gamma_4 \rho_{44} + i\Omega_4 \rho_{34} - i\Delta_{lf} \rho_{43},$$

$$\frac{d\rho_{21}}{dt} = -\left(\frac{\gamma_2}{2} + i\Delta_{lf}\right) \rho_{21} + i\Omega_4 \rho_{23} - i\Omega_p (\rho_{22} - \rho_{11}),$$

$$\frac{d\rho_{31}}{dt} = -\left[\frac{\gamma_3}{2} + i \left(\Delta_{xyz} + \Delta_{lf}\right)\right] \rho_{31} + i\Omega_s (x, y, z) \rho_{21} + i\Delta_{lf} \rho_{41} - i\Omega_p \rho_{32},$$

$$\frac{d\rho_{41}}{dt} = -\left[\frac{\gamma_4}{2} + i \left(\Delta_{xyz} + \Delta_{lf}\right)\right] \rho_{41} + i\Omega_s (x, y, z) \rho_{21},$$

$$\frac{d\rho_{32}}{dt} = -\left[\frac{\gamma_3}{2} + i \Delta_{xyz}\right] \rho_{32} + i\Omega_4 \rho_{31} - i\Omega_p \rho_{32} + i\Delta_{lf} \rho_{42},$$

$$\frac{d\rho_{42}}{dt} = -\left[\frac{\gamma_4}{2} + i \Delta_{xyz}\right] \rho_{42} + i\Omega_4 \rho_{41}.$$
where $\rho_{11} + \rho_{22} + \rho_{33} + \rho_{44} = 1$, $\gamma_2$, $\gamma_3$ and $\gamma_4$ are decay rates for $|2\rangle \rightarrow |1\rangle$, $|3\rangle \rightarrow |2\rangle$ and $|4\rangle \rightarrow |2\rangle$, respectively. Based on the theory of light propagating in atomic medium, we get the expression of the complex susceptibility for the probe field

$$\chi_p = \frac{N|\mu_{21}|^2}{\epsilon_0 i\Omega_0}$$

where physical parameter $\epsilon_0$ is the free space permittivity and $N$ is the atom number density of four-level atomic system.

According to the theory about light propagating in medium, we know the imaginary part $\text{Im}(\chi_p)$ of the susceptibility can be applied to describe absorption of probe field. To simplify the formula, we let $\delta_2 = \frac{\gamma_2}{2} + i\Delta_p$, $\delta_3 = \frac{\gamma_3}{2} + i(\Delta_{xyz} + \Delta_p)$, and $\delta_4 = \frac{\gamma_4}{2} + i(\Delta_{xyz} + \Delta_p + \Delta_{rf})$. So from Eqs. (3) and (4), the expression about $\text{Im}(\chi_p)$ is given by

$$\text{Im}(\chi_p) = \text{Im}\left(\frac{\delta_3\delta_4 + \Omega_{rf}^2}{\delta_2\Omega_{rf}^2 + \delta_4\Omega_{xyz}^2(x,y,z) + \delta_4\delta_2} - i\right)$$

$$= \frac{A(\frac{\gamma_3}{2} - \Delta_4\Delta_3 + \Omega_{rf}^2)}{A^2 + B^2}$$

where $\Delta_3 = \Delta_{xyz} + \Delta_p$, $\Delta_4 = \Delta_{rf} + \Delta_{xyz}$, $A = (\gamma_4\Omega_{xyz}^2(x,y,z) + \gamma_2\Omega_{xyz}^2 - \Delta_4\Delta_3 - \Delta_3\Delta_4 - \Delta_4\gamma_3 - \Delta_3\gamma_2 - \Delta_4\Delta_p\gamma_3 + \frac{\gamma_4\gamma_2}{4})/2$, $B = \Delta_4\Omega_{xyz}^2(x,y,z) + \Delta_p\Omega_{xyz}^2 - \Delta_4\Delta_p - (\Delta_3\gamma_2 + \Delta_p\gamma_3 + \Delta_4\gamma_3\gamma_4)/4$.

Eq. (5) tells us that the imaginary part $\text{Im}(\chi_p)$ depends on not only space coordinates $(x,y,z)$ and the detunings $\Delta_p, \Delta_{xyz}, \Delta_{rf}$ between natural frequency of atom and light fields, but also the Rabi frequencies $\Omega_{rf}$ and $\Omega_0(x,y,z)$. Accordingly, we can use absorption of the probe field to detect distribution of atoms in space. In the process of numerical calculations, we will determine probe absorption $\text{Im}(\mu_{21}/\Omega_0)$ and measure the probe absorption depending on space position. Then informations about the atomic space distribution are obtained, and finally realize atom localization in three dimensional. Through choosing appropriate parameters of system, we can control the localization behavior of the atom in 3D space and improve the precision of detection of atomic position.

### 3 Localization structures

In this section, we will give some results of numerical calculations based on the imaginary part of the probe absorption under the condition of choosing different parameters of the system. Numerical analyses show measuring probe absorption can help one to achieve high-efficiency and high-precision 3D atom localization under the condition of choices of appropriate parameters. In the following process of numerical calculations, we keep $\gamma_2, \gamma_3$, and $\gamma_4$ at the same values. Here, an atomic structure can be realized in cold $^{87}$Rb atoms using the transitions $5S_{1/2} - 5P_{1/2} - 5D_{3/2}$. In our system, the selections of atomic levels are: $|1\rangle = |5S_{1/2}, F = 2\rangle$, let $|5P_{1/2}, F = 2\rangle = |2\rangle$, $|5D_{3/2}, F = 1\rangle$ is set to $|3\rangle$, and $|5D_{3/2}, F = 2\rangle$ is arranged as level $|4\rangle$. In this real atomic system, the decay rates $\gamma_2 = 5.3\text{Mhz}$ and $\gamma_3 = \gamma_4 = 0.67\text{Mhz}$, respectively.

![Figure 1: Schematic diagram of a four-level atomic system](image-url)
more and more smaller. We continue to increase the value of $\Delta p$ until detuning $\Delta p = 6.5\gamma_2$ (Figure 2(d)), the sphere occupies a very small volume in space becomes very small, and here our spatial resolution is about $0.1\lambda$. From Figure 2(d), we find we only carry out measurement on absorption of probe field in a very small space, that is position of atoms is confined in this narrow space. Figure 2 shows that the precision of detecting position of atoms can be improved if we increase gradually the value of detuning of weak probe light. We can use the quantum coherence to explain the phenomenon of atom localization. Under the quantum coherent manipulation, one can build a quantum correlation between the detuning and absorption for the weak probe light. This type of strong correlation evidently affects the properties of absorption of medium when changing the detuning of the weak probe field, and then distribution of absorption of probe light in three-dimensional space will be influenced. So choosing appropriate parameters and by means of quantum coherence, various structure diagrams about atom localization in three-dimensional space are obtained in Figure 2.

Numerical results about atom localization in three-dimensional space is shown in Figure 3 when we consider the case with changing detuning of standing-wave field $\Delta_{xyz}$. Firstly we let $\Delta_{xyz} = \gamma_2$, and structure diagram about distribution of the probe absorption is plotted in the three-dimensional space ($-1 \leq k/j/\pi \leq 1, j=x, y, z$) [see Figure 3(a)]. The shape of structure diagram is a sphere and its volume is biggest in Figure 3, which means we do not obtain atom localization. Next, we increase the values of $\Delta_{xyz}$, and its values are set to be $2\gamma_2$ [Figure 3(b)] and $4\gamma_2$ [Figure 3(c)], respectively, we can still plot a sphere in the three-dimensional space. However, comparing with Figure 3(a), the size of spheres grow smaller. Which means, increasing the detuning of standing-wave field, the probability of detecting the atom can approach 1 all the time and the precision has been improved. When values of $\Delta_{xyz}$ reaches $5\gamma_2$ [Figure 3(d)], a very small sphere appears in the whole three-dimensional space, that is motion of atoms is confined in the tight range. Here the spatial resolution is still about $0.1\lambda$, so atom localization in three-dimensional is realized with high probability and high precision.

In this paper, a cascade four-level atom is considered, and its top two levels are fine structures and can be treated as two-folded levels. The coupling between two-folded levels can be carried out with the help of radio-frequency driving field or a microwave field. Here we use radio-frequency. Relative to a laser field, radio-frequency source has some advantages such as easy to control and easy to adjust. In final part of this paper, we numerically analyze how the intensity of radio-frequency field affect atom localization in the three-dimensional space. Here we only adjust Larmor frequency $\Omega_{rf}$ of radio-frequency field and fix values of other parameters. When the $\Omega_{rf} = \gamma_2$, we can see a small sphere in the three-dimensional space ($-1 \leq k/j/\pi \leq 1, j=x, y, z$) [see Figure 4(a)]. Going on changing the value of $\Omega_{rf}$ and letting its values equal to $3\gamma_2$ and $5\gamma_2$, respectively,
there is remain only one sphere, respectively showed in Figure 4(b) and Figure 4(c), in each space but its size is getting bigger and bigger. When the value of Larmor frequency $\Omega_{rf}$ continues to be increased and reaches $6\gamma_2$, we get a bigger sphere in three-dimensional space, and it is the biggest sphere in Figure 4, which means the precision of the localization becomes low step by step with increasing the intensity of radio-frequency field. So if we want to realize achieve the three-dimensional atom localization with high precision and high efficiency, choices of intensity of radio-frequency field needs to be carefully considered and we should keep it with an appropriate value.

**4 Conclusion**

To sum up, through detecting the size of spatial region of probe absorption, the three-dimensional atom localization in a coherently driven atomic system with cascade four levels is investigated in detail. Through analyzing situations with different physical parameters, we find one can significantly improve the precision and efficiency of atom localization in three-dimensional space under the condition with appropriate physical parameters. Our results show that we can confine the motion of atom in a narrow space, and we can find atom in this region with 100% probability. Our scheme is more precise than that in other research [28]. In our atomic system, we use a radio-frequency field to couple with a transition between two hyperfine levels, and numerical results show weak radio-frequency field can help us to get efficient 3D atom localization. Compared with laser, the radio-frequency is easier to operate in experiment, so our scheme has feasibility in the experiment. In addition, atom localization with high precision and high efficiency has some potential practical value in some fields such as atom lithography, laser cooling and so on.

**Acknowledgement:** This work is supported by the National Natural Science Foundation of China (Grant Nos. 11365009 and 11775190), and by Science Foundation of Zhejiang SCI-TECH University under Grant No. 17062071-Y.

**References**

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