Aircraft Automatic Control System Failure and Flight Safety

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Abstract – This article presents a mathematical model estimating the probability of successful completion of the aircraft’s flight in case of aviation equipment failure in flight. This paper shows the relationship between the aircraft’s automatic control system and flight safety. The calculations of probability are made for the successful completion of the flight on Boeing 737 aircraft when the automatic control system has failed.

Keywords – Aircraft, automatic control system, flight safety, negative factors, probability of failure.

I. INTRODUCTION

The aircraft automatic control system was initially designed to maintain main performance functions in case of failure. For this purpose, it is designed to be multi-channel, i.e. two, three or four completely identical control channels run as parallel steering control circuits, and the failure of one or two channels cannot affect the overall performance of the system. However, even a full pre-flight check of the automatic control system by means of a software-controlled test cannot give an absolute guarantee of system integrity. One of the most important characteristics of the airplane’s successful flight is an adequate position of bank angle [1]–[8].

The aircraft is banked when it is turning around its longitudinal axis. There is a restriction for each aircraft roll, which is calculated on the basis of structure, strength, aerodynamics and speed of the civil aircraft: 25 degrees is a limit allowed in flight, 30 degrees is a limit allowed when the aircraft has to avoid an obstacle. In critical situations, pilots are permitted to exceed the maximum angle of bank with the aim of saving passengers’ lives. The limits of the permitted bank angle are usually exceeded due to the human factor.

For example:

1. During the maintenance of B-737 aircraft, cable connections of the same type, which transmit signals from bank angle and direction angle sensors, were mixed up. The maintenance error occurred because the connectors of both circuits were of the same size. Accordingly, the ailerons tried to reduce fluctuations in the direction coordinate plane, while the direction rudder tried to reduce the oscillations of bank angle. As a result, volatility increased and the design restrictions were exceeded leading to the collapse of the aircraft fuselage under overload conditions.

2. The automatic functional control system on the aircraft is powered by three-phase voltage (36 V). The maintenance mechanic connected wires “colour to colour”, i.e. yellow to yellow and so on. As a result the automatic functional control system was powered with incorrect polarity, which led to the reverse polarity of pulses. This means that the system rocked the aircraft instead of damping it. This resulted in an accident.

This article presents the calculation results of the probability of successful completion of the flight on B-737 type aircraft with autopilot failure on a channel providing signals of bank angle.
II. A MATHEMATICAL MODEL ENABLING TO ESTIMATE THE PROBABILITY OF THE SUCCESSFUL COMPLETION OF THE FLIGHT WHEN THE FLYING AIRCRAFT IS EXPOSED TO NEGATIVE FACTORS

To estimate the probability of safe flight, let us look at the conditions of accident development process such as consecutive changes of aviation system conditions, i.e. transition from one status to another (Fig. 1).

The conditions of “crew-aircraft” system are marked as node points on the scheme.

where

- \( AF / \overline{AF} \) – the emergency factor does not manifest itself / manifest itself;
- \( AFN / \overline{AFN} \) – the emergency factor has been eliminated / has not been eliminated;
- \( AN / \overline{AN} \) – successful flight / accident.

The possibility of transitioning from one status to another is indicated by the arrows. The transition probability is indicated next to arrows. To determine the probability of the successful completion of the flight \( P_{LD}(t) \), it is necessary to consider various cases.

1. The emergency factor in flight does not manifest itself (event \( AF \)). In this case (1), the probability of \( P_{LD}(t) \) would be consistent with the accident probability factor.

\[
P_{AF} = P_{AF}
\] (1)

2. During the flight, emergency factor \( AF \) with probability \( Q_{AF} \) may manifest itself in the system. Its appearance means only the possibility of an accident rather than its inevitability. At the beginning of the event the aircraft parameters begin to differ at some rate from the given parameters [9]. Stating this fact with the help of some sources of information, the pilot will try to remedy the consequences of factor emergence and, first of all, will try to avoid parameter \( X_i \) restricted by the flight safety conditions, he/she will also try to avoid going beyond the permissible values \( (\alpha \leq \alpha_{allowable}, \ n_y \leq n_y_{allowable}, \ M \leq M_{allowable}, \ and\ others) \) [10]. In the general case, the pilot is able to remedy the factor \( (AF) \) or fails to deal with its consequences.
Let us denote the conditional probability of these events with \( p(AN/AF_i) \) and \( q(AN/AF_i) \) (2) respectively. With account of the pilot’s intervention and elimination of the effects of the factor, the probability of the successful completion of the flight in case of emergency factor manifestation is

\[
P_{LD}(AF_i/AN) = q_{AF_i} \cdot p(AN/AF_i). \tag{2}
\]

3. If the determining parameters have gone beyond the permissible limits in spite of the pilot’s intervention, this does not always lead to an aviation accident [11].

Let us suppose that as a result of exposure to the emergency factor, the allowable angle of attack has been exceeded and the aircraft is stalling. In this case, the pilot by taking correct actions can prevent the aircraft from spinning and avoid the aviation accident. Let us denote the probability of aviation accident prevention after the aircraft has exceeded the permissible operating parameters by \( p(AN/AFN) \) (3).

The probability of the successful completion of the flight after the emergency factor has manifested itself and the flight determinant has gone beyond the allowable limits is

\[
P_{LD}(AF_i, AFN, AN_i) = q_{AF_i} \cdot q(AFN/AF_i) \cdot p(AN_i/AFN). \tag{3}
\]

The probability that the flight will end in an aviation accident \( i \) due to the emergency factor in accordance with the probability multiplication theory may look as

\[
P_{LD}(AF_i, AFN, AN_i) = Q_{LD}(t) = q_{AF_i} \cdot q(AFN/AF_i) \cdot q(AN_i/AFN), \tag{4}
\]

so the probability of the successful completion of the flight is

\[
P_{LD}(t) = 1 - Q_{LD}(t) = 1 - q_{AF_i} \cdot q(AFN/AF_i) \cdot q(AN_i/AFN) =
\]

\[
= 1 - (1 - p_{AF_i}) \cdot \{1 - (AFN/AF_i)\} \cdot \{1 - (AN_i/AFN)\}. \tag{5}
\]

In most cases, it is believed that the conditional probability of accident prevention is \( p(AN_i/AFN) \). Thus, the probability of the successful completion of the flight will be

\[
P_{LD}(t) = p_{AF_i} + (1 - p_{AF_i}) \cdot p(AN_i/AF_i). \tag{6}
\]

So the probability of the successful completion of the flight after emergency factor \( i \) depends on the absence of the emergency factor during the flight and the conditional probability that the pilot will prevent the determinants from going beyond the allowable limits.

With the help of separate analytical flight safety criteria it is possible to determine the probability of the successful completion of the flight in case of emergency factors related to aviation equipment failure, personnel error or adverse external environmental effects [12].

For example, the probability of the successful completion of the flight in case of aviation equipment failure is determined by a partial analytical criterion, which for some possible failure \( j \) can be written down as:

\[
P_{TLD}(t) = p_{Tj}(t) + [1 - p_{Tj}(t)] \cdot p_{Tj}(AFN/ATT), \tag{7}
\]

where

\[ P_{Tj}(t) \] – probability of the absence of aviation equipment failure \( j \) in flight (the probability of the fail-safe operation of aircraft functional systems);

\[ P_{Tj}(AFN/ATT) \] – conditional probability that the pilot will avoid the consequences of failure \( j \).

If during the flight there appears \( m \), i.e. aviation equipment failures which are independent from one another, the probability of the successful completion of the flight will be:
III. The Probability of the Impact of Boeing 737/500 Autopilot System Failure on Flight Safety, Analysis and Calculation of the Probability

Description of the situation: in the case of autopilot failure with some bank angle, the aileron will deviate by a certain angle and the aircraft will exceed the critical angle of bank [13]. Bank angle γ and angular velocity ωx will be regarded as determining parameters. The example is given for a hypothetical aircraft.

Initial data:
- the coefficient of aerodynamic damping moment along the x axis \( m_{x}^{\alpha x} = -0.52 \);
- wing length \( l = 37.5 \) m;
- the coefficient of aerodynamic moment \( m_{x}^{\delta x} = -0.0042 \);
- wing cross-sectional area \( 39.2 \) m\(^2\);
- the moment of inertia along the x axis \( I_x = 10 \times 105 \) kg·m\(^2\);
- the angle of aileron deflection \( \delta_3 = 9^\circ \);
- aircraft weight \( G = 70 \) 000 kg;
- allowable aircraft rotational speed (specified in regulatory documentation for each type of aircraft) \( X_{\text{allowable}} = 20^\circ = 0.349 \) rad;
- autopilot failure rate \( \lambda = 0.0004 \);
- flight duration 3 h;
- height \( H = 9000 \) m;
- flight speed \( V = 240 \) m/s.

Solution path: let us determine the probability of the successful completion of the flight by using the specific (particular) analytical criterion (9):

\[
P_{\text{TLD}}(t) = P_T(t) + [1 - P_T(t)] \cdot P_{\text{ATT}}(AFN/\text{ATT}),
\]

where
- \( P_T(t) \) – probability of the successful completion of the flight depending on time \( t \);
- \( P_T \) – probability of autopilot failure-safe operation during time \( t \);
- \( P_{\text{ATT}}(AFN/\text{ATT}) \) – certain probability of the pilot’s reaction and failure prevention in case of failure (pilots are trained on simulators and training devices during certain periods of time; this value depends on the pilot’s qualification);
- \((1 - P_T)\) – probability of failure.

1. Let us determine the value of \( P_T \) (10):

\[
P_T = e^{-\lambda t} = e^{-0.0004 \cdot 3} = 0.9988 .
\]

2. Let us determine the value of \( P(PRIFAIL) \) (11):

Isolated bank movement is described by the equation:

\[
I_x \frac{d\alpha_x}{dt} = M_x^\delta_3 + M_x^{\alpha x} \cdot \omega_x ,
\]

where
- \( I_x \) – moment of inertia against axis \( x \);
- \( \frac{d\omega_x}{dt} \) – aircraft’s rotation acceleration against axis \( x \);
- \( M_x^\delta_3 \) – aerodynamic moment against axis \( x \) (aileron deflection per unit);
\( \delta_3 \) – aileron deflection angle;
\( M_x^{\omega x} \) – an aerodynamic damping moment;
\( \omega_x \) – angular velocity against axis \( x \),
or
\[ T \dot{\omega}_x + \omega_x = k \delta_3, \]  
(12)

where
\( T \) – rotation period;
\( \dot{\omega} \) – angular velocity acceleration against axis \( x \);
\( \omega \) – angular speed against axis \( x \);
\( K \) – damping coefficient;
\( \delta_3 \) – aileron deflection angle,
where the constant time (period) \( T \) equal to
\[ T = \frac{l_x}{-M_x^{\omega x}} = \frac{l_x}{-m_x^2 \cdot \omega_x \cdot \frac{t}{2V} \cdot \frac{\rho V_x^2}{2}}, \]  
(13)

where
\( V \) – aircraft speed;
\( \rho \) – air density.
Damping coefficient \( K \) is equal to
\[ K = \frac{M_x^{\omega x}}{M_x^{\omega x}} = \frac{m_x^2}{m_x^{\omega x}} = \frac{m_x^2}{m_x^{\omega x} \cdot \frac{t}{2V}}, \]  
(14)

where
\( M_x^{\omega x} \) – aerodynamic damping moment.
Solution to equation (12) is expressed as follows:
\[ \omega_x = \omega_{x \ past} (1 - e^{-\frac{t}{T}}), \]  
(15)

where
\( \omega_{x \ past} \) – constant angular speed along axis \( x \);
\( e^{-\frac{t}{T}} \) – exponent of the total time to the period.
If \( \omega_{x \ allow} > \omega_{x \ past} \), parameter \( \omega_x \) at a specific failure is not a determining one, i.e. angular velocity will not exceed the permissible value [14].

If \( \omega_{x \ allow} < \omega_{x \ past} \), the failure can be dangerous and angular speed can go beyond the permissible norms (this case is characteristic of a manoeuvrable aircraft).

To find the dependence of bank angle \( \gamma \) on time \( t \), let us transform equation (15):
\[ \omega_x = \frac{dy}{dt} = \omega_{x \ past} (1 - e^{-\frac{t}{T}}), \]  
(16)

and
\[ \gamma = \int \omega_{x \ past} (1 - e^{-\frac{t}{T}})dt, \]  
(17)
from this it follows that
\[ \gamma = \omega_{x \ past} \left( t + T \cdot e^{-\frac{t}{T}} \right) + c, \]  
(18)

where
\( c \) – constant value received as a result of function integration.
If we assume that \( t = 0, \gamma = 0 \), it follows that:
\[ c = -\omega_{x \ past} \cdot T, \]  
(19)
and
\[ \gamma = \omega_{x \text{ past}} \cdot t + \omega_{x \text{ past}} \cdot T e^{-\frac{t}{T}} - \omega_{x \text{ past}} = \omega_{x \text{ past}} \cdot t + \omega_{x \text{ past}} \cdot T \left( e^{-\frac{t}{T}} - 1 \right). \] (20)

In this task, the bank angle is regarded as a determining parameter. The bank angle will stop increasing at the moment when the pilot returns the ailerons to initial basic position [15]. Consequently, the pilot’s activity time \( t_p \) – time during which bank angle \( \gamma \) reaches \( \gamma_{\text{allowable}} \) value. This \( \gamma_{\text{allowable}} \) is determined by equation (20) by solving it graphically (Fig. 2).

Let us determine the required values:

\[ T = \frac{19 \cdot 10^5}{0.52 \cdot \frac{37.5}{2} \cdot 240} = 0.269 \text{ (s)}, \] (21)

\[ K = \frac{0.0042}{0.52 \cdot \frac{37.5}{2} \cdot 240} = 0.1 \left( \frac{1}{s} \right), \] (22)

\[ \omega_{x \text{ past}} = 0.1 \cdot 9 = 0.9 \left( \frac{1}{s} \right). \] (23)

Assuming some values of \( t \), it is possible to determine the value of bank angle \( \gamma \). The calculation results are shown in Table I.

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>RESULTS OF CALCULATIONS DEPENDING ON ( T ) VALUES</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>( T, \text{ s} )</td>
</tr>
<tr>
<td>1</td>
<td>( \omega_{x \text{ past}} \cdot t )</td>
</tr>
<tr>
<td>2</td>
<td>( \omega_{x \text{ past}} \cdot T )</td>
</tr>
<tr>
<td>3</td>
<td>( e^{-\frac{t}{T}} )</td>
</tr>
<tr>
<td>4</td>
<td>( e^{-\frac{t}{T}} - 1 )</td>
</tr>
<tr>
<td>5</td>
<td>( \omega_{x \text{ past}} \cdot T \left( e^{-\frac{t}{T}} - 1 \right) )</td>
</tr>
<tr>
<td>6</td>
<td>( \gamma, \text{ rad} )</td>
</tr>
<tr>
<td>7</td>
<td>( \gamma, \text{ °} )</td>
</tr>
</tbody>
</table>
The mathematical expected time $m_{tb}$ when the pilot starts to control the aircraft can be determined by using the relationships from this task:

$$
\begin{align*}
\dot{\omega}_x \text{average} &= \frac{\omega_x \text{allowable}}{t_b} \left(1 - e^{-\frac{t}{T}}\right) \\

m_{tb} &= 0.308 + \frac{0.142}{\dot{\omega}_x \text{average}},
\end{align*}
$$

where

- $\dot{\omega}_x \text{average}$ – average value of angular speed acceleration;
- $t_t$ – current time before bank angle stabilization;
- $m_{tb}$ – mathematical expected time.

This set of equations can be presented with the help of the graphical method. Having set several $t_t$ values, let us determine $m_{tb}$. On the basis of the calculation results, it is possible to create a graph with $m_{tb} = f(\dot{\omega}_x \text{average})$ and $\omega_x \text{average} = f(t_t)$, as shown in Fig. 3.
From Fig. 3 it follows that $m_{th} = 0.380$ s. The probability determined by the pilot’s backup activity in case of aircraft failure can be determined by using the Laplace function [16]:

$$P\left(\frac{m}{att}\right) = dt \int_0^T f(t) = 0.5 + \phi \left(\frac{t_p - m_{th}}{6_{th}}\right) = 0.5 + \phi \left(\frac{0.62 - 0.38}{0.34 \cdot 0.38}\right) = 0.9678. \quad (25)$$

In addition to that:

$$6_{th} = c_T \cdot m_{th} = 0.34 \cdot m_{th}, \quad (26)$$

where

- $P\left(\frac{m}{att}\right)$ – probability of the pilot’s reaction and failure prevention in case of failure;
- $\phi$ – Laplace function coefficient;
- $c_T$ – table coefficient;
- $6_{th}$ – mean square value.

The probability that the flight will be completed without accident is calculated by using equation (9):

$$P_v(t) = P_T + (1 - P_T) \cdot P\left(\frac{m}{att}\right) = 0.998 + (1 - 0.998) \cdot 0.9678 = 0.99996. \quad (27)$$

IV. CONCLUSION

As a result of autopilot failure on the bank angle channel, the permissible bank angle is exceeded, i.e. the consequences of failure cannot be prevented through the pilot’s backup activity in the process of controlling the aircraft during the flight.

From a hundred thousand flights on this type of aircraft with flight duration of three hours, the allowable value of bank angle will be exceeded due to autopilot failure.

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