Research Article

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Multivariable polynomial fitting of controlled single-phase nonlinear load of input current total harmonic distortion

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Abstract: The power systems usually include a number of nonlinear receivers. Nonlinear receivers are the source of disturbances generated to the power system in the form of higher harmonics. The level of these disturbances describes the total harmonic distortion coefficient THD. Its value depends on many factors. One of them are the deformation and change in RMS value of supply voltage. A modern LED luminaire is a nonlinear receiver as well. The paper presents the results of the analysis of the influence of change in RMS value of supply voltage and the level of dimming of the tested luminaire on the value of the current THD. The analysis was made using a mathematical model based on multivariable polynomial fitting.

Keywords: curve fitting, total harmonics distortion, single-phase nonlinear loads, LED luminaire

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1 Introduction

Nonlinear receivers are the source of many negative phenomena occurring in the power system. They receive the distorted currents from the power system which, when flowing by the power systems elements, result in voltage drops caused by higher harmonics of current. Consequently, this causes distortion of the supply voltage. The distorted current causes increased heating of wires and other power systems elements, as well as insulation aging, increased power losses, increased probability of resonance, and many more. In three-phase systems, another problem concern the occurrence of current in neutral conductor.

Nowadays, more devices are equipped with SMPS power supplies. Such devices are receivers with nonlinear current-voltage characteristics e.g. LED luminaires that are equipped with SMPS power supplies. Modern luminaire solutions allow to control the luminous flux. This is accomplished by controlling the current or output voltage of the power supply. Usually, most of power supplies are equipped with PFC systems which are used to improve the power factor. They are generally designed for increasing the rated power of the power supply. In the case of power supplies, the influence of the level of control on the generated disturbances is observed. With the decrease of the dimming level of control gear, THD coefficient of the current increases and the power factor decreases. In addition, the $THD_I$ is influenced by a number of factors: the power supply design and algorithm, the level of the supply voltage distortion and its RMS value, the control level.

The voltage and current $THD$ coefficients are described by (1) and (2).

$$THD_U = \frac{U_H}{U_1} = \sqrt{\left(\frac{U}{U_1}\right)^2 - 1} \quad (1)$$

$$THD_I = \frac{I_H}{I_1} = \sqrt{\left(\frac{I}{I_1}\right)^2 - 1} \quad (2)$$

According to standard [23] for steady-state and non-sinusoidal currents and voltages, the RMS values of voltage $U$ and current $I$ have two components:

$$U^2 = U_0^2 + U_H^2 \quad (3)$$

$$I^2 = I_0^2 + I_H^2 \quad (4)$$

A component with a lower index 0 denotes a constant component. For other components, the index is the actual harmonic or voltage number.

$$U_H^2 = U_0^2 + \sum_{H=1} U_H^2 = U^2 - U_1^2 \quad (5)$$
$$P_H^2 = P_0^2 + \sum_{H \neq 1} P_H^2 = I^2 - I_1^2$$  \hfill (6)

A new problem with the generation of harmonics is the fact that increasingly responsible for this phenomenon are the receivers with low power although present in large quantities. In particular, it concerns adjustable light sources and luminaires dimmed light sources and luminaires.

Many analytical methods are used to evaluate this effect. One method uses a simplified schematic diagram of a nonlinear load described by time differential equations based on the knowledge of basic electrical laws [1]. Another method include replacing the nonlinear receiver with a nonlinear resistance [4]. In the literature [12, 17, 19], a method of creating a mathematical model of a nonlinear equipment based on measurements is presented. It is based on accepting the input and output quantities which characterize the given nonlinear equipment in the studied phenomena. The method described was used to assess the impact of computer power supplies on the quality of electricity in the mains [12, 14, 17, 18, 20].

As a tested object the LED luminaire with controlled power and light flux was selected. For the analysis, the methods described in [6–8] using the measurement results of a real object were used. Using these methods a mathematical model was developed. Input values of the model are the RMS value of the supply voltage and the control level. The output value of the model is the value of the THD coefficient. Mathematical model is a n-th degree polynomial of two input variables. The model was developed using the Curve Fitting Toolbox available in MATLAB™.

2 Laboratory results

LED luminaire equipped with power supply with analogue control input made of standard 1·10V has been subjected to the laboratory tests. This luminaire has rated power equal to 32W. The RMS value of the supply voltage was varied in the range 230V ± 10% (from 207 to 253 V). The limits of voltage variations are described in EN 50160 [21] for acceptable voltage changes in low voltage networks.

The second input values was the input control level of the luminaire, determined by the value of the control voltage from 1V to 10V, with a step equal to 1 volt. The output value was the value of the THD coefficient. Measurements were made using the FLUKE 1760 which is an electrical power quality analyzer. The laboratory power supply as a source of DC control voltage was used.

Figure 1 shows the relation of the power of the lighting fixture $P$ as a function of the supply voltage $U_S$ and control voltage $U_C$.

![Figure 1](image1.png)

Figure 2 shows two different areas of variation. For control voltages greater than approx. 5V, the THD coefficient shows a strong upward trend.

![Figure 2](image2.png)

The presented dependence is characteristic for the tested luminaire. Other types of luminaires with control...
systems may had different dependencies while the general nature of the dependency changes are similar.

For the illuminated luminaire, the characteristics of the THD₁ coefficient are described by two polynomials according to (7):

\[ THD_1 = \begin{cases} 
  THD_{I-15}(U_c, U_S) \quad \forall U_c \in (1; 5) \land \forall U_S \in (207; 253) \\
  THD_{I-510}(U_c, U_S) \quad \forall U_c \in (5; 10) \land \forall U_S \in (207; 253) 
\end{cases} \]

Polynomials \( THD_{I-15} \) and \( THD_{I-510} \) are presented as the sum of the product of the supply voltage \( U_S \), the control voltage \( U_c \) and the coefficients \( a_{ij} \) and \( b_{ij} \) (8) (9), respectively:

\[
THD_{I-15}(U_c, U_S) = \sum_{i=0}^{m} \sum_{j=0}^{m} a_{ij} U_c^i U_S^j 
\]

(8)

\[
THD_{I-510}(U_c, U_S) = \sum_{i=0}^{m} \sum_{j=0}^{m} b_{ij} U_c^i U_S^j 
\]

(9)

The coefficients \( a_{ij} \) and \( b_{ij} \) can be written in matrix form as A and B (4):

\[
A = \begin{bmatrix} 
  a_{00} & a_{01} & a_{02} & a_{03} & a_{04} & a_{05} \\
  a_{10} & a_{11} & a_{12} & a_{13} & a_{14} & 0 \\
  a_{20} & a_{21} & a_{22} & a_{23} & 0 & 0 \\
  a_{30} & a_{31} & a_{32} & 0 & 0 & 0 \\
  a_{40} & a_{41} & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 
\end{bmatrix}_{m \times m}
\]

(10)

\[
B = \begin{bmatrix} 
  b_{00} & b_{01} & b_{02} & b_{03} & b_{04} & b_{05} \\
  b_{10} & b_{11} & b_{12} & b_{13} & b_{14} & 0 \\
  b_{20} & b_{21} & b_{22} & b_{23} & 0 & 0 \\
  b_{30} & b_{31} & b_{32} & 0 & 0 & 0 \\
  b_{40} & b_{41} & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 
\end{bmatrix}_{m \times m}
\]

For the study, the luminaire coefficients \( a_{ij} \) and \( b_{ij} \) take the following values.

\[
A = \begin{bmatrix} 
  \ldots & -1.306 \cdot 10^7 & 115.40 & -0.5098 & 0.001277 & -9.97 \cdot 10^5 \\
  -1792 & 1235 & -0.04816 & -0.0001527 & 0 & 0 \\
  81.91 & -0.3671 & 0.0001717 & 0 & 0 & 0 \\
  -6.322 & 0.02564 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 
\end{bmatrix}_{10 \times 10}
\]

\[
B = \begin{bmatrix} 
  \ldots & -1.639 \cdot 10^7 & -5.156 & 0.142 & -0.0001527 & 2.749 \cdot 10^7 \\
  -1.639 & 20.32 & -0.1216 & 0.0001384 & -7.28 \cdot 10^7 & 0 \\
  -0.04816 & -0.00113 & 0 & 0 & 0 & 0 \\
  0 & 0.000142 & 0 & 0 & 0 & 0 \\
  -0.01304 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 
\end{bmatrix}_{10 \times 10}
\]

The polynomials \( THD_{I-15} \) and \( THD_{I-510} \) from the dependence (1) were obtained using the two-variable polynomial fitting method to the measurement results using the Curve Fitting Toolbox [7]. The proposed tool for data vectors \( U_S \), \( U_C \) and \( THD_1 \) have found the vector of polynomial coefficients of the assumed degrees. Polynomial coefficients were selected to obtain a minimum value of the mean square deviation between the given function \( THD_1 = f(U_S, U_C) \), and approximating polynomial \( THD_{I-15}(U_S, U_C) \) for \( U_S \in (1; 5) \) and \( THD_{I-510}(U_S, U_C) \) for \( U_S \in (5; 10) \).

Figure 3: Graph presenting the approximating polynomial \( THD_{I-15}(U_S, U_C) \)

For the obtained approximating polynomials (Figures 3 and 4) the mean square errors were respectively 0.95 and 0.99.

Verification of the correctness of the model has been made in order to determine the error values at the calculation points. The calculation results are shown in Figure 5 for the \( THD_{I-15} \) polynomial approximation and Figure 6 for the \( THD_{I-510} \) polynomial.

Due to the high nonlinearity according to \( THD_1 = f(U_S, U_C) \) in range of the control voltage from 1V to 5V the error values are within the range of 0.10% to 26.73%.

The maximum error value occurred for \( U_C = 3 \) V and \( U_S = 245 \) V. An error graph is shown in Figure 5. Within the control voltage range of 5 to 10V, the maximum error value is 2.52%, as illustrated in Figure 6.
3 Computational example
designation of \( \text{THD}_I \) coefficient

The proposed method of estimating the value of \( \text{THD}_I \) of the power supply was used to calculate this factor for the tested luminaire and the assumed daily schedule of its operation. The luminaire works at full power from 16.00 to 23.00 and from 5.00 to 7.00. At night from 23.00 to 5.00 in the morning it is operated at 50% of the rated power. In order to implement the adopted work schedule of luminaire, it is necessary to know the power \( P \) depending on the input value \( U_C \). This relationship is shown in Figure 7 and it is practically linear. For analytical purposes according to \( U_C = f(P) \) it may therefore take advantage of the approximation as a linear function.

The obtained approximation function \( U_C = f(P) \) is represented by the expression (11) and obtained for the mean square error is equal to 0.9984.

\[
U_C(P) = \begin{cases} 
10 & \text{for } P = 1 \text{ (pu)} \\
8.541 \cdot P - 0.4562 & \text{for } 0.1932 \leq P < 1 \text{ (pu)} 
\end{cases}
\]

(11)

For the further calculations it is assumed that in order to achieve the full power of the luminaire \( U_C = 10 \text{V} \) and for the remaining control levels, the \( U_C \) value is calculated from the relationship (11). The dependence (11) allows to determine the value of the \( U_C \) control voltage to obtain the assumed power of the luminaire in any schedule of its operation.

The second input value (supply voltage \( U_S \)) was derived from measurements made in an existing lighting installation.

Figure 8 shows the time dependent waveforms of RMS supply voltage from measurements and the \( \text{THD}_I \) values obtained from the calculations. These results correspond to a full daylight cycle with a scheduled power reduction. For the full power of the luminaire, the predicted \( \text{THD}_I \) value 14%. The reduction of the power of the luminaire causes it to increase to a range of 15.5 to 17%. This corresponds to a 19% increase in \( \text{THD}_I \).
4 Conclusions

The problem of the quality of electricity is one of the most important aspects related to the operation of power devices. According to EN 61000-3-2 [21], class C receivers cannot generate harmonic currents exceeding their limit values. One of the indicators of the level of disturbances generated to the power supply in the form of higher harmonic current is the total harmonic distortion factor $THD_I$. On the basis of the measurements it was proved that for the LED luminaire with power regulation its value depends on the level of control and the value of the supply voltage. From an operational point of view, it is important to predict $THD_I$, values under given power conditions at a given level of control.

The applied mathematical model of the nonlinear receiver based on the results of the measurements allows for accurate estimation of the $THD_I$ level of the given receiver. The knowledge of the $THD_I$ characteristics as a function of changes in the input values allows to determine permissible changes of these values to maintain the current deformation at a given level. For the tested luminaire, this method determined the acceptable range of power regulation and luminous flux. The analysis can be carried out for the assumed range of voltage supply variations, ranging from 230V ± 10% (from 207V to 253V and full range of power $P$ control).

References


[21] EN 50160:2010 Voltage characteristics of electricity supplied by public electricity

[22] EN 61000-3-2:2006 Electromagnetic compatibility (EMC) - Part 3-2: Limits - Limits for harmonic current emissions (equipment input current < 16 A per phase) networks