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MATHEMATICS AND GOD'S POINT OF VIEW[1]

Abstract. In this paper the final stages of the historical process of the emergence of actual infinity in mathematics are considered. The application of *God's point of view* – i.e. the possibility to create mathematics from a divine perspective, i.e. from the point of view of an eternal, timeless, omniscience and unlimited subject of cognition – is one of the main factors in this process. Nicole Oresme is the first man who systematically used actual infinity in mathematical reasoning, constructions and proofs in geometry.

1. Introductory remarks

Who creates mathematics? The answer seems to be obvious: mathematics is created by mathematicians. However, the subject of cognition in mathematics possesses some ideal properties because it is able to grasp actually infinite objects or to perform infinite operations in "Cantor's paradise". Even during the creation of a formalized theory, an ideal mathematician has at his disposal an infinite number of symbols and variables, or he can create eternal theorems being true in a timeless environment and forever. How is all of this possible? Man is a mortal being. However, there are many possible answers and many of them concern *mathematical Platonism*.

In this paper, I would like to present the historical process in which the so-called "God's point of view", and some Platonic objects as well as methods, started to reign supreme in mathematics. The most important author in this is Nicole Oresme who was – as we will see – the first in the effective application of God's point of view in mathematics. Today, the possibility of operating with an actually infinite straight line or a numerical axis is seen as straightforward and absolutely non-problematic. Infinite space in Euclidean geometry is also a relatively new discovery, unknown in Antiquity.

The most important goal of our present considerations is to find where, when, and why the concepts of infinity and infinite objects were used in strict mathematical reasoning for the first time, e.g. in mathematical proofs

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in geometry and in arithmetic. This point is independent from the possibility of analysing the concept of infinity in philosophy, theology, astronomy, etc. The *mathematical* possibility of exact analysis with the use of infinite objects and a concept of infinity, in a sense, is *ideal* or *purely logical*, i.e. it is independent from the philosophical beliefs of philosophers and mathematicians concerning, for instance, the existence of actual infinity. Obviously, the move towards effective and fruitful use of the infinite objects in mathematics was possible because of many – previous and subsequent – philosophical or theological discussions.

At the beginning of this paper, it is necessary to remark briefly on some ancient views concerning the void physical space and the possibility of the existence of an infinite extra-mundial empty infinite space.

Let us remind ourselves that Aristotle denied the existence of an empty place, a place with no body in it, or a vacuum, cf. for instance his *De caelo* 279a 12–14, 17–18, and his definition of the void in *Physics* 214a 8–19 and the *De caelo* 279a 14–15. The first to change the Aristotelian definition of the vacuum was Roger Bacon. It became "a space in which there is absolutely no body, nor is there a natural aptitude for receiving any body; but to assume vacuum in this way, is to assume it beyond heaven."[2] Bacon invented the purely conceptual idea of an empty place beyond the heavens because the definition involved a place admitting no body.

One fragment from Archytas of Tarentum, preserved in Simplicius' *Commentary on Aristotle's Physics*, describes a thought experiment arguing for the extension of heaven without limit; cf. (Cornford 1936), p. 233.

Archytas' argument was unknown to the Middle Ages, cf. (Grant 1982) p. 106. Instead, there was a fragment of Symplicius' *Commentary on De caelo* in the Latin translation by Wilhem of Moerbecke, 1271, where almost the same argument was ascribed to the Stoics; cf. (Grant 1982), p. 106–107. The Stoics were mostly inclined to Aristotle's physics and accepted its finitism. We know, however, the hypothetical reasoning of Cleomedes where an infinity of space surrounding the spherical world is argued from the proposition that no container exists. The vacuum must be infinite, otherwise it should be delimited by a body. Yet, there is no body outside the world. Therefore, the vacuum, if it exists, must be infinite.[3]

Some treatises were composed in a quasi-mathematical manner, not unlike Euclid's *Elements*, with theorems, enunciations, proofs, and corollaries, for instance Bradwardine's *De continuo*; cf. (Murdoch 1957), (Murdoch 1987). There was a distinct philosophical body of work about the problems of continuum or infinity in the 13^{th} and 14^{th} centuries: Gerard of Odo's *De continuo*, Adam Wodo's *Tractatus de invisibilibus*, John Gedo's *Tracta-*

tus de continuo and William Collingham's *De infinito*; cf. (Murdoch 1957), pp. 14–17. The list of treatises considering the problems of the continuum is extensive. The most important are those by William of Ockham (e.g. *De sacramento altaris*), John Buridan (*De puncto*) and Robert Grosseteste. The latter made some very interesting remarks concerning the concept of infinity in *De luce*, writing about different types of infinity and infinite numbers; cf. (Grosseteste 1942), pp. 11–12, and (Grosseteste 1912). Some other works in the context of Oresme's strictly mathematical achievements are mentioned below.

Important for the philosophical discussion of infinity and continuum in the 13th century were the concepts of *cathegorematic* or actual infinity ("bigger than any number"), and *syncathegorematic* or potential infinity, introduced by the late Pope John XXI Peter Hispanus in his *Summulae logicales*; see (Duhem 1906–13), vol. II, pp. 1–53.[4]

Philosophers started their analysis from *God's point of view*, i.e. they considered logical situations in which the assumptions concerned the omnipotent, eternal, and all-knowing subject of cognition. For a human, it was impossible to live infinitely long, to divide a continuum actually to infinity etc. However, it was possible for him to analyze the logical consequences of such premises. This last point is exemplified by the medieval discussions concerning the concept of God's omnipotence or some properties of the continuum. Without the consideration of God's properties and divine possibilities, the use of actually infinite objects in mathematics would not have been possible.

Various notions of infinite and void extra-cosmic space are described by (Grant 1982) and (Grant 1969) in detail. Here we would only recall the proponents of the existence of the infinite imaginary void space: Bradwardine (cf. his *De causa Dei contra Pelagium*) and Nicole Oresme (cf. his *Questiones super De coelo*; cf. (Oresme 1965) and *La Livre du ciel et le monde*; cf. (Oresme 1968)).

A famous argument from shortly after Oresme's time by Henry of Harclay is reported by William of Alnwick in his *Determinationes*, quoted by Adam Wodeham, cf. *Tractatus de indivisibilibus*, more examples in (Grant 1982), see (Wodeham 1988) p. 289:

> God actually sees or knows the first beginning point of a line, and any other point which it is possible to pick out in the same line. Therefore, either [i] God sees that, in between this beginning point of the line and any other point in the same line, a line can intervene, or [ii] not. If not [i.e. (ii)], then he sees a point immediate to the point, which is what we propose. If so [i.e. (i)], then, since it is possible to assign points in the intermediate line, those points will

not be seen by God, which is false. This consequence is clear, for according to what we have posited, a line falls between the first point and any other point (of the same line) seen by God, and consequently there is some midpoint between this point and any other point seen by God. Therefore this midpoint is not seen by God.

The above argument applies God's point of view. This was widely discussed and led to three main groups of 13th century theories about continuum: 1. it is composed of infinitely many indivisible parts, for instance, points; 2. it is composed of infinitely many parts, each part being a continuum (cf. the views of Gregory of Rimini)[5]; 3. no points exist at all.

It is necessary to differentiate between God's point of view and the process of divinization of space, i.e. from the identification of space with the infinity of God or with God himself, which was described in great detail by E. Grant; cf. (Grant 1982). However, as we will see, the problem is purely mathematical and, even from the historical point of view, it is independent from the invention of an infinite void space in physics and cosmology.[6]

We can now consider the development of Platonic methods in mathematics and not physics, cosmology, philosophy, or theology. Such methods involve certain infinite objects in Euclidean geometry such as infinite lines, surfaces, and space.

2. Nicole Oresme and the application of God's point of view in XIV century mathematics

The works of Nicole Oresme (c. 1320–1387) are of the highest importance from the *mathematical* point of view. There is very mature and conscious use of the concept of infinity in them, both in arithmetic and in geometry. Thus, God's point of view enters into mathematics.

Nicole Oresme applies infinite concepts in the two domains.[7] The first one is a summation of infinite series, and the second, theorems concerning the commensurability and incommensurability of circular motions. Oresme's arithmetical theorems involve mainly a geometrical point of view, contrary to the purely logical and arithmetical style of the Oxford *calculatores.* In Oresme, one can find the first prototypes of infinite lines, surfaces, and even of an infinite three-dimensional space.

The term "summation" of infinite series can be misleading because Oresme does not sum up the series, but instead uses a *division* of given *finite* magnitude into an actually infinite number of proportional parts. Oresme's main mathematical innovation is the adoption of God's point of view: the divisions employed in proofs are made *actually* in infinity. This is revolutionary. Such a small yet revolutionary step is made later by Descartes, demonstrating on the first few pages of *Geometry* how to assign a linear representation to arithmetical operations, a possibility previously invisible.

Descartes shows, for instance, that multiplication can be represented by a line of given length. The ancient limitations to spatial representation of that and other operations are thus overcome. Until Descartes, the multiplication of either numbers or magnitudes is represented by a two-dimensional figure, e.g. a parallelogram. Descartes' use of the concept of the mathematical infinite is, however, limited compared to Oresme.

Such small steps as those by Oresme and Descartes seem inessential to a modern mathematician: nowadays even a child can think this way. However, from a historical or philosophical point of view, such steps are the crux of the matter in the development of mathematics. From the technical standpoint, they are "small" because they are "easy", yet are changing the intuitive foundations of mathematics and are actually great and revolutionary.

It is worth noting that, in this way, the method of Oresme differs from the ancient method of exhaustion, for instance, since:

> The Greek [as well as the Arabian; cf. for instance (Edwards 1979), pp. 81–86 – Z.K.] mathematicians, however, never considered the process as being literally carried out to an infinite number of steps, as we [and Oresme – Z.K.] do in passing to the limit ... . (See (Boyer 1968), p. 34.)

A detailed analysis of infinite series is contained in two of Oresme's works: *Questiones super geometriam Euclidis* and *Tractatus de configurationibus qualitatum et motuum*, the first written between 1343 and 1351, see the critical edition by Busard and Folkerts (Busard 2010) p. 2, the second a little older from somewhere around 1350, cf. the critical edition by Clagett, (Clagett 1968), p. 14.

In *Questiones super geometriam Euclidis*, infinity and infinite series are considered in questions I–V. Writing about the concepts of commensurability and incommensurability in question VIII (see questions VI–IX), Oresme states that "continuum non compositur ex indivisibilibus infinitis", and that some continua can be augmented continuously, such that the augmented magnitudes become greater than the other without ever being equal to it. In questions I and II, Oresme shows how to divide *every* continuous magnitude into infinitely many *proportional* parts, i.e. the parts given by a clear principle of division: a ratio. In modern notation, the division may be described as *a sum* of an infinite series 1). $\mathbf{1/2 + 1/4 + \ldots + 1/(2^n) + \ldots = 1}$, and 2). $\mathbf{1/1000 + 1/1000(1 - 1/1000) + \ldots + 1/1000(1 - 1/1000)^n + \ldots = 1}$.

The famous Bradwardine rule[8] is extended in question III to the case of infinite quantities by division of time ("an hour") into an infinite number of proportional parts. Question IV removes ancient finitism concerning asymptotes. Oresme gives an affirmative answer to the question of the possibility of extending two lines *in infinitum*, such that they will always be closer but never meet. The fifth Corollary of that question shows the transformation of an infinite geometrical object (a body with a finite base of one square foot, but infinitely high) into another one (an infinite sphere) by cutting off a cube of one cubic foot, which is next transformed into a one-cubic foot sphere, then wrapped around by subsequent spheres of one-cubic foot being cut repeatedly from the first body in an infinite process.

Tractatus de configurationibus qualitatum and motuum constitutes a more mature body of work. There, Oresme states that mathematical objects such as points or lines are non-existent. They can only be imagined, hence they have only an imaginary existence; cf. *op. cit.*, chapter I.i. The chapter provides evidence that mathematical objects and operations such as extensions of lines have to be imaginary. This allows Oresme to manipulate actually infinite objects. Similar arguments to those indicated by Clagett are also present in Oresme's Treatise *Qustiones de sphere*. However, in the latter, Oresme does not decide whether geometrical objects are fictions or "indivisible accidents of the soul"; cf. *Questio I*, p. 23, 25 and 26 in (Droppers 1966) and Clagett's comments to Chapters I.i and III.iv in (Clagett 1968), pp. 438–439 and 492–493.

Oresme specifically states that those mathematical objects are imaginary non-existent fictions only in *De configurationibus …*, in Questions III.iv (34–43, *op. cit.*) and III.xii (37–41, *op. cit.*), and in *Questiones super de coelo* Book II, Question 7, see (Clagett 1968) p. 543, 187–189; for more information, see (Clagett 1968), p. 438–439. The last six questions, III.viii – III.xiii, in *De configurationibus …* are about infinite geometrical objects and infinite series. Oresme applies the series only in geometrical problems, the series corresponding to some geometrical objects. In question III.viii, for instance, he examines the statement, "A finite surface can be made as long as we wish, or as high, by varying the extension without increasing the size", cf. (Clagett 1968), p. 413. In order to show how it is possible in the case of surfaces, Oresme considers two, one-square foot identical surfaces, e.g. the squares. He divides both squares into infinitely many proportional parts, the sub-surfaces of area equal to 1/2, 1/4, 1/8,

..., named ***E***, ***F***, ***G***, etc.) The sum of those is visibly equal to the one foot square in both cases. Then, taking the infinite number of the parts of the second square, Oresme puts them on top of the proportional parts of the first square. He then demonstrates that the area of such a figure "stepped" to infinity and containing infinitely many parts with known areas is finite and equal to a two square foot figure. Thus, Oresme geometrically sums up the following series: $\mathbf{1 + 1/2 + 1/4 + \ldots + 1/(2^n) + \ldots = 4\ (1/2) = 2}$. Oresme writes:

> Then upon this whole let the second part, namely ***F***, be placed, and again upon the whole let the third part, namely ***G***, be placed, and so on for the others to infinity. When this has been done ("Quo facto"), let the base line ***AB*** be imagined as being divided into parts continually proportional according to the ratio of 2 to 1 ... . (See (Clagett 1968), p. 415.)

"When this has been done" refers to the simple but revolutionary step into the strict analysis of infinite geometrical objects. In every case, the summations, (cf. the next questions of Part III) are based on infinite divisions of some *finite* magnitudes into infinitely many proportional parts. In the next question, III.ix, Oresme divides in this way a finite line and a finite surface, creating an infinitely "high" surface of a finite area from the parts of the surface on the parts of the line. A similar construction is considered in *Questio* III.ix. In the last three questions, i.e. III.xi, III.xii and III.xiii, Oresme shows how to extend a finite magnitude (a line, a surface, and a body) to infinity.

In the following and last chapter of the treatise, he writes, cf. (Clagett 1968), pp. 431, 433, 435:

> A finite quality can be imagined as being extended infinitely in an absolute way ["in infinitum extendi simpliciter"] and in every dimension without its augmentation ... . (...) For let ***A*** be a body which is absolutely infinite on all sides, i.e. in every direction, and which occupies everything. ["Sit enim A unum corpus simpliciter infinitum undique, scilicet ad omnem partem, et quod omnum occupet."]

In the above fragment, for the first time ever there is a prototype of an infinite three-dimensional space, a container for other geometrical objects, imagined and used for some mathematical reasons and from God's point of view. It was only a matter of time for such reasoning to be repeated and to come to fruition in mathematics.[9]

Describing certain "series" and reconstructing them with the use of modern algebraical notation, we need to remember that Oresme worked with a quite different intuitive model, closely related to geometrical intuitions and objects of Euclid's geometry. The divisions and summations concern objects displaying some intensities of qualities. For instance, a body moving with constant velocity is described by a rectangle with a base line representing the span of time.

Oresme uses mainly geometrical series in many places in his works. However, even a little before Oresme, an algebraical approach to these series was also present, as we will see. The origins of the geometrical method for representing intensities of qualities can be traced back to medieval medicine and especially to Roger Bacon's, *De graduatione medicinarum compositarum*.[10] Oresme is the first to represent some qualities by surfaces since Bacon and earlier approaches only applied one-dimensional lines.[11] Some ideas of the method were suggested by Jean Buridan in *Questions on the Physics* and by Richard Swineshead (Calculator) in the second treatise from the famous *Liber calculationum*, entitled *De difformibus* and written at Merton College in Oxford around 1340. Swineshead sums up infinite series in an algebraical way.

Calculator sums up the following series: $\mathbf{1 + 1/2\ 2 + 1/4\ 3 + \ldots + (1/(2^n))n + \ldots = 4}$. Boyer, who studied the mathematical technique of the *Liber calculationum*, notes:

> There is in the entire *Liber calculationum* no diagram or reference to geometrical intuition, the reasoning being purely verbal and arithmetical. On the other hand, Oresme felt that the multiplicity of types of variation involved in the latitude of forms is discerned with difficulty, unless reference is made to geometrical figures. The work of Oresme therefore makes most effective use of geometrical diagrams and intuition ... . (Cf. (Boyer 1968), p. 80. The analysis of *Liber calculationum*, see pp. 75–80.)

In one Parisian manuscript of *Liber calculationum* a scribe inserted (in 1375) a single diagram made with the use of Oresme's geometrical method to explain the text of the quoted fragment of Calculator; cf. (Clagett 1968), pp. 61, 78–81, 495–496. However, *Liber calculationum* came a decade earlier than *Tractatus de configurationibus qualitatum et motuum*. Clagett writes that he is not sure if Oresme read Calculator's work before writing *Tractatus de configurationibus ...*; cf. (Clagett 1968), p. 59. Nevertheless, the method of Oresme is different.

Intuition and preliminary remarks anticipating Oresme's approach to infinite series are present in his earlier work. For instance, in *Questions on Generation and Corruption* and *Questiones super libros physicorum*, Book I, *Questio* 20; see (Clagett 1968), p. 63. As Clagett reports, *op. cit.*, p. 65, footnote 19, an analogue of this method is found in *De motibus naturalibus* by Swineshead, which is an earlier work. It seems that these approaches were independent, and that Robert Swineshead was the first to use infinite geometrical objects in mathematical studies.[12]

Another 14$^{\text{th}}$ century fragment concerning the infinite series is present in a Parisian manuscript, BN lat. 16134, pp. 79v–80r, possibly containing the work of Benedictine Johannes Bode *A est unum calidum*. It is absent, however, from any other manuscript of the treatise, as noted by H. L. L. Busard; cf. (Busard 1965), p. 387, also (Clagett 1968), p. 499. Clagett argues that this is a fragment of Oresme's lost treatise, *Sophismata*.

The text contains a proof concerning the sum of the series: $\mathbf{1 + (1/2)\ 2 + (1/4)\ 3 + \ldots + (1/(2^n))n + \ldots = 4}$. Clagett renders the idea of the proof (cf. the Latin text in (Duhem 1906–13), pp. 499–501) such that the series is transformed into the following: $\mathbf{1 + 1 + {+1} + (1/2 + 1/4 + \ldots + 1/(2^n) + \ldots}$; *ibidem*, pp. 501–502.

The ideas of *Liber calculationum* and Nicole Oresme about infinite series had a direct influence on two treatises from the 15$^{\text{th}}$ and 16$^{\text{th}}$ centuries. The above algebraical transformation was copied with minor changes by Bernardus Torni of Florence in his commentary, *In capitulum de motu locali Hentisberi* (i.e. of Heytesbury), published in 1494. Torni ascribes the idea of the proof to Oresme; cf. the Latin text of Torni's proof and the historical analysis in (Clagett 1968), pp. 502–510. However, the approach is purely algebraical and the method is rather atypical for Oresme.

The second work using the above operations on infinite series is *Liber de triplici motu*, by Thomas Alvarus and comes from the 16$^{\text{th}}$ century. Thomas Alvarus provides some clarification and generalization of the ideas contained in *Liber calculationum*. He also applies Oresme's series from *Tractatus de configurationibus qualitatum et motuum*, *Questio* III.x; cf. Tract II, Chapter 3, Conclusio 9, (Clagett 1968), pp. 514–516.

Oresme's ideas concerning infinite series may have been spreading, along with the reception of his configuration doctrine.[13] Still, the doctrine, basically without the infinitary methods, was applied in the works of Johannes de Casali (*Questio de velocitate motus alterationis*; probably because of some differences from the original Oresme method, cf. (Clagett 1968), pp. 66–70)), and in the Parisian school: Albert of Saxony (*Questiones in octo libros Physicorum*), Henry of Hesse (*De reductione effectuum particularium in causas universales*), Symon de Castello (*De proportionibus velocitatum in motibus*), Petrus de Candia (later Pope Alexander V; *Lectura super sententias*), in Italy: Jacobus de Sancto Martino (around 1390 A.D., *Tractatus de latitudinibus formarum*).

The latter was popular in Italy and was used by scientists like, for instance, Masino Condronchi, in his *Questiones super questionem Johannis de Casali*, and in the response to it by Blasius of Parma titled *Questiones super tractatum de latitudinibus formarum*, or in an anonymous work entitled, *Questio utrum omnis forma habeat latitudinem nobis presentabilem par figuras geometricas*. Another example is a manuscript from Venice, Bibl. Marc. Lat. VIII, 19. In the reception and application of Oresme's ideas, Italy was the main territory which explains why the development of infinite methods emerged exactly there.

Yet another application of Oresme's configuration doctrine is a work by Roger Thomas, *Tractatus proportionum*, bearing some secondary traces of infinitary reasoning. The most important, however, is "the paraphrase and commentary of the *De configurationibus* found in a Florence manuscript of the Biblioteca Nazionale Centrale" (Clagett 1968), p. 100–101, and the text (with a translation) in Appendix III. Clagett describes examples of the use of Oresme's diagrams, e.g. Antonius de Scarparia, Angelus de Fossambruno, Jacopo de Forli; cf. (Clagett 1968), pp. 101–102. The doctrine was also known to Nicolaus of Cusa (cf. Book II of *De mathematicis complementis*).

A very curious case is Galileo who, for instance, applies the doctrine in his proof of the Merton Rule (i.e. in the uniform acceleration theorem) given in *Discorsi e dimonstrazioni mathematiche intorno à due nuove scienze*, Third Day, Theorem I, Proposition I. There is a striking similarity between Galileo's and Oresme's treatment of the problem; cf. *Questiones super geometriam Euclidis*, Question 10, Question 15, and *De configurationibus*, III.vii. Clagett concludes that Galileo "almost certainly knew of the medieval configuration doctrine" from the works of Heytesbury and Swineshead, i.e. not directly from Oresme's works; cf. *ibidem*, pp. 106, 105.

The doctrine also influenced Descartes and Beeckman. However, the most important is the impact of the infinitary methods on the work of John Wallis, due to Thomas Harriot who "called his attention" to the method;[14] cf. *Mechanica sive de motu tractatus geometricus*, London 1670 A.D., Part III, Chapter X, Propositions II–III, especially Prop. II, quoted by Clagett, *ibidem*, pp. 106–107) and Christiaan Huygens (*Horologium oscillatorium sive de motu pendulorum ad horologia demonstrationes geometricae*, Paris 1673 A.D., Part II, Proposition V).

As has already been pointed out, the next domain of application of infinitary methods in Oresme's work is the problem of commensurability and incommensurability of circular motions. His work considered so far is concerned with geometric, or spatial infinity. In the theorems about circular motions of celestial bodies, Oresme uses the actual infinity of time. The prototypes of the essential qualities of Newtonian absolute space emerge for the first time.

There are two main works of Oresme on the problem of circular motions: *Ad pauca rescipientes*, cf. the critical edition of the text by E. Grant in (Grant 1966), and the unpublished *Tractatus de commensurabilitate vel incommensurabilitate motuum coeli*, cf. critical edition also by E. Grant, in (Grant 1971) and (Grant 1961), both written between 1340 and 1370. The first criticizes the views of astrologers and the possibility for them to make predictions about the future. Infinity is present in theorems of the type of Proposition II, Part One, the quotation following Grant's translation in (Grant 1961), p. 391:

> Any mobile arranged as before [i.e. "Let circle ***A*** be double circle ***B***" and they have the same centre – Z.K.] have a finite number of places or points on their circles in which, through an eternal motion, they have been conjuncted an infinite number of times, and in which they will be conjuncted an infinite number of times in the future.

A similar language and method of (or hints for) the proofs is present in Propositions III, VI, Part One, and V, VI, Part Two of the treatise. The infinity of time is expressed in many theorems by the use of the words "never" (e.g. Proposition VII, VIII, Part One, and Propositions XI, XII, XIII, XV), "at any time" ("no time", "other times" etc.; e.g. Propositions VII, IX, Part Two), "through all eternity" (e.g. Proposition IX, Part One, and XIV). As in Proposition II above, Oresme speaks about an "infinite number of points" or "times" (of a conjunction), for example in Propositions II, IV, V, VI, III, Part One, and VI, Part Two or also about an "eternal motion" (Propositions II and VI, Part One).

This new infinite time span (God's perspective) is present in the second treatise from this group, *Tractatus de commensurabilitate vel incommensurabilitate motuum coeli*. The "infinite landscape" of time is an ideal one because Oresme believed that the universe was created by God. Therefore, the infinity of the past is purely theoretical and possible for God only. In some places, Oresme speaks about an actually infinite division of a continuum, as in Proposition 2, Part I; cf. (Grant 1971), p. 183. The proofs involve the infinity of time.

Oresme considers infinitely small arcs and angles, wonders about the paradoxical properties of infinite divisions, regular though incommensurable motions, and speaks about "rational irrationality" or "harmonious discord". In the proof of Proposition 12 in Part II, Oresme uses, for the first time, an infinite spiral without a beginning or an end; cf. *ibidem*, p. 277. However, it does not follow from all this that for making the constructions, we actually need infinite space because the spiral emerges in a determined area; cf. (Grant 1961), pp. 454–456, and (Grant 1971), pp. 59–60.

The use of the "infinite" divine methods by Oresme is unique regarding the problem of the commensurability and incommensurability of circular motions. He had some predecessors, however, in the "finite part" of the methods, Theodosius of Tripoli (*De diebus et noctibus*) for instance; cf. (Grant 1971), pp. 78–86. It is unclear whether Johannes de Muris, the author of *Quadripartitum numerorum* (1343), was a predecessor of Oresme. Grant comes close to the conclusion that they had both read the lost treatise by Campanus of Novara (we know about the treatise from Cardano; cf. (Grant 1971), p. 101, footnote 58, and p. 158), a common source for them; cf. (Grant 1971), pp. 98–102.

Some theorems by Johannes de Muris are similar to those by Oresme but Johannes employs finite methods. In some places, he mentions infinity, as, for instance, in Chapter 13 ("the circle has been subtracted as many times as possible"[15]), in Chapter 14 (where he speaks about two bodies which "will never, through all eternity, conjunct again in the same point"; cf. (Grant 1971), p. 89, and p. 363. Cf. also similar remarks in Chapters 24. Some other example can be found in Chapter 25 ("the conjunction is repeated an infinite number of times", *ibidem*, p. 371, p. 373 ("time had no beginning").

Oresme's research on circular motions influenced some other scholars, either from Paris, such as Henry of Hesse, Marsilius of Inghen, Pierre d'Ailly (*Tractatus contra astronomos*), Jean Gerson (*Trilogium astrologiae theologizatae*), or from Italy, i.e. Paul of Venice, John de Fundis, and Jerome Cardano. The Parisian school was mainly interested in the commensurability versus incommensurability of the motions, without use of infinitary methods. Some traces of infinitary concepts are present in Marsilius of Inghen (died 1396). Grant quotes two relevant examples from his *Questiones super octo libros Physicirum*, (Book 8, Questio 3; see (Grant 1971), p. 128 and 129).

Paul of Venice (died 1429) speaks of the Great Year and the cyclic return of history. John de Fundis (in the XV century), as an astrologer, wrote a critical commentary of Oresme's *Ad pauca respicientes*. "(...) [H]e reveals a complete lack of understanding of Oresme's objectives and, together with numerous errors in his version of Oresme's text, the result is not a happy one" writes Grant; (Grant 1971), p. 138.

Oresme's main successor (though probably not aware of the fact) was Jerome Cardano (1501–1571), who, in *Opus novum de proportionibus,* gives seven propositions on circular motions. He tries to substitute the Euclidean language of proportions by a new algebraic one. In one corollary (Corollary 2 to the Theorem 48), Cardano demonstrates that the three given bodies have to conjunct "through all eternity" at one single point only; cf. (Grant 1971), p. 146–147.[16] Thus, Cardano uses new algebra without the infinite Oresmian methods.

Actual infinity is also considered by philosophers. Some traces of Oresme's methods can be found in a treatise by John Major (1467–1550), *Propositum de infinito*, who argues for cathegorematic (actual) infinity in certain cases; cf. (Mair 1938), pp. 6–16. The considerations of John Major are purely philosophical but he was inspired by the achievements of mathematicians.

We are at the end of the story concerning the introduction of God's point of view in mathematics. The above-mentioned historical facts are known. However, the connection between them and the role of God's point of view *in mathematics* is new. The next steps in the development of infinitary methods in mathematics were made by Descartes, Cavalieri, Torricelli, Wallis, Newton *et al.* The reader can find the relevant information about these steps in my book, *Platonism and the Development of Mathematics. Geometry and Infinity.* One can say that modern, or even contemporary mathematics, has been created from God's point of view and the adoption of *God's perspective* in mathematics was one of the most important factors in the emergence of modern mathematics, especially Calculus, and of modern science, especially of Newtonian mechanics.

NOTES

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[2.] Cf. Roger Bacon *Physica*, Book IV, p. 108 in (Bacon 1928). I quote the English translation by E. Grant in (Grant 1982), p. 106.

[3] Cf. (Cleomedes 1891), pp. 14, 16, (Czwalina 1927), pp. 5–6 and (Grant 1982), p. 107.

[4] For more information about the problems analyzed in this section; see (Król 2015).

[5] Analysis and literature on the subject, see (Cross 1998).

[6] The information containing a few paragraphs from this section were also published in 2013 in my paper; cf. (Król 2013).

[7] Further information, together with the relevant quotations and analyses, can be found in Chapter 14 (Król 2015) on which this section is based.

[8] "The proportion of the speeds of motions varies in accordance with the proportion of the power of the mover to the power of the thing moved"; cf. (Busard 2010), p. 9, and (Crosby 1961), p. 111.

[9] The same kind of over-wrapping sphere is used in some other works of Oresme; cf. *Questions on the Physics*, the question III.12, in the *Questiones super geometriam Euclidis*, *Questio* 4, and *Livre du ciel et du monde*, pp. 234–236 in (Oresme 1968); cf. also (Murdoch 1968), p. 517.

[10] Clagett doubts if the author is Roger Bacon. He thinks that the treatise is a work by an early fourteenth century author; cf. (Clagett 1968), p. 57, footnote 15.

[11] For more historical details concerning the origins of Oresme's geometrical method with the translation of the relevant passage from Roger Bacon, Arnald of Villanova and Jean Buridan; see (Clagett 1968), pp. 54–58.

[12] Clagett decides that one more manuscript, i.e. Codex A.50, Bern, Stadtbibliothek, 172r–176r, with a text of the treatise titled *De proportione dyametri quadratiiad costam eiusdem* is a work of Oresme's disciple rather than of Oresme himself or Albert of Saxony. In this work, infinite line is in use. The method applies to a mathematical problem that is very similar to the method used in the above-quoted fragments of *Questions on the Generation and Corruption* and *Questions on the Physics*. Cf. the Latin text in (Clagett 1968), p. 65, footnote 18. The author of this work also transforms an infinite sphere into a finite body which is, however, in contradiction to Oresme's conclusions in *Questions on the Physics* and *Livre du ciel et le monde*; cf. *op. cit.*, p. 66.

[13] The reader can find the necessary details concerning the works quoted below in (Clagett 1968), pp. 66–97.

[14] Cf. the manuscript in London, Brit. Mus., Add. Ms 6789, f. 62r.

[15] Cf. the Latin text and an English translation of *Quadripartitum* in (Grant 1971), p. 363.

[16] In Proposition 52, Cardano speaks about bodies where the bodies "will never meet"; cf. (Grant 1971), p. 157. Cardano indicates as his source not Oresme but a small (unknown to us) work of Campanus of Novara.

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