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CANTOR, GOD, AND INCONSISTENT MULTIPLICITIES*

Abstract. The importance of Georg Cantor's religious convictions is often neglected in discussions of his mathematics and metaphysics. Herein I argue, *pace* Jané (1995), that due to the importance of Christianity to Cantor, he would have never thought of absolutely infinite collections/inconsistent multiplicities, as being merely potential, or as being purely mathematical entities. I begin by considering and rejecting two arguments due to Ignacio Jané based on letters to Hilbert and the generating principles for ordinals, respectively, showing that my reading of Cantor is consistent with that evidence. I then argue that evidence from Cantor's later writings shows that he was still very religious later in his career, and thus would not have given up on the reality of the absolute, as that would imply an imperfection on the part of God.

The theological acceptance of his set theory was very important to Cantor. Despite this, the influence of theology on his conception of absolutely infinite collections, or inconsistent multiplicities, is often ignored in contemporary literature.[1] I will be arguing that due in part to his religious convictions, and despite an apparent tension between his earlier and later writings, Cantor would never have considered inconsistent multiplicities (similar to what we now call proper classes) as completed in a *mathematical* sense, though they are completed in *Intellectus Divino*.

Before delving into the issue of the actuality or otherwise of certain infinite collections, it will be informative to give an explanation of Cantor's terminology, as well a sketch of Cantor's relationship with religion and religious figures. Such will comprise the first part of this paper, after which I will argue that although there is tension between how Cantor discusses the absolute infinite before roughly 1896, and inconsistent multiplicities thereafter, due to his continuing and even strengthening religious convictions, Cantor would have maintained his earlier position that inconsistent multiplicities are not *mathematically* extant, but also not merely potential. I achieve this aim by first pointing out that the evidence taken by Jané (1995) to show

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that Cantor changed his view on absolute infinities is in fact consistent with my opposing thesis, and then pointing to additional evidence from Cantor's later writings that supports my view. I will do this by showing that Cantor's continued religiosity, as evidenced by his later correspondence, is inconsistent with a fundamental change in his metaphysical view of the absolute. Before concluding, I point to a few ways in which this interpretative project might influence the current philosophy of mathematics.

1. Cantor and Religion

Historically interesting, but also important in understanding Cantor's way of thinking is the fact that, despite his copious correspondence with Catholic theologians (cf. Tapp, 2005), he was in fact Protestant (Meschkowski & Nilson, 1991, p. 444). His familiarity with Catholicism was due in large part to his reading of Aquinas, as well as his mother's Catholicism. Perhaps because of the tension between Catholicism and Protestantism in Cantor's life, he was not adherent to any of the organised churches, and thus was comfortable questioning Catholic dogma. Further evidence of Cantor's willingness to question Catholic teaching is his (pseudonymous) publication, in 1905, of a pamphlet entitled *Ex oriente lux*,[2] in which he argues that Joseph of Arimathea was the biological father of Jesus, thereby denying the virgin birth (Tapp, 2005, §6.9). All of this is not to say that Cantor wasn't religious, quite the contrary, but more on this in (especially) §6.

Despite his willingness to question the Church, it was very important to Cantor to show that his theory of actually infinite sets could be rectified with Catholic teaching which traditionally held that the only completed infinite was the infinite of God. This may have been partly a result of Cantor's apparent belief that set theory was given to him directly by God. This belief is evidenced by letters to Gösta Mittag-Leffler from the winter of 1883–4 in which Cantor claimed explicitly to have been given the content of his articles by God, having only provided the organisation and style himself (see Dauben, 1990, p. 146). Further evidence of Cantor's perceived connection to God comes from a letter to his father early in his mathematical career where he speaks of an "unknown, secret voice [compelling] him to study mathematics" (Dauben, 1990, p. 288); and a letter to Hermite in 1894 in which he thanks God for constraining him to Halle (by denying him a position in Göttingen or Berlin) so that he could better serve Him and the Catholic Church (Dauben, 1990; Meschkowski, 1967, p. 124).

Combined with the publication of *Ex oriente lux*, the letter to Hermite provides good evidence, outside of Cantor's mathematical correspondence with theologians, of his continued or even deepening connection to God and the Catholic church later in his career. Additionally, it is worth noting that the vast majority of Cantor's theological correspondence was written after his first psychological breakdown of 1884. In fact Dauben (1990) and Meschkowski (1967) hint at an explicit connection between Cantor's declining mental health and his increased interest in religion. The correlation may also be explained in part by the lack of non-religious reading material available to him during his incapacitations (Meschkowski 1967).

2. Metaphysics of Sets

When thinking about Cantor's philosophy and theology of mathematics it is important to remember that Cantor was a platonist about mathematics, which is to say a realist about independently existent mathematical objects. This is not the platonism of Frege or Gödel however, as Cantor's platonism is based on what we now might call *theological psychologism*. He believed that any coherent mathematical object (mathematical object quantified over in a coherent mathematical system) must exist necessarily due to God's omniscience, omnipotence, and magnificence. The existence 'proof'

> is based on the concept of God and concludes first from the highest perfection of God's essence the possibility of the creation of a transfinitum ordinatum, then from his benevolence and his magnificence the necessity of the actual creation of the transfinitum. (Meschkowski & Nilson, 1991, p. 255 translated in Jané, 2010, p. 212)

For a fuller picture of Cantor's theologico-metaphysical conception of mathematical objects later in his career, the following passage from an 1895 letter to Jeiler is instructive.

> ... there are transfinite cardinal numbers and transfinite ordinal numbers, which possess a mathematical regularity as definite and as humanly researchable as the finite numbers and forms. All these particular modes of the transfinite exist from eternity as ideas in the divine intellect. (Tapp, 2005, p. 427 translated in Jané, 2010, p. 218)

This is particularly telling, as it confirms that Cantor maintained his theological psychologism until, at worst, just before the period that will be the focus in the second half of this paper.

The reasoning is that any coherent mathematical object possibly exists, and every possibly existent object *already* exists in the mind of God, thus, due to the above mentioned qualities of God, any coherent mathematical object exists.[3]

Another aspect of Cantor's platonism is picked out well by Micheal Hallett who paraphrases what he calls Cantor's 'Platonic principle' which is that "the 'creation' of a consistent coherent concept in the human mind is actually the uncovering or discovering of a permanently and independently existing real abstract idea." (Hallett (1984, p. 18), from Cantor (1962, III.4)) This is a nice formulation in Cantorian terms of the thesis that the mathematical universe exists objectively, so that mathematics is in the business of discovering mathematical truths rather than e.g. constructing tools useful to science.

What is important for the purposes of this investigation is that for Cantor, all of the cardinals and ordinals exist independently of mortal minds, and therefore there is a meaningful way in which the $\aleph$-sequence (say) is completed. I will argue below however, that the completion of such sequences, alternatively, the existence of what are now called proper classes, was to not to be considered a *mathematical* completion, but rather metaphysical and/or theological.

3. Actual, Potential, and Absolute Infinities

In his *Grundlagen einer allgemeinen Mannigfaltigkeitslehre* (Foundations of a General Theory of Manifolds, hereafter *Grundlagen*) (Cantor, 1962, originally published in 1883), Cantor makes the distinction between *Eigentlich-* and *Uneigentlichunendlichen*, usually translated as "proper" and "improper infinities". Later, in *Über die verschiedenen Standpunkte in bezug auf das aktuelle Unendliche* (first published in 1886), Cantor identifies these with actual and potential infinities respectively (Tapp, 2005, §3). This essentially boils down to the difference between infinities that cannot be 'increased' or added to, and those that can. It turns out however, that due to an idea of Cantor's that Hallett (1984, p. 7–8) calls the "domain principle," which says that any potential or increasable series must have a domain into which it increases, the only actual infinite is the completed infinite. In other words, potential infinities may be heuristically useful, but are not actual.[4]

A second distinction made by Cantor is between two kinds of actual infinities: the transfinite and the absolute. It is this distinction that will be central to the rest of the paper, as it is, in an important sense, the distinction between the mathematical and the divine. The transfinite can be likened to domains of discourse —what potential infinities lead to, or increase into, while the absolute is the domain of God, embracing both the finite and the transfinite, and hence is unknowable. Cantor puts it like this: "the absolute can be acknowledged, but never known, nor approximately known" (Tapp, 2012, pp. 10–11).

It was the explication of this distinction that convinced many Catholic theologians, in particular Cardinal Franzelin, a papal theologian to the Vatican Council who was initially worried about the threat of pantheism often raised by theologians with respect to the actual infinite, that transfinite set theory was not a threat to Catholic doctrine (Dauben, 1990, p. 145).

4. (In)Consistent Multiplicities

4.1. Epistolary Evidence

Beginning around 1896, Cantor makes a terminological shift from writing about the absolute as opposed to the transfinite, and begins instead writing of consistent and inconsistent multiplicities. These were to be identified with sets and the absolute, respectively. Cantor introduces inconsistent multiplicities in his July 1899 letter to Dedekind thus:

> A multiplicity can be such that the assumption of the "togetherness" of *all* of its elements leads to a contradiction, so that it is impossible to regard the multiplicity as a unity, "a completed thing". Such collections I call *absolute infinities* or *inconsistent multiplicities* [all emphasis original] (Meschkowski & Nilson, 1991, p. 407).

A paragraph later he defines consistent multiplicities as those that can be thought of as *one* thing, and identifies these with sets. At first glance this may appear to be an innocent change in terminology, lacking any deep conceptual significance. However, this change, combined with Cantor's apparent use of Ω (the ordinal sequence) and the $\aleph$-sequence in mathematical argumentation (albeit in proofs that said such sequences cannot be considered sets, see §4), and Cantor's renewed emphasis on the difference between the absolute and the transfinite, has led some to argue that beginning around 1897, Cantor began to think of absolute infinities/inconsistent multiplicities as merely potential infinities. Jané (1995) makes this point explicitly, citing especially Cantor's letters to Hilbert between 1897 and 1899.[5]

I take it that Jané's argument rests particularly on three phrases from Cantor's letters to Hilbert from September 2, 1897 and October 6, 1898 (published in Meschkowski & Nilson, 1991, pp. 390, 393–5, respectively[6]).

First is the contrast of "absolutely infinite sets" (i.e. inconsistent multiplicities[7]) to the transfinite sets, of which "it is possible to think without contradiction... of all of their elements as being together, and consequently, of the set itself as a thing in itself; or again (in other words) if it is possible to think of the set together with the totality of their [sic] elements as actually existing." The second important passage says that "the totality of all alephs cannot be conceived as a definite and also completed set" (both of these quotations are from the 1897 letter, translated in Jané 1995, p. 389).

Note that in both of these passages, it is the possibility of *thinking of* or *conceiving* of an inconsistent multiplicity as a completed object that is questioned. No explicit metaphysical claim is being made. This *thinking of* or *conceiving* can easily be thought of as implicitly including only human thought, as opposed to divine thought. Read in this way, it is still possible for inconsistent multiplicities to exist in *Intellectus Divino*, the Mind of God. This, in turn, is consistent with Cantor's earlier view that the Absolute was beyond human comprehension, but nevertheless actual, due to its existence in the Mind of God.

The third passage, consisting of two paragraphs from the 1898 letter, and paraphrased by Jané (1995, p. 390), says

> that all of the Alephs are not coexistent, cannot be brought together (*zusammengefasst*) as a 'thing in itself', in other words regarded as a completed set.
>
> [...]
>
> The *absolute unboundedness* of the set[8] of all Alephs appears as grounds for the impossibility that they can be brought together as a completed thing in itself. (Meschowski & Nilson, 1991, p. 395, my translation, all emphasis original)

At first it may seem that this passage vindicates the reading of Cantor as taking inconsistent multiplicities to be potential, but this is not the only way to understand it. That 'all of the Alephs are not coexistent' must be seen only as a contrast with sets, whose members *are* coexistent. This is natural because Cantor says explicitly that this is meant to mean that all of the alephs cannot be *thought of* as a completed set. Furthermore the use of the word 'completed' (*fertig*) is likely meant to differentiate from the non-standard use of the word 'set' elsewhere in the passage, so that 'completed set' just means transfinite set.[9]

The second of the above quoted passages can simply be taken to say that it is impossible to consider the $\aleph$-sequence as a single *mathematical* unit. This seems natural, as Cantor goes on to explain why the antinomies are not problematic for his theory. Most notable is the Burali-Forti paradox which shows that the collection of all ordinals cannot be a set (cf. Copi, 1958, for an exposition and interesting discussion). The publication of said paradox in 1897 may have been what prompted Cantor to discuss the antinomies, as he did not think that they were applicable to his theory. It is also clear that Cantor was aware of the Burali-Forti paradox, as well as the analogous cases for the cardinals, and the entire set-theoretic universe. To avoid the antinomies, we need not take inconsistent multiplicities as potential, but only as non-mathematical. One might object to this by saying that, even earlier in this letter, Cantor makes mathematical use of the $\aleph$-sequence, but upon closer inspection we realise that he can be read as only taking it as mathematical in so far as to show that that assumption leads to a contradiction —that if the $\aleph$-sequence is a set, then there must be a cardinal number larger than itself.

4.2. The Generating Principles

Perhaps more convincing is the implicit reliance of Cantor's proof of the inconsistency (i.e. non-sethood) of Ω on the generating principles from the *Grundlagen*. The first generating principle for the ordinals says that for any ordinal α there exists a next greater ordinal equal to $\alpha + 1$. The second says that for any sequence of ordinals with no greatest element, there is an ordinal greater than all of them, which is called a limit ordinal. Because of these principles, Ω, as the system of all ordinals, cannot itself be an ordinal as that would imply that there is an ordinal greater than Ω that is also *in* Ω. In other words, if Ω is a set, then $\Omega < \Omega$, a contradiction.

Jané (1995) argues that this is problematic because, if the two generating principles are presented more formally, the second principle, which Jané correctly takes to be necessary for the derivation of the contradiction, either depends on the independent existence of ordinals, or collapses to circularity.

He mathematises[10] the second principle of generation thus:

> If A is a set of ordinals without a largest element, there is a (unique) ordinal β such that (i) $A < \beta$ (i.e., for all $\alpha \in A$, $\alpha < \beta$) and (ii) for no $\gamma : A < \gamma < \beta$. We put $\beta = \lim A$ (p. 395).

Jané points out that this definition relies on an independent determination of which collections of ordinals can be called sets, and concludes that the generating principles must be extra-mathematical, metaphysical principles (*ibid.*).[11] There are two problems with this argument. First, there is no good reason to believe that the generating principles were meant to be purely mathematical, especially given Cantor's platonism (see §2). And second, even if we take Jané's mathematisation as faithful to Cantor's intent, we can take the generating principles to be (at least part of) the definition of what is is to be an ordinal/set of ordinals. Ordinals are (transitive) well-ordered sets that are discovered via the two generating principles. This definition is circular, but not viciously so, assuming that we can identify the finite ordinals independently (and accept the domain principle), which would give us, by the second principle of generation, the smallest transfinite ordinal, ω.

Given the above analysis, there is at worst an unresolved tension between Cantor's later use of inconsistent multiplicities in seemingly mathematical contexts, and his earlier insistence that the absolute is the domain of God, and unmathematisable. What is lacking in Jané's analysis of this tension is a thorough consideration of Cantor's theology with respect to his conception of the absolute, which will be the focus of the next section.

5. Back to Religion

Given his continued, and even growing connection to the Catholic church later in his career, it seems unlikely that Cantor would have completely given up his conception of the absolute, and therefore inconsistent multiplicities, as being actual, i.e. having objective existence. This is especially true given the explicit identification of the absolute with God. If inconsistent multiplicities are meant to be potentially infinite, as Jané would have us believe,[12] then either the identification with God must be thrown out, or there must be some imperfection in God's knowledge or power. The second option goes directly against the Catholic conception of God that Cantor seems to be working with, so either inconsistent multiplicities were no longer the domain of God, or remained actual and unmathematisable despite having some quasi-mathematical content. It is the latter I find more plausible, as it does not attribute a radical change in thought to Cantor. I will thus argue that, taking Cantor's words at face value and appealing to the fact that he never takes a definite stand on the issue in his later writings, combined with epistolary evidence that Cantor's religious convictions never waned, Cantor is unlikely to have ever thought of inconsistent multiplicities as merely potential.

Having shown that Jané's arguments in favour of a radical change in Cantor's conception of the absolute are insufficient, we need only to note that after 1896, Cantor writes very little else about the nature of inconsistent multiplicities, mentioning them only in contrast to consistent multiplicities, and without explicit appeals to metaphysics or theology.[13] This lack of evidence, cited by Jané (1995), is certainly not evidence of anything, and is thus consistent with the interpretation that Cantor never gave up his realism about the Absolute. Coupled with this is the copious evidence from Cantor's later writings that his religious convictions remained strong into his later years. I will provide what I take to be a representative sample.

Perhaps first and foremost is a short letter to Pope Leo XIII from February 1896 in which he offers the Pope seven copies of the *confessionem fidei* of Francis Bacon, to which Cantor attached particular significance, as well as three copies of the works of Bacon.[14] Cantor then expresses his love for the Pope and the Holy Roman Catholic Church, signing the letter "Your Holiness's humblest and most highly devoted servant" (Meschkowski & Nilson, 1991, p. 383 – my translation, from Meschkowski's translation from the Latin into German).[15]

The closing seems only appropriate for a letter to the Pope, but the importance put on Bacon's confession of faith, as well as the fact that Cantor was sending gifts to the Pope in the first place, is strong evidence that his faith wasn't waning, especially as he says that the gifts were meant as token of his love for the Pope and the Church.

In March 1896 Cantor wrote a letter to Father Thomas Esser, a Jesuit priest in Rome. In this letter, Cantor emphasizes the necessary connection between metaphysics and theology, before turning to the subject of his own mathematics, saying first that "Every extension of our insight into what is possible in creation leads necessarily to an extended cognition of God" (translation from Tapp, 2012, p. 9).[16] This principle, elaborated in this letter and elsewhere, can be taken to say that furthering our knowledge of the transfinite gets us closer to an (unobtainable) knowledge of the absolute, which is identified with God.

This letter is also relevant as an example of Cantor's wider correspondence with theologians between roughly 1894 and 1896 in which he asks that they confirm the acceptability of his theory of the transfinite theologically (Tapp, 2005, §6.4). This is noteworthy not just because of the date (at worst just before the period we are considering), but also because it shows just how important it was to Cantor that his theory be accepted, not just mathematically, but theologically as well. This is further supported by Cantor's apparent pride, and continual citation of the fact that Cardinal Franzelin had not found his transfinite sets, nor his identification of the Absolute with the divine, as theologically problematic (Meschkowski, 1967; Dauben, 1990; Tapp, 2005).

The final letter I will cite here in favour of Cantor's continued religiosity is a passage from a letter to Grace Chisholm-Young from June 1908 where he states:

> I have never assumed a 'Genus Supremum' of the actual infinite. Quite on the contrary I have rigorously proved that there can be no such 'Genus Supremum' of the actual infinite. What lies beyond all that is finite and transfinite is not a 'Genus'; it is the unique, completely individual unity, in which everything is, which contains everything, the 'Absolute', unfathomable for human intelligence, thus not subject to mathematics, unmeasurable, the 'ens simplicissimum,' the 'Actus purissimus,' which is by many called 'God.' (Meschkowski & Nilson, 1991, p. 454)[17]

This quotation speaks more directly to Cantor's conception of the absolute later in his career, and confirms the thesis that the absolute is incomprehensible and beyond the domain of mathematics. That Cantor says that the absolute is unmeasurable further reinforces the idea that inconsistent multiplicities cannot be investigated mathematically. Perhaps more importantly, the claim that the absolute 'contains everything' (*die Alles umfasst*), combined with Cantor's realist conception of sets, suggests very strongly that Cantor was still thinking of the set-theoretic universe as being completed in a metaphysically robust sense. This reading is particularly compelling given the intrinsic connection between metaphysics and theology expounded by Cantor.[18]

On the other hand, and since I am attempting to establish Cantor's continued religiosity in connection with his metaphysics of sets, we might wonder at the locution 'which is by many called "God"' which certainly seems to suggest a hesitation to endorse that particular identification. That may be so, but it is clear nonetheless that whether or not the absolute is to be directly identified with God, it is certainly divine in nature. One possible way to read this then, is that Cantor is allowing for there to be more to God than how he has characterised the absolute in this passage.

These passages (and many others besides) make a strong case that Cantor was still very religious later in life, which in turn supports my contention that it is unlikely that he would have ever considered inconsistent multiplicities as merely potential, as that would imply an imperfection on the part of God, with whom such infinities were identified in Cantor's earlier writings.

6. Concluding Remarks

In keeping with the theme of this issue, this piece can simply be seen as a historical case study showing the interaction between mathematics and theology, and the importance of considering the theological commitments of mathematicians when evaluating their (often implicit) metaphysical positions, but I would like to suggest that there may be further implications of this work for the contemporary philosophy of mathematics. Although there are few philosophers of mathematics who would take theological arguments about the metaphysics of mathematics seriously, we can nevertheless take the Cantorian view that proper-classes are not mathematical in the same sense as sets. Welch (2012), and Welch & Horsten (UM) are certainly in this vein, taking a Cantorian view of the mathematical universe, but substituting mereology for the theological absolute, and it seems possible that other mathematical realists could make use of such approaches. Certainly also in this vein are proponents of the limitation of size conception of set theory (see for example, Hallett, 1984), who take a Cantorian view of the set-theoretic universe as being limited mathematically to things that are not "too big to be sets."

Additionally, a more thorough understanding of Cantor's views (to which I hope this piece contributes) will facilitate a better understanding of Cantor's influence on more recent philosophy including the above mentioned realists and limitation of size-ists, as well as Husserl, Gödel, and their intellectual descendants.[19] This, in turn, will help our understanding of the development of the philosophy of mathematics in the twentieth century.

To recap, in §4 I argued that neither Jané's (1995) argument from Cantor's letters to Hilbert, nor his argument from the mathematisation of the generating principles for ordinals are sufficient to show that Cantor moved from thinking of the absolutely infinite as actual to thinking of inconsistent multiplicities as potential. In the first case it is merely a matter of a slight variation in interpretation, while in the latter Jané fails to take into account Cantor's platonism, or his view that mathematics, metaphysics and theology are intimately and necessarily related.

I then argued from letters and other late writings of Cantor that his religious convictions were just as strong late in his career as they were earlier on, which would make it unlikely that Cantor would have ever taken the absolute as potential, as this would involve questioning God's perfection. In combination with the negative result just mentioned, we have good reason to believe that Cantor maintained his realism well past 1897.

I will end by noting that, although it may be impossible to definitely settle the question (without talking to Cantor), the above arguments show that the proposed interpretation of Cantor is more charitable, and nearer the truth than the attribution of a radical metaphysical shift in Cantor's thinking.

NOTES

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[1] But see van der Veen & Horsten (2013) for an interesting discussion and use of this connection, as well as, of course, (Tapp, 2005).

[2] Subtitled: "Gespräche eines Meisters mit seinem Schüler über wesentliche Punkte des urkundlichen Christentums." In a letter to Jourdain later the same year (reprinted in Meschkowski & Nilson, 1991, pp. 442-3), Cantor elaborates on the contents of the pamphlet showing deep knowledge and respect for the biblical texts.

[3] Note here that Cantor is not precise about what he means by coherence, though intra-theoretic consistency seems to be a necessary, but likely not sufficient, condition. Consistency with established mathematics is also a continuing theme, though this is also weaker than consistency in the modern, technical sense.

[4] Compare this to Ernst Zermelo's conception of the set theoretic universe as consisting in an ever increasing series of normal domains.

[5] It is worth noting at this point that Welch & Horsten (UM, p. 5) also seem to disagree on this point, though it is not central to their project, so discussion is quite limited.

[6] The dates referenced by Jané and Meschowski & Nilson for the first letter differ by exactly one month, but it it clear that it is the same letter.

[7] The use of the word *Mengen*, translated as 'sets' here, should not be taken to signify a commitment to the $\aleph$-sequence as a set, but rather a lapse in terminological consistency. This is supported by footnotes 59 and 60 in Jané (2010), as well as by Cantor himself in various letters (see, for example Meschkowski & Nilson, pp. 399, 446).

[8] See previous note.

[9] The distinction between 'set (*Menge*)' and 'completed set (*fertige Menge*)' in this letter and elsewhere looks to be analogous to the distinction between 'proper class' and 'class' sometimes made in contemporary set-theory.

[10] By 'mathematise,' I mean, in this context, the (in this particular case anachronistic) formalisation of Cantor's notions in purely mathematical (as opposed to philosophical) language.

[11] In fact, Jané is even more explicit about this in his (2010) where he states:

> As Cantor acknowledged, the generating approach to numbers does not belong to mathematics proper. This notwithstanding, it is precisely this approach that brought to light the riches of the absolutely infinite number sequence and provided fruitful insight into its structure. (p. 224)

[12] Although he backs away from this position somewhat in his (2010).

[13] See Meschkowski & Nilson (1991, pp. 393–5, 409, 433–4) for letters to Hilbert, Dedekind and Jourdain respectively.

[14] Frank Jankunis suggested to me that the numbers of copies may be related to the creation story in Genesis, and the Holy Trinity, respectively.

[15] But thanks to Arlin Daniel for double checking this with the original Latin.

[16] Letter reprinted in full in (Tapp, 2005, pp. 307–312).

[17] Translation based on that in (van der Veen & Horsten, 2013, p. 9), with minor modifications based on the original text.

[18] See especially the above mentioned 1896 letter to Esser, in which Cantor expounds at length, and with rhetorical skill, the close connections between theology, metaphysics, mathematics (including set-theory) and natural science. (Tapp, 2005, pp. 307–312).

[19] See (Ternullo, TA) for an in depth and thorough investigation of Cantor's influence on Gödel via Husserl.

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