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Complementary wave solutions for the long-short wave resonance model via the extended trial equation method and the generalized Kudryashov method

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Abstract: In this paper the nonlinear long-short (LS) wave resonance model is analyzed through a new perspective. We obtain the classification of exact solutions by making use of the complete discrimination system for the trial equation method and through the generalized Kudryashov method. These methods do generate complementary wave solutions such as bright and dark solitons, rational functions, Jacobi elliptic functions as well as singular and periodic wave solutions. Some among them extend the already reported solutions through other techniques. For some types of solutions adequately graphical representations are displayed. The concerned methods could also be used in order to study other interesting nonlinear evolution processes in n dimensions.

Keywords: Long-short wave resonance model, solitary wave solutions, extended trial equation method, generalized Kudryashov method, symbolic computation

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1 Introduction

It is well known that nonlinear phenomena occur in various areas of science and engineering, such as fluid mechanics, plasma, solid-state physics, biophysics, optical fibers, technology of space, control engineering problems, hydrodynamics etc. They could be modeled through nonlinear partial differential equations (NPDEs). Thus to search for various solutions of the NPDEs does become one of the most exciting and extremely active areas of investigation of these complex physical phenomena. During the recent years, a lot of powerful methods such as the inverse scattering method [1, 2], the Hirota bilinear transformation [3], the generalized Riccati equation method [4], the \((G')G\)-expansion method [5, 6], the Lie symmetry method [7–11], the soliton ansatz method [12, 13], the generalized conditional symmetry approach [14, 15] and other techniques were established in order to obtain exact travelling wave solutions of nonlinear physical problems [16–19].

A general theory able to explain the interactions between short and long waves has been developed (see Ref. [20]). It is known that the long-wave–short-wave resonance is a special case of the three wave resonances which does appear when the phase velocity of a long wave does match with the group velocity of a short wave and it could also appear when the second order nonlinearity would occur in the process. The physical importance of the so called (LS) type equations [21] consists in the fact that the dispersion of the short wave is balanced by the nonlinear interaction between the long wave and the short wave, while the evolution of the long wave is driven by the self-interaction of the short wave. The (LS) wave resonance model is described by the following equations:

\[
\begin{align*}
\imath S_t + \alpha S_{2x} - LS &= 0, \\
L_t + \beta(|S|^2)_x &= 0.
\end{align*}
\]

(1)

where \(L\) represents the amplitude of the long wave and it is a real function, \(S\) is the envelope of the short wave and it is a complex function, while \(\alpha\) and \(\beta\) are real constants.

Some works such as [22, 23] have been published about the qualitative research of global solutions for the long–short wave resonance equations. In [24] Lax’s formulation for Eqs. (1) have been provided and Cauchy problem has been solved through the inverse scattering method. The orbital stability of the solitary waves associated to (LS) type equations has been proven in [25, 26]. The first integral method [27] has been recently applied [28] to the govern-
ing system. The obtained solutions have been expressed in terms of trigonometric, hyperbolic and exponential functions. Multiple exact travelling wave solutions have been also derived in [29] by making use of the extended hyperbolic functions method for nonlinear wave equations. This approach is explicitly related to the solutions of the projective Riccati equations. Another perspective is offered in [30], where the solutions of the (LS) system are assumed as polynomial expansions in two variables which do satisfy to the projective Riccati equations yet their explicit solutions are not sought for.

However, in our contribution, we will classify the types of solutions related to Eq. (1) according to the values of some parameters through two distinct approaches. In Sections 2 and 3 we will provide the fundamental usefulness of the extended trial function method [31] and respectively of the generalized Kudryashov method [32]. We will study in Section 4 how some types of travelling wave solutions are successfully pointed out by making use of the concerned methods. We will outline some solutions such as (a) the solitary wave solution of bell-type, (b) the solitary wave solution of kink-type for $S$ and bell-type for $L$, (c) the compound solitary wave solution of the bell-type and the kink-type for $S$ and $L$, (d) the singular travelling wave solutions and (e) the solitary wave solution of Jacobi elliptic function type. Some of them are more general than the ones derived through other algorithms. Section 5 is dedicated to some conclusions and final remarks.

2 Basics on the extended trial equation method

In this section we will describe the extended trial equation method for finding the travelling wave solutions of NPDEs. The main steps of the method are the following:

**Step 1:** For a given NPDE:

$$E(u, u_t, u_x, u_{tt}, u_{xx}, \ldots) = 0,$$  \hspace{1cm} (2)

the wave transformation is applied:

$$u(t, x) = u(\eta), \eta = kx + wt + \xi_0.$$  \hspace{1cm} (3)

Here $k, w$ and $\xi_0$ are constants. The travelling wave variable (3) allows us to reduce Eq. (2) into an ODE for $u = u(\eta)$:

$$F(u, u', u'', \ldots) = 0.$$  \hspace{1cm} (4)

**Step 2:** Suppose that the solution of Eq. (4) can be expressed as follows:

$$u(\rho) = \sum_{i=0}^{\delta} a_i \Gamma^i(\eta),$$  \hspace{1cm} (5)

where $\Gamma(\eta)$ satisfies the trial equation:

$$(\Gamma')^2 + \Lambda(\Gamma) = \frac{\Phi(\Gamma)}{\Psi(\Gamma)} = \frac{\xi_p \Gamma^n + \ldots + \xi_1 \Gamma + \xi_0}{\nu_n \Gamma^n + \ldots + \nu_1 \Gamma + \nu_0}.$$  \hspace{1cm} (6)

Here $\Phi(\Gamma)$ and $\Psi(\Gamma)$ are polynomials.

The coefficients $a_i, \ i = \delta, \delta, k = \delta, \delta, p, \nu, \ r = \delta, \delta$ are constants to be determined later. A relationship between $p, n, \delta$ could be defined by taking into consideration the homogeneous balance between the highest order derivatives and the nonlinear terms that do appear in the ODE (4).

**Step 3:** When the solution (5) together with (6) is substituted into (4) an equation for the polynomial $\Omega(\Gamma)$ is generated:

$$\Omega(\Gamma) = \sum_{i=0}^{\delta} \sigma_i \Gamma^i = 0.$$  \hspace{1cm} (7)

Equating the coefficients of each power $\Gamma^i, \ j = \delta, \delta$ to zero we generate a set of algebraic equations for $a_i, \xi_k$ and $\nu_r$.

**Step 4:** Simplify Eq. (6) to the elementary integral shape:

$$\frac{d\Gamma}{\sqrt{\Lambda(\Gamma)}} = \int \sqrt{\frac{\Psi(\Gamma)}{\Phi(\Gamma)}} d\Gamma.$$  \hspace{1cm} (8)

By making use of a complete discrimination system [33] for polynomials in order to classify the roots of $\Phi(\Gamma)$, we can solve (8) and we can as well derive the travelling wave solutions of underlying Eq. (2).

3 Basics on the generalized Kudryashov method

In this section we will describe a generalized version of the Kudryashov method for finding solitary wave solutions of NPDEs. When taking into consideration the NPDE:

$$P(u, u_t, u_x, u_{tt}, u_{xx}, \ldots) = 0,$$  \hspace{1cm} (9)

the main steps of the method can be summarized as follows:

**Step 1:** We are looking for travelling wave solutions of (9) under the form:

$$u(t, x) = e^{\mu t} u(\rho), \mu = \mu_1 x + \mu_2 t, \rho = \rho_1 x + \rho_2 t,$$  \hspace{1cm} (10)
where \( \mu_k, \rho_k, k = 1, 2 \) are arbitrary constants. The wave variables (10) allow us to convert (9) into an ODE:

\[
N(u, u', u'', \ldots) = 0. \tag{11}
\]

**Step 2:** We assume that the exact solutions of Eq. (11) can be expressed as follows:

\[
u(\rho) = \sum_{i=0}^{N} a_i Q(\rho) = \frac{A[Q(\rho)]}{B[Q(\rho)]}, \tag{12}\]

where \( a_i, i = 0, N, b_j, j = 0, M \) are arbitrary constants and \( Q(\rho) \) is the solution of the Kudryashov equation [34]:

\[Q_\rho = Q^2 - Q. \tag{13}\]

The relationship between the integers \( M \) and \( N \) can be found by taking into consideration the homogeneous balance between the highest order derivatives and the nonlinear terms which appear in the travelling wave Eq. (11).

**Step 3:** For each couple of values respectively taken by \( M \) and \( N \), by substituting Eqs. (12) and (13) into Eq. (11) yields to a polynomial \( R(Q) \). By setting all the coefficients of \( R(Q) \) to zero, the parameters \( a_i, b_j, \mu_k, \rho_k \) can be explicitly determined by solving the equations of their algebraic relations.

### 4 Travelling wave solutions related to (LS)-type equations

In this section, we will apply the methods which we have summarized in the previous section in order to construct the travelling wave solutions for the system (1). Let us present in the next two subsections some preliminary results.

#### 4.1 Relationship between the real functions in the case of (LS)-type equations

Suppose that the governing system (1) admits travelling wave solutions under the form:

\[
S(t, x) = e^{\mu t}S(\rho), \quad L(t, x) = L(\rho), \tag{14}\]

where \( \mu = \mu_1 x + \mu_2 t, \mu_1 x + \mu_2 t \) are wave variables and \( S(\rho), L(\rho) \) are real-valued functions.

By substituting (14) into the system (1) the following ODEs are generated:

\[
\begin{align*}
(p_2 + 2\mu_1 \rho_1)S &= 0, \\
(\mu_2 - \mu_1)S + p_1^2S + SL &= 0, \\
p_2S' + p_1(S^2)' &= 0. \tag{15}
\end{align*}
\]

where the prime does indicate the derivative with respect to \( \rho \).

From the first equation of (15) we derive the parameter relation:

\[
\rho_2 = -2\mu_1 \rho_1. \tag{16}
\]

When integrating the last equation from (15) with respect to \( \rho \), then taking into consideration (16), we derive the relationship between functions \( S \) and \( L \) under the form:

\[
L(\rho) = \frac{1}{2\mu_1}S^2(\rho), \tag{17}
\]

where the integration constant is chosen to be zero.

The previous result together with the second equation from system (15) generate the Liénard ODE:

\[
S''(\rho) + AS(\rho) + BS^3(\rho) = 0, \tag{18}\]

where we use the denotations:

\[A = -\frac{\mu_1}{\mu_2}, \quad B = \frac{1}{2\mu_1\rho_1^2}. \tag{19}\]

Consequently the search for travelling wave solutions in the case of (LS)-type equations is reduced to a discussion about travelling wave solutions related to the ODE (18). This discussion will be the object of the next subsection.

#### 4.2 Travelling wave solutions of (1) via the extended trial equation method

Let us suppose that the solutions of Eq. (18) are expressed in accordance with (5)-(6). Taking into consideration the homogeneous balance between \( S'' \) and \( S^3 \), we should require that:

\[
p = 2\delta + n + 2. \tag{20}\]

In order to obtain the travelling wave solutions for (1) let us choose \( n = 0, \delta = 1 \) and \( p = 4 \). This choice corresponds to solutions under the form:

\[
S(\rho) = a_0 + a_1 \Gamma \tag{21}\]

where the constants \( a_0 \) and \( a_1 \) do satisfy to \( a_0^2 + a_1^2 \neq 0 \) and have to be determined later. Relying upon the above description of the method, we can obtain:

\[
S'' = \frac{\Phi'}{2\Psi} = \frac{(4\xi_4\Gamma^3 + 3\xi_3\Gamma^2 + 2\xi_2\Gamma + \xi_1)a_1}{2\nu_0} \tag{22}\]

By substituting (21) and (22) into (18), the left-hand side of the underlying ODE becomes a third order polynomial in \( \Gamma \). Setting the coefficients of power \( \Gamma^m, m = 0, 3 \) to zero
yields an algebraic system for parameters \(a_i, i = 0, 1, 2, \xi_k, k = 0, 1, 4, \mu_j, j = 1, 2, \rho_1, \nu_0\). The solution given by the Maple program is:
\[
a_0 = \frac{a_1 \xi_1}{4 \xi_4}, \quad \xi_1 = \frac{2a_0(\xi_2 a_1^2 - 4a_0^2 \xi_4)}{a_1^2}, \quad \nu_0 = \frac{-4a_0^2 \mu_1 \xi_4}{a_1^2},
\]
\[
\mu_2 = \frac{4 \xi_4 \mu_1^2 + \xi_2 a_1^2 - 6a_0^2 \xi_4}{4 \mu_1 \xi_4},
\]
\[
\forall \xi_1, \forall \xi_k, \forall k = 0, 2, 3, 4, \forall \mu_1, \forall \rho_1.
\]

In the above mentioned particular case, the integral (8) becomes:
\[
z\rho - \rho_0 = K \int \frac{d\Gamma}{\Gamma^4 + \frac{\xi_4}{4} \Gamma^3 + \frac{\xi_4}{4} \Gamma^2 + \frac{\xi_4}{4} \Gamma + \frac{\xi_4}{4}},
\]
where \(K = \sqrt{\nu_0/\xi_4}\). By substituting the solution (23) into (24), we get:
\[
z\rho - \rho_0 = K \int \frac{d\Gamma}{\Gamma^4 + \frac{\xi_4}{4} \Gamma^3 + \frac{\xi_4}{4} \Gamma^2 + 2a_0 \left(\frac{\xi_2}{a_1} - \frac{4a_0^2}{a_1^2}\right) \Gamma + \frac{\xi_4}{4}},
\]
where \(K = \sqrt{\frac{-4a_0^2 \mu_1}{\nu_0}}, \) with \(\mu_1 < 0\).

If we denote by \(a_n, n = 1, 4\) the roots of the polynomial involved in (25), we reach five distinct cases which allow us to find the unknown \(\Gamma\) function. They are:

1) When \(a_1\) is a one and only multiple root of the polynomial, we get
\[
z\rho - \rho_0 = -\frac{K}{\Gamma - a_1},
\]
(26)

2)–3) When the polynomial admits two distinct roots \(a_n, n = 1, 2\), we obtain:
\[
z\rho - \rho_0 = \frac{2K}{a_1 - a_2} \sqrt{\Gamma - a_2} - \Gamma - a_1, \quad a_2 > a_1,
\]
\[
z\rho - \rho_0 = \frac{K}{a_1 - a_2} \ln \left(\frac{\Gamma - a_1}{\Gamma - a_2}\right), \quad a_1 > a_2.
\]
(27)

4) When the polynomial admits three distinct roots with \(a_1 > a_2 > a_3\), we have:
\[
z\rho - \rho_0 = \frac{2K}{(a_1 - a_2)(a_2 - a_3)} F(\varphi, l),
\]
(28)

5) When all the roots of the polynomial are distinct with \(a_1 > a_2 > a_3 > a_4\), the result is:
\[
z\rho - \rho_0 = \frac{2K}{(a_1 - a_3)(a_2 - a_4)} F(\varphi, l),
\]
(29)

where
\[
F(\varphi, l) = \int_0^\varphi \frac{d\psi}{\sqrt{1 - l^2 \sin^2 \psi}}, \quad l^2 = \frac{(a_2 - a_3)(a_1 - a_4)}{(a_1 - a_3)(a_2 - a_4)}
\]
\[
\varphi = \arcsin \sqrt{\frac{\Gamma - a_1}{\Gamma - a_2}} \frac{(a_2 - a_3)}{(a_1 - a_4)}.
\]

Let us choose for simplicity \(\rho_0 = 0, a_0 = -a_1 a_1\). After introducing \(\Gamma(\rho)\) from (26)-(30) into (21) and (17), we can classify the solutions of the (LS) system (1) as follows:

(i) solitary wave solutions of bell-type given by
\[
S_1(t, x) = e^{i(\mu_1 x + \mu_2)} \left(\pm \frac{K_1}{\rho_1 x + \rho_2 t}\right)^{\frac{1}{2}},
\]
(31)

\[
L_1(t, x) = \frac{1}{2\mu_1} \left(\frac{K_1}{\rho_1 x + \rho_2 t}\right)^2,
\]
where \(K_1 = a_1 K\).

(ii) singular wave solutions
\[
S_3(t, x) = \frac{(a_1 - a_2)a_1}{2} \left[1 \pm \cosh \left(\frac{a_1 - a_2}{K} (\rho_1 x + \rho_2 t)\right)\right]^{\frac{1}{2}}
\]
\[
L_3(t, x) = \frac{1}{2\mu_1} \left[\frac{K_1}{\rho_1 x + \rho_2 t}\right]^2,
\]
(33)

\[
S_4(t, x) = e^{i(\mu_1 x + \mu_2)} \frac{K_2}{P + \cosh[R(\rho_1 x + \rho_2 t)]}
\]
\[
L_4(t, x) = \frac{1}{2\mu_1} \left[\frac{K_2}{P + \cosh[R(\rho_1 x + \rho_2 t)]}\right]^2,
\]
(34)

where \(K_2 = \frac{2(a_1 - a_3)(a_1 - a_4)}{a_1 - a_2}, \) \(P = \frac{2(a_1 - a_3 - a_2)}{a_1 - a_2} \neq 0, R = \sqrt{[a_1 - a_3]/(a_1 - a_4)}\). Here \(K_2\) is the amplitude of the soliton, and \(R\) is the inverse width of the soliton. Thus, we can assert that solitons exist for \(a_1 < 0\). In our situation \(P\) cannot be zero, because \(R\) does not admit real values. However, the solutions \(S(\rho)\) proportional with such function could be derived in (30) through another technique, namely an improved \((\xi^\infty)\) expansion method.

(iii) Jacobi elliptic function type solutions:
\[
S_5(t, x) = e^{i(\mu_1 x + \mu_2)} \frac{K_3}{M + N \left[\sin(\varphi, l)\right]^2},
\]
\[
L_5(t, x) = \frac{1}{2\mu_1} \left[\frac{K_3}{M + N \left[\sin(\varphi, l)\right]^2}\right]^2,
\]
(35)

where \(K_3 = 2a_1(a_1 - a_3)(a_4 - a_2), M = a_4 - a_2, N = a_1 - a_4, \)
\[
\varphi = \pm \frac{\sqrt{(a_1 - a_3)(a_4 - a_2)}}{2k} (\rho_1 x + \rho_2 t), \quad l^2 = \frac{(a_1 - a_3)(a_1 - a_4)}{(a_1 - a_2)}.
\]
4.3 Travelling wave solutions of (1) via the generalized Kudryashov method

Let us now expand $S(\rho)$ from (18) in accordance with (12) and (13). By balancing the terms $S^r$ and $S\ell$, we require that:

$$N = M + 1$$

(38)

If we choose $N = 2$, $M = 1$, the solution of (18) will be sought under the form:

$$S(\rho) = \frac{a_0 + a_1 Q + a_2 Q^2}{b_0 + b_1 Q}$$

(39)

where $a_i, i = 0, 2, b_j, j = 0, 1$ are constants to be determined later and $Q(\rho) = \frac{1}{1 + \rho^2}$ are the solutions of the auxiliary Eq. (13).

When substituting (39) together with (13) into the (18) and gathering all the terms with the same power of $Q$, the left-hand side of (18) is converted into a polynomial with $Q^r, r = 0, 1, 2$. When equating each coefficient of this polynomial to zero, a set of 7 algebraic equations for $a_i$, $i = 0, 2$, $b_j, j = 0, 1, 2, \mu_1, \mu_2, \rho_1$ is derived. By solving this algebraic system with the help of Maple program, we get solitary wave solutions for the (LS)-system as follows:

(i) The solitary waves solution of kink-type:

$$S_6(t, x) = \frac{a_2 \left(1 - \tanh \left(\frac{\rho_1 x + \rho_2 t}{2}\right)\right)^2}{2 \tanh^2 \left(\frac{\rho_1 x + \rho_2 t}{2}\right) - \left(\rho_1 + \rho_2\right)\tanh \left(\frac{\rho_1 x + \rho_2 t}{2}\right) - a_1},$$

$$L_6(t, x) = \frac{a_2 \left(1 - \tanh \left(\frac{\rho_1 x + \rho_2 t}{2}\right)\right)^2}{2 \tanh^2 \left(\frac{\rho_1 x + \rho_2 t}{2}\right) - \left(\rho_1 + \rho_2\right)\tanh \left(\frac{\rho_1 x + \rho_2 t}{2}\right) - a_1}$$

(40)

generated through the choice of $Q(\rho) = \frac{1}{1 + \rho^2}$ and through taking under consideration the parameters:

$$a_0 = a_1 = 0, \quad a_2 = 2\mu_1 b_1 (1 - b_1)$$

$$b_0 = 2, \quad b_2 = \mu_2, \quad \forall b_1 \neq 1, \forall \mu_1, \forall \rho_1.$$  

(41)

(ii) Dark soliton solution of kink-type for $S$ and bell-type for $L$ expressed under the form:

$$S_7(t, x) = -e^{i(\mu_1 x + \mu_2 t)} \frac{a_2}{2} \tanh \left(\frac{\rho_1 x + \rho_2 t}{2}\right),$$

$$L_7(t, x) = \frac{a_2^2}{8 \mu_1} \left[\tanh \left(\frac{\rho_1 x + \rho_2 t}{2}\right)\right]^2,$$

(42)

derived by assuming that $Q(\rho) = \frac{1}{1 + \rho^2}$ as well as the relationships among parameters:

$$a_0 = \frac{a_2}{4}, \quad a_1 = -a_2, \quad b_1 = -2b_0 = 1, \quad \mu_1 = \frac{1}{10} \left(\frac{a_2}{\rho_1}\right)^2$$

$$\mu_2 = \frac{1}{100} \left(\frac{a_2}{\rho_1}\right)^4 + \frac{5}{4} \rho_1^2, \quad \forall a_2 \neq 0, \forall \rho_1.$$

(43)
The dynamics of dark soliton solution (42) of kink-type for $S_9$ and of bell-type for $L_9$ with suitable parametric choices is shown in Figure 2. The imaginary values of $S_9$ and respectively of the long wave’s amplitude $L_9$, are shown at parametric choices $a_2 = 1, \rho_1 = 2, \rho_2 = -0, 1, \mu_1 = 0.025, \mu_2 = 5, 001$. The imaginary values of $S_9$ do present similar graphs except for a phase difference.

By choosing $Q(\rho) = \frac{1}{1+\rho^2}$, singular travelling wave solutions expressed in terms of coth function can be found. 

(iii) Compound wave solution of the bell-type and the kink-type for $S$ and $L$ with the explicit expressions:

$$S_{10}(t, x) = \frac{a_2 e^{i(\mu_1 x - \mu_2 t)}}{2} \tanh \left( \frac{\mu_1 x + \mu_2 t}{2} \right) \pm \sech \left( \frac{\mu_1 x + \mu_2 t}{2} \right) - 3,$$

$$L_{10}(t, x) = \frac{a_2^2}{8 \mu_1} \left[ \tanh \left( \frac{\mu_1 x + \mu_2 t}{2} \right) \pm \sech \left( \frac{\mu_1 x + \mu_2 t}{2} \right) - 3 \right]^2,$$

which are related to $Q(\rho) = \frac{1}{1+\rho^2}$ and to the parameter relationships:

$$a_0 = -a_1 = \frac{a_2}{2}, \quad b_0 = b_1 = 1, \quad \mu_1 = -\left( \frac{a_2}{2 \rho_1} \right)^2,$$

$$\mu_2 = \left( \frac{a_2}{2 \rho_1} \right)^4 + \frac{1}{2 \rho_1^2}, \quad \forall a_2 \neq 0, \forall \rho_1.$$

5 Concluding remarks

In this paper we have discussed about the possibilities offered by the extended trial equation method and by the generalized Kudryashov method in order to find new wave solutions for the long–short (LS) wave resonance Eq. (1). At least for the considered cases (21) respectively (39), these two methods lead us to complementary classes of solutions.

The key idea of the trial equation method is to reduce an ODE into a solvable integral Eq. (8) which involves polynomial functions. For the (LS)-type equations we have chosen to take advantage of the solutions to the trial function (forth degree polynomial) by making use of a mathematical tool denominated as the complete discrimination system. We have pointed out solutions such as: solitary wave solution of the bell-type, bright soliton solution, Jacobi elliptic function type solution, singular wave solution, periodic function type solution. To our knowledge, such classification involving the complete discrimination system for polynomials has not been reported. According to the specific parameters, some wave solutions (32), (34)-(37) do extend the ones obtained by the means of other techniques [27, 35, 36].

The generalized Kudryashov method is useful in order to reach the analytical solutions for the (LS) wave resonance model. We have been able to derive wave solution of kink-type (40), dark soliton solution of the kink-type for $S$ and the bell-type for $L$ (42) as well as a compound wave solution of the bell-type and the kink-type for $S$ and $L$ (44).

When we have taken into consideration some other values of parameters which should verify (20) and respectively (38), some new solitary wave solutions could be identified. The proposed methods have their own advantages. They offer both a concise and an efficient way for solving NPDEs, in two or more dimensions. Other methods, such as multi-symplectic method [37, 38], or general-
ized multi-symplectic method [39, 40] will be employed in future works.

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