Research Article

Akram Masoud, Walid Ali Rahoma*, Elamira Hend Khattab, and Fawzy Ahmed Abd El-Salam

Design of Artificial Sun-Synchronous Orbits with Main Zonal Harmonics and Solar Radiation Pressure Using Continuous Low-Thrust Control Strategies

<https://doi.org/10.1515/astro-2019-0012>

Received Oct 16, 2018; accepted Feb 13, 2019

Abstract: Artificial sun-synchronous orbits are suitable for remote sensing satellites and useful in giving accurate surface mapping. To design such orbits accurately with arbitrary orbital elements, three control strategies are provided with the consideration of main zonal harmonics up to J_4 and solar radiation pressure (SRP). In this paper, the continuous variable low-thrust control is used as a way to achieve these artificial orbits and given by electric propulsions rather than chemical engines to enlarge lifespan of the spacecraft. The normal continuous low-thrust control is used to illustrate the control strategies. Furthermore, formulas for refinement of normal control thrusts are applied to overcome errors due to approximations. The results of the simulation show that the control strategies explained in this paper can realize sun-synchronous orbits with arbitrary orbital parameters without side effects and the effect of solar radiation pressure is very small relative to main zonal harmonics. A new technique is suggested, ASSOT-3, to minimize fuel consumption within one orbital period more than others. This technique is based on computing the root mean square of the rate of ascending node longitude instead of the average.

Keywords: Synchronous orbit – Solar Radiation Pressure - Control Design - Low-thrust

List of Abbreviations

SSO Sun-Synchronous Orbits

ASSOT	Artificial SSO Technique
avg	average
LAN	Longitude of Ascending Node
ob	oblateness
res	residuals
rms	root mean square
SRP	Solar Radiation Pressure

1 Introduction

At the equator, a satellite can pass overhead at the same local time in each revolution through what is called Sun-synchronous orbits (SSO) (or heliosynchronous orbits) by a combination of altitude and inclination in a certain way. In other words, the plane of the SSO rotates at the same rate of the rotation of the Earth around the Sun. The angle of SSO and the Sun is constant (Wang *et al.* 2011). SSO are based on the secular variation of the angle of ascending node due to a central body of a non-spherical shape. This asphericity can be used for a useful purpose that is maintaining the orbital plane in a fixed orientation to the Sun. Consequently, SSO can be achieved with a little expulsion of propellant. The period of the orbit is synchronized with the rotation of a central body around the Sun. Such orbits are useful for giving accurate surface mapping. (Wu *et al.* 2015) and enables Earth observation platforms to repeat region observations over an extended time under similar

Akram Masoud: Department of Astronomy and Space Science, Faculty of Science, Cairo University, Cairo, Egypt, 12613; Email: akram_2007_omar@yahoo.com

Corresponding Author: Walid Ali Rahoma: Department of Astronomy and Space Science, Faculty of Science, Cairo University, Cairo, Egypt, 12613; Laboratoire d'Astronomie de Lille/Université de Lille, 59000 Lille, France; Email: rahoma@sci.cu.edu.eg

Elamira Hend Khattab: Department of Astronomy and Space Science, Faculty of Science, Cairo University, Cairo, Egypt, 12613; Email: elamira_hend@yahoo.com

Fawzy Ahmed Abd El-Salam: Department of Astronomy and Space Science, Faculty of Science, Cairo University, Cairo, Egypt, 12613; Taibah University, Faculty of Science, Mathematics Department, Madinah, Saudi Arabia; Email: f.a.abdelsalam@gmail.com

illumination conditions (Macdonald and Badescu 2014). Because of the regular precession of orbital plane, the SSO is suitable for remote sensing satellites (Wu *et al.* 2015). SSOs that can be realized by control strategies are called artificial sun-synchronous orbits. There are two control ways to realize such orbits: multi-impulsive control and continuous variable low-thrust control (Wu *et al.* 2014). The electric propulsions or chemical engines can provide these controls (Weeden *et al.* 2008) and (Macdonald *et al.* 2010). The electric propulsion systems are more efficient because they can enlarge the lifespan of spacecrafts and they have high specific impulse (Folcik 2008). As a proper example, refer to the Gravity Field and Steady-State Ocean Circulation Explorer (GOCE) (Enrico 2008). A wide range of orbital elements can be selected by using of control strategies compared with natural SSOs. This is shown by simulation. J_2 is the dominant perturbation that causes the precession of ascending node (Abd El-Salam *et al.* 2006; Rahoma and Deleflie 2014). However, perturbations due to J_3 , J_4 and SRP are taken into account in this paper. The fuel consumption is related to increment (characteristic) velocity. For this reason, the increment velocity for each control strategy must be calculated in order to optimize the fuel consumption. According to the mean element method, the change averaged rate of ascending node longitude is deduced from Gauss' variation of orbital elements. Then, the normal acceleration must be used to realize SSOs. Finally, we must eliminate residual secular growth by amending methods. The control strategies can achieve SSOs with arbitrary orbital elements. Furthermore, more energy can be saved without any side effect on other orbital elements (Wang *et al.* 2011).

There are many modern research papers about SSOs and frozen orbits. (Wang *et al.* 2011) studied SSOs and frozen orbits around the Earth. They provided several control strategies to realize these orbits using continuous low-thrust. Only J_2 perturbation was taken into account. They used amending methods to eliminate the residual secular growth and proposed that the normal control thrust is a function of semimajor axis and eccentricity. Despite (Wu *et al.* 2014) did not use amending methods to obtain accurate values, they developed their research to include J_3 and J_4 . Furthermore, they studied orbits around the Earth and Mars. They proposed that the magnitude of control thrust around the Earth is larger than that of Mars. After that, they developed their studies to include orbits around terrestrial planets Mercury, Venus, the Earth, and Mars with J_2 , J_3 and J_4 perturbations using continuous low-thrust control (Wu *et al.* 2015). Earth and Mars have natural SSOs while Venus and Mercury do not have. The normal control acceleration of the artificial SSO around the Mars is less than those of the others. (Kuznetsov and Jasim 2016) researched

the dynamic evolution on the initial value of LAN over an interval of twenty years. This was for two families of SSOs with altitudes of 1191 and 751 km. Celestial Mechanics software package were used for this simulation. They proposed that the dynamic evolutions were depending on the initial values of LAN and passages through the shadow of Earth lead to long-period perturbations of the semimajor axis.

This paper focuses on artificial SSOs around the Earth with main zonal harmonics up to J_4 (Rahoma 2014; Rahoma and El-Salam 2014) and SRP. Three control strategies are provided. A new technique, ASSOT-3, is used by a calculation of root mean square of the rate of LAN. The Simulation shows that ASSOT-3 is the best selection to save more energy with no side effect on other parameters.

2 Secular Growth of LAN due to Constant Thrusts

Assume that $\vec{F}$ is a constant thrust vector and let S , T and W be radial, transverse and normal disturbing acceleration components of the thrust $\vec{F}$, respectively. To express the relation between such components and the time derivative of the variation of LAN, Gaussian variation of the orbital elements equations can be used. We need only the change rate of LAN,

$$\frac{d\Omega}{dt} = \frac{\sqrt{1-e^2}}{\sin i} \sqrt{\frac{a}{\mu}} \frac{\sin(\omega+f)}{1+e\cos f} W \quad (1)$$

From Eq. (1), $\dot{\Omega}$ depends only on the normal disturbing acceleration component W . According to the mean element method (Zhou *et al.* 2008; Brouwer 1959; Masoud *et al.* 2018), we deal with the secular change of the LAN. If $\dot{\Omega}_{avg}$ represents the average of $\dot{\Omega}$ over one orbital period T_o , then the formula that expresses this average can be written as,

$$\dot{\Omega}_{avg} = \frac{1}{T_o} \int_0^{T_o} \dot{\Omega} dt \quad (2)$$

According to Eq. (1), the LAN movement is not a function of time t but rather of the true anomaly f . Therefore, the conversion from explicit time dependence dt to the true anomaly dependence df in unperturbed case is formulated as:

$$dt = \frac{T_o}{2\pi} (1-e^2)^{3/2} \frac{df}{(1+e\cos f)^2} \quad (3)$$

Substitute Eq. (3) into Eq. (2), (Wilkins and Kapila 2007) yields,

$$\dot{\Omega}_{avg} = \frac{(1-e^2)^{3/2}}{2\pi} \int_0^{2\pi} \frac{\dot{\Omega}}{(1+e \cos f)^2} df \quad (4)$$

Because of the existence of cyclic quantities such as $\sin(f + \omega)$ in the movement of ascending node equation, the direct average calculations may produce dissipation in the absolute average. Therefore, the root mean square of $\dot{\Omega}$ may be needed.

$$\dot{\Omega}_{rms} = \sqrt{\frac{1}{T_o} \int_0^{T_o} \dot{\Omega}^2 dt} \quad (5)$$

Substitute Eq. (3) into Eq. (5), yields,

$$\dot{\Omega}_{rms} = \frac{(1-e^2)^{3/4}}{\sqrt{2\pi}} \sqrt{\int_0^{2\pi} \frac{\dot{\Omega}^2}{(1+e \cos f)^2} df} \quad (6)$$

3 Secular Variation of LAN Caused by Main Zonal Harmonics and SRP

Due to the Earth asphericity perturbations, there is a secular change of LAN position. This analysis of asphericity perturbation on orbits through main zonal harmonics J_2 , J_3 and J_4 was proposed by (Kozai et al. 1959). According to mean element theory, the mean nodal precession rate due to oblateness is given by

$$\begin{aligned} \dot{\Omega}_{ob} = & -\frac{3\mu^{1/2}J_2R_e^2}{2a^{7/2}(1-e^2)^2} \cos i \\ & -\frac{9\mu^{1/2}J_2^2R_e^4}{4a^{11/2}(1-e^2)^4} \left\{ \left(\frac{3}{2} + \frac{e^2}{6} - 2\sqrt{1-e^2} \right) \right. \\ & - \sin^2 i \left(\frac{5}{3} - \frac{5}{24}e^2 - 3\sqrt{1-e^2} \right) \\ & \left. - \frac{35J_4}{18J_2^2} \left[\left(\frac{6}{7} + \frac{9}{7}e^2 \right) - \sin^2 i \left(\frac{3}{2} + \frac{9}{4}e^2 \right) \right] \right\} \cos i \end{aligned} \quad (7)$$

The equation of motion of LAN due to SRP is given from Sun-Synodic Equations of Motion as follows (Lantukh et al. 2015),

$$\begin{aligned} \dot{\Omega}_{SRP} = & -1.9143 \\ & \times 10^{-10} \sqrt{\frac{a}{\mu}} \frac{e}{\sqrt{1-e^2}} \sin \omega \sin (\Omega - f) \end{aligned} \quad (8)$$

4 Artificial Sun-Synchronous Orbits

The planes of the SSOs is subjected to a synchronous precession with the rotation of the central planet around the Sun (Boain 2004). The natural SSOs are formed due to many perturbations. The most important perturbation is J_2 . In this paper, also J_4 and the SRP have been taken into account in control strategies to design artificial SSOs. The natural SSOs satisfy the following relation:

$$\dot{\Omega} = \dot{\Omega}_{ob} + \dot{\Omega}_{SRP} \quad (9)$$

where $\dot{\Omega}$ is the angular speed of the Earth about the Sun and it equals to $\frac{0.9865^\circ}{day}$. Since the natural SSOs have to strict constraints on orbital elements, the artificial orbits are needed. There are similar characteristics between the natural and the artificial orbits. However, the orbital parameters can be selected arbitrarily in the case of the artificial orbits. From Gaussian variation of parameters equations (Wu et al. 2014), the mean nodal precession rate can be changed by normal control thrust. Let the average variation rate consists of three parts $\dot{\Omega}_{ob}$, $\dot{\Omega}_{SRP}$ and $\dot{\Omega}_{avg/rms}$ from normal control thrust. Then the artificial SSOs will be satisfied,

$$\dot{\Omega}_{avg/rms} = \dot{\Omega} - \dot{\Omega}_{ob} - \dot{\Omega}_{SRP} \quad (10)$$

Substitute Eq. (1) into Eq. (4) and Eq. (6),

$$\dot{\Omega}_{avg} = \frac{(1-e^2)^2}{2\pi \sin i} \sqrt{\frac{a}{\mu}} \int_0^{2\pi} \frac{\sin(\omega + f)}{(1+e \cos f)^3} W df \quad (11)$$

$$\dot{\Omega}_{rms} = \frac{(1-e^2)^{5/4}}{\sqrt{2\pi} \sin i} \sqrt{\frac{a}{\mu}} \sqrt{\int_0^{2\pi} \frac{\sin^2(\omega + f)}{(1+e \cos f)^4} W df} \quad (12)$$

The characteristic (increment) velocity can express the fuel consumption of an orbital period,

$$V = 2\pi \mu^{-1/2} a^{3/2} W \quad (13)$$

There are three diverse control techniques designed to regulate the nodal precession rate. They are ASSOT-1, ASSOT-2 and ASSOT-3. The term "ASSOT" is originated as an acronym for "Artificial Sun-Synchronous Orbit Technique". Only normal thrust is responsible for realizing SSO. a and e have been assumed to be constants in Eq. (11) and Eq. (12). However, there are short and long periodic variations in both a and e . Therefore, the calculation of the average or root mean square of $\dot{\Omega}$ will not give accurate values for normal continuous low-thrust control. To overcome the effect of

constancy of a and e , such a correction must be done. From Eq. (10), (Wang *et al.* 2011),

$$\dot{\Omega}_{avg/rms} + (-\dot{\Omega} + \dot{\Omega}_{ob} + \dot{\Omega}_{SRP}) = 0 \quad (14)$$

For every control strategy, the average or root mean square of the nodal precession rate will be expressed as a factor $C_{ASSOT-i}$ of normal continuous low-thrust control $W_{ASSOT-i}$ where $i = 1; 2$ or 3 .

$$\dot{\Omega}_{avg/rms} = C_{ASSOT-i} W_{ASSOT-i} \quad (15)$$

From Eq. (14) and Eq. (15) yields,

$$C_{ASSOT-i} W_{ASSOT-i} + (-\dot{\Omega} + \dot{\Omega}_{ob} + \dot{\Omega}_{SRP}) = 0 \quad (16)$$

Due to the non-included perturbations, Eq. (16) can be corrected with some residuals in the rate of change of LAN as,

$$\bar{C}_{ASSOT-i} W_{ASSOT-i} + (-\dot{\Omega} + \dot{\Omega}_{ob} + \dot{\Omega}_{SRP}) = \dot{\Omega}_{res} \quad (17)$$

By inserting corrected thrust,

$$\bar{C}_{ASSOT-i} \bar{W}_{ASSOT-i} + (-\dot{\Omega} + \dot{\Omega}_{ob} + \dot{\Omega}_{SRP}) = 0 \quad (18)$$

From Eq. (16), Eq. (17) and Eq. (18), the corrected thrust will be,

$$\bar{W}_{ASSOT-i} = W_{ASSOT-i} - \frac{\dot{\Omega}_{res}}{C_{ASSOT-i}} \quad (19)$$

4.1 ASSOT-1

In this control strategy, the normal control thrust is constant. The integral in Eq. (11) is independent of the normal thrust (Wang *et al.* 2011).

$$\begin{aligned} \dot{\Omega}_{avg} &= -\frac{3}{2} \frac{e}{\sqrt{1-e^2}} \frac{\sin \omega}{\sin i} \sqrt{\frac{a}{\mu}} W_{ASSOT-1} \\ &= C_{ASSOT-1} W_{ASSOT-1} \end{aligned} \quad (20)$$

Substitute Eq. (20) into Eq. (10), the normal control thrust will be,

$$W_{ASSOT-1} = -\frac{2\sqrt{1-e^2} \sin i}{3e \sin \omega} \sqrt{\frac{\mu}{a}} (\dot{\Omega} - \dot{\Omega}_{ob} - \dot{\Omega}_{SRP}) \quad (21)$$

Substitute Eq. (21) into Eq. (13), the increment velocity will be,

$$V_{ASSOT-1} = -\frac{4a\pi\sqrt{1-e^2} \sin i}{3e \sin \omega} (\dot{\Omega} - \dot{\Omega}_{ob} - \dot{\Omega}_{SRP}) \quad (22)$$

Figure 1. Reversing the direction of $\vec{W}$ at $f \in [\pi, 2\pi]$

4.2 ASSOT-2

The normal component has a constant magnitude whereas the direction is reversed (Wilkins and Kapila 2007) to eliminate the effect of the sign change of the integrand in Eq. (11) at $f \in [\pi, 2\pi]$ Figure 1 and consequently, the average of the LAN rate will be,

$$\begin{aligned} \dot{\Omega}_{avg} &= \frac{(1-e^2)^2}{2\pi \sin i} \sqrt{\frac{a}{\mu}} \\ &\cdot W \left[\int_0^\pi \frac{\sin(\omega+f)}{(1+e \cos f)^3} df - \int_\pi^{2\pi} \frac{\sin(\omega+f)}{(1+e \cos f)^3} df \right] \end{aligned} \quad (23)$$

$\dot{\Omega}_{avg}$ is given approximately as, (Wu *et al.* 2014),

$$\begin{aligned} \dot{\Omega}_{avg} &= \frac{2}{\pi \sin i} \\ &\cdot \sqrt{\frac{a}{\mu}} \left[\frac{1}{(1-e^2)^3} - \frac{e^2}{(1-e^2)^{2.5}} \cos(2\omega) \right] W_{ASSOT-2} \\ &= C_{ASSOT-2} W_{ASSOT-2} \end{aligned} \quad (24)$$

Substitute Eq. (24) into Eq. (14), the normal thrust will be,

$$\begin{aligned} W_{ASSOT-2} &= \frac{\pi \sin i}{2} \sqrt{\frac{\mu}{a}} \left[(1-3e^2+3e^4) \right. \\ &\quad \left. + (e^2-3.5e^4) \cos(2\omega) + e^4 \cos^2(2\omega) \right] \\ &\quad \times (\dot{\Omega} - \dot{\Omega}_{ob} - \dot{\Omega}_{SRP}) \end{aligned} \quad (25)$$

Moreover, the characteristic velocity will be,

$$\begin{aligned} V_{ASSOT-2} &= \pi^2 a \sin i \left[(1-3e^2+3e^4) \right. \\ &\quad \left. + (e^2-3.5e^4) \cos(2\omega) + e^4 \cos^2(2\omega) \right] \\ &\quad \times (\dot{\Omega} - \dot{\Omega}_{ob} - \dot{\Omega}_{SRP}) \end{aligned} \quad (26)$$

4.3 ASSOT-3

In this control strategy, the normal control thrust is constant. The integral in Eq. (11) is independent of the normal thrust.

$$\begin{aligned}\dot{\Omega}_{rms} &= \frac{(1-e^2)^{5/4}}{\sqrt{2} \sin i} \sqrt{\frac{a}{\mu}} \left\{ 1 + \left[\frac{5}{2} - \frac{5}{4} \cos(2\omega) \right] e^2 \right. \\ &\quad \left. + \left[\frac{55}{16} - \frac{5}{4} \cos(2\omega) - \frac{25}{32} \cos^2(2\omega) \right] e^4 \right\} W_{ASSOT-3} \\ &= C_{ASSOT-3} W_{ASSOT-3}\end{aligned}\quad (27)$$

Substitute Eq. (27) into Eq. (10), the normal thrust will be,

$$\begin{aligned}W_{ASSOT-3} &= \sqrt{2} \sin i \sqrt{\frac{\mu}{a}} \left\{ 1 + \left[-\frac{5}{4} + \frac{5}{4} \cos(2\omega) \right] e^2 \right. \\ &\quad \left. + \left[\frac{35}{32} - \frac{55}{16} \cos(2\omega) + \frac{75}{32} \cos^2(2\omega) \right] e^4 \right\} \\ &\quad (\dot{\Omega} - \dot{\Omega}_{ob} - \dot{\Omega}_{SRP})\end{aligned}\quad (28)$$

From Eq. (13) the characteristic velocity will be,

$$\begin{aligned}V_{ASSOT-3} &= 2\sqrt{2} \pi a \sin i \sqrt{\frac{\mu}{a}} \left\{ 1 \right. \\ &\quad \left. + \left[-\frac{5}{4} + \frac{5}{4} \cos(2\omega) \right] e^2 \right. \\ &\quad \left. + \left[\frac{35}{32} - \frac{55}{16} \cos(2\omega) + \frac{75}{32} \cos^2(2\omega) \right] e^4 \right\} \\ &\quad (\dot{\Omega} - \dot{\Omega}_{ob} - \dot{\Omega}_{SRP})\end{aligned}$$

5 Simulation

The Earth's data and the initial orbital parameters of reference orbit used for the simulation of SSOs are mentioned in Table 1 (Wu *et al.* 2014).

Table 1. Data of Earth and Reference Orbit

Quantity for Earth	Value	Orbital Parameter	Value
J_2	1.083×10^{-3}	a	2.5×10^7 m
J_3	-2.53266×10^{-3}	e	0.1
J_4	-2×10^{-6}	i	45°
μ	3.968×10^{14} m ³ /s ²	ω	270°
R_e	6.378145×10^6 m	Ω	100°
$\dot{\Omega}$	$\frac{0.9865^\circ}{d} = 1.993 \times 10^{-7}$ rad/s	f	0

Figure 2. $\dot{\Omega}_{ob}/\dot{\Omega}_{SRP}$ vs. semimajor axis and eccentricity

Figure 3. $\dot{\Omega}_{ob}/\dot{\Omega}_{SRP}$ vs. semimajor axis and inclination.

Figure 4. Control thrust with the different considered strategies vs. semimajor axis.

To compare the effect of main zonal harmonics and SRP, the ratio $\frac{\dot{\Omega}_{ob}}{\dot{\Omega}_{SRP}}$ is 3D plotted twice.

One picture represents vs. semimajor axis and eccentricity Figure 2 and the other is vs. semimajor axis and inclination Figure 3.

It is clear that the effect of main zonal harmonics is roughly at least three orders of magnitude more massive than that of SRP.

As it appears in Figure 4, the control strategy ASSOT-3 is very close to control strategy ASSOT-2 than that of control

Table 2. Control Thrusts and Characteristic Speeds of all Control Strategies

Technique	Control Thrust m/s^2	Control Thrust without SRP m/s^2	Increment Speed m/s	Increment Speed without SRP m/s
ASSOT-1	3.9509×10^{-3}	3.9510×10^{-3}	155.777	155.780
ASSOT-2	9.1294×10^{-4}	9.1296×10^{-4}	35.995	35.996
ASSOT-3	8.2186×10^{-4}	8.2188×10^{-4}	32.404	32.405

Figure 5. Control thrust vs. semimajor axis and eccentricity for ASSOT-2 and ASSOT-3 **Figure 7.** Control thrust vs. semimajor axis and inclination for ASSOT-2 and ASSOT-3 **Figure 6.** Control thrust vs. semimajor axis and eccentricity for all techniques **Figure 8.** Control thrust vs. semimajor axis and inclination for all techniques

strategy ASSOT-1, so the comparison of ASSOT-2 with ASSOT-1 is separated than that ASSOT-2 with ASSOT-3.

For each technique, the control thrusts are 3D plotted twice. One is vs. semimajor axis and eccentricity, see Figure 5 (in which we compare ASSOT-2 with ASSOT-3) and the Figure 6 (in which we compare ASSOT-1 with ASSOT-2) and the other is vs. semimajor axis and inclination, see Figure 7 (in which we compare ASSOT-2 with ASSOT-3) and Figure 8 (in which we compare ASSOT-1 with ASSOT-2).

Furthermore, the control thrusts and characteristic speeds for all control strategies are fixed for the given data of the Earth and reference orbit in Table 2.

The simulation shows the following:

1. The effect of SRP is very small compared with that of main zonal harmonics.
2. The technique that is illustrated using root mean square, ASSOT-3, saves more energy than those illustrated using averages, ASSOT-1 and ASSOT-2.

6 Conclusion

There are three suggested control strategies to realize SSOs around the Earth based on main zonal harmonics and SRP. They are ASSOT-1, ASSOT-2 and ASSOT-3. All of these control strategies can achieve SSOs with arbitrary orbital parameters. The magnitudes of their normal control thrusts are different. ASSOT-1 has the largest normal control thrust magnitude while ASSOT-2 and ASSOT-3 have close magnitudes and ASSOT-3 has the smallest one. For this reason, ASSOT-3 is the best option for minimizing fuel consumption. The effect of SRP is much less than main zonal harmonics.

References

- Abd El-Salam, F.A., El-Tohamy, I.A., Ahmed, M.K., Rahoma, W.A., Rassem, M. A. 2006 *Applied Mathematics and Computation*, 181, 6-20. DOI:10.1016/j.amc.2006.01.004
- Masoud, A., Rahoma, W. A., Khattab, E. H., El-Salam, F. A. 2018, *The Journal of the Astronautical Sciences* 65: 448–469. DOI: 10.1007/s40295-018-0135-y.
- Boain, R. J. 2004, *Adv. Astronaut. Sci.* 119(1), 85-104. DOI:10.1007/s10569-014-9596-x
- Brouwer, D. 1959, *Astron. J.* 64(1274), 378. DOI: 10.1086/107958
- Enrico C. 2008, *Automatica* 44, 1766–1780. DOI: 10.1016/j.automatica.2007.11.023
- Folcik, Z. J. 2008, M.S. Thesis. Department of Aeronautics and Astronautics, MIT.
- Wang, G., Meng, Y., Zhen, W., Tang, G. 2011, *Acta Mech. Sin.* 27(5), 809–816. DOI: 10.1007/s10409-011-0489-x
- Kozai, Y. 1959, *Astron. J.* 64(9), 367-377.
- Kuznetsov, E.D., Jasim, A.T. 2016, *Sol Syst Res.* 50(3), 197-203. DOI: 10.1134/S0038094616030059
- Lantukh, D., Russell, R.P., Broschart, S. 2015, *Celest. Mech. Dyn. Astr.*, 121(2), 171-190. DOI:10.1007/s10569-014-9596-x.
- Macdonald, M., McKay, R.J., Vasile, M. and de Frescheville, F.B. 2010, *Journal of Guidance, Control and Dynamics* 33(6), 1935-1940. DOI: http://dx.doi.org/10.2514/1.49011.
- Macdonald, M., Badescu, V. 2014. *The International Handbook of Space Technology*. University of Strathclyde, Glasgow, Scotland. Polytechnic University of Bucharest, Bucharest, Romania.
- Rahoma, W. A., Deleflie, F. 2014, *Indian Journal of Science and Technology*. 7(7), 968-974.
- Rahoma, W.A. 2014, *Journal of Astronomy and Space Science*. 31(3), 199-204. DOI: 10.5140/JASS.2014.31.3.199
- Rahoma, W.A., El-Salam, F.A. 2014, *Journal of Astronomy and Space Science*. 31(4), 285-294. DOI: 10.5140/JASS.2014.31.4.285
- Wilkins C., Kapila, V. 2007, *AIAA Guidance, Navigation, and Control Conference*, Hilton Head, SC, AIAA-2007-6844.
- Weeden, B., Shortt, K. 2008, Paper No. 2008-3547, *AIAA SpaceOps 2008 Conference*.
- Wu, Z., Jiang, F., Li, J. 2014, *Astrophys Space Sci.* 352(2), 503-514. DOI:10.1007/s10509-014-1962-3
- Wu, Z., Jiang, F., Li, J. 2015, *Astrophys. Space Sci.* 360, 12. DOI: 10.1007/s10509-015-2529-7
- Zhou, J.B., Yuan, J.P., Luo, J.J. 2008, *Journal of Astronautic*. 29(5), 1536–1539 (In Chinese).