Research Article

Ireneusz Włodarczyk*

The potentially hazardous NEA 2001 BB16

<https://doi.org/10.1515/astro-2019-0016>

Received Apr 10, 2019; accepted Aug 05, 2019

Abstract: We computed the impact solutions of the potentially dangerous Near Earth Asteroid (NEA) 2001 BB16 based on 47 optical observations from January 20.08316 UTC, 2001, through February 09.15740 UTC, 2016, and one radar observation from January 19.90347 UTC, 2016. We used two methods to sample the starting Line of Variation (*LOV*). First method, called thereafter *LOV1*, with the uniform sampling of the *LOV* parameter, out to $\sigma_{LOV} = 5$ computing 3000 virtual asteroids (VAs) on both sides of the *LOV*, which gives 6001 VAs and propagated their orbits to JD2525000.5 TDT=February 12, 2201. We computed the non-gravitational parameter $A2 = (34.55 \pm 7.38) \cdot 10^{-14} \text{ au/d}^2$ for nominal orbit of 2001 BB16 and possible impacts with the Earth until 2201. For potential impact in 2195 we find $A2 = 20.0 \cdot 10^{-14} \text{ au/d}^2$. With a positive value of $A2$, 2001 BB16 can be prograde rotator. Moreover, we computed Lyapunov Time (*LT*) for 2001 BB16, which for all VAs, has a mean value of about 25 y. We showed that impact solutions, including the calculated probability of a possible collision of a 2001 BB16 asteroid with the Earth depends on how to calculate and take into account the appropriate gravitational model, including the number of perturbing massive asteroids. In some complicated cases, it may depend also on the number of clones calculated for a given sigma *LOV1*. The second method of computing the impact solutions, called thereafter *LOV2*, is based on a non-uniformly sampling of the *LOV*. We showed that different methods of sampling the *LOV* can give different impact solutions, but all computed dates of possible impacts of the asteroid 2001 BB16 with the Earth occur in accordance at the end of the 22nd century.

Keywords: minor planets, asteroids: search, astrometry, orbits

1 Introduction

Asteroid 2001 BB16 was first observed at the Lincoln Laboratory ETS, New Mexico (MPC 704), on January 20, 2001. The discovery will be defined when the object is numbered, as the IAU Minor Planet Center noticed at https://minorplanetcenter.net/db_search. Asteroid 2001 BB16 belongs to the Aten group, comprising 1516 members as of April 7, 2019. Atens are Near-earth asteroids (NEA) orbit similar to that of 2062 Aten (semimajor axis, $a < 1.0 \text{ au}$; aphelion distance, $Q > 0.983 \text{ au}$). Currently, there are 19876 cataloged Near-earth asteroids. According to the JPL (<https://ssd.jpl.nasa.gov/sbdb.cgi#top>), its absolute magnitude is 23.2 mag.

Asteroid 2001 BB16 is not listed as a potential impactor in the next 100 years in both Impact Monitoring Centers: the JPL Sentry - Earth Impact Monitoring (<https://cneos.jpl.nasa.gov/sentry/>) and the NEODys consortium (<https://newton.spacedys.com/neodys/>). Partially possible impact of 2001 BB16 with other potential impactors are discussed in Del Vigna et al. (2019). In this paper, we have

analyzed this problem in many aspects using two methods of the sampling of the Line Of Variation (*LOV*), including computations of Lyapunov Time (*LT*) of the asteroid 2001 BB16.

2 Orbit

We computed the orbit of the asteroid 2001 BB16 based on all observations using the OrbFit software (<http://adams.dm.unipi.it/orbfit/>). Seventeen perturbing asteroids were used according to Farnocchia et al. (2013a,b) and similar to Włodarczyk (2015). We also used the new version of the OrbFit Software, namely OrbFit v.5.0.5, which has the new error model based upon Chesley et al. (2010) and the debiasing and weighting scheme described in Farnocchia et al. (2015). Moreover, we used the DE431 version of the JPL's planetary ephemerides.

The inclusion of a non-gravitational dynamical model to perform the impact monitoring of a Near-Earth Asteroid is necessary in case the orbit is highly constrained and a small change causes visible changes in the computed of impact risk. It is visible in the computations of the impact risk of asteroid 2001 BB16, also in the motion of four aster-

Corresponding Author: Ireneusz Włodarczyk: Chorzow Astronomical Observatory, 41-500 Chorzow, Poland; Email: astrobit@ka.onet.pl

Table 1. Initial nominal orbital elements of the asteroid 2001 BB16: a denotes semimajor axis, e - eccentricity, angles i , Ω and ω refer to the Equinox J2000.0, M - mean anomaly. Epoch: 2018-March-23=JD2458200.5 TDB. Orbital elements are computed with non-gravitational parameter $A2$ and for different value of σ .

a (au)	e	i (deg)	Ω (deg)	ω (deg)	M (deg)
Orbital parameters: non-gravitational $A2=(34.551\pm7.384)E-14$ au/d ²					
17 ast., 3 sigma, 4 sigma, 5 sigma					
8.559142081701E-01	0.171651671680	2.0302540991	122.4423453735	195.892885271	55.341107138
2.87778E-08	1.57945E-07	1.34142E-05	6.25689E-05	1.12492E-04	1.77622E-04

Figure 1. Close approaches of the asteroid 2001 BB16 with the Earth and with Venus.

oids from so called Special group: <https://newton.spacedys.com/neodys/index.php?pc=4.1> as was described in Włodarczyk (2020). Additionally, the object has many deep close approaches and hence, we used the Yarkovsky effect which manifests in a secular semimajor axis drift. We performed the hazard assessment for the NEA 2001 BB16 using the OrbFit software v.5.0.5, including the Yarkovsky effect and using the *LOV* method (with two different sampling techniques).

The easiest method of computing non-gravitational effects in the motion of asteroids can be taken by assuming the existence of the non-gravitational parameter $A2$ connected with the Yarkovsky effect. In the new versions of the OrbFit software 5.0. we can compute $A2$ as a model uncertainty, one of 7 parameters instead of 6 in the past, *i.e.* solving only pure gravitational orbital elements. This allows us to determine the orbit and propagate it with $A2$ in the future to the day of possible impact.

Table 1 presents starting orbital elements of the asteroid 2001 BB16 computed with non-gravitational parameter $A2$. We computed the value of the non-gravitational parameter equal to $A2=(34.551\pm7.384)\cdot10^{-14}$ au/d². It is only one value of $A2$ computed for all cases, *i.e.* for different sigma of Line of Variation (*LOV*).

A positive value of $A2$ for asteroid 2001 BB16 denotes that the mean semimajor axis drift $da/dt > 0$ and hence asteroid can be prograde rotator.

3 Close approaches

To compute close approaches of asteroid 2001 BB16 with the Earth, we first compute its Virtual Asteroids (VAs). To do this we computed orbital elements of 1201 clones (VAs) with the use of the OrbFit software v. 5.0.5 and the method of Milani (Milani, 2006). Following this method, we computed 600 clones of both sides of the *LOV* with the first sampling method of the *LOV*, called thereafter *LOV1* - *i.e.* computed with the uniform sampling of the *LOV* parameter. Then we propagate all the VAs till 2200 and search for close approaches with the Earth and other planets.

Figure 1 presents close approaches of the asteroid 2001 BB16 with the Earth and Venus with non-gravitational parameter $A2=(34.551\pm7.384)\cdot10^{-14}$ au/d² and starting orbital elements from Table 1 with 17 perturbing massive asteroids, 3 sigma and 1201 VAs. We observe deep close approaches (CAs) with the Earth in 2035.0288 at 0.01417 au, in 2082.0366

Table 2. The asteroid 2001 BB16. The *LOV1* method. The earliest possible impacts were computed with a different number of perturbing asteroids, different sigma and for computed non-gravitational parameter $A2=(34.551\pm7.384)\cdot10^{-14}$ au/d². The JPL DE431 Solar System model is used.

date YYYY/MM/DD	MJD	sigma	distance and width RE	stretch RE/sigma	impact probability	exp. en. MT	PS
17 asteroids, 3 sigma							
2195/01/15.544	122781.544	-2.281	3.24±0.003	8.31E+04	1.43E-06	2.05E-05	-5.65
2198/01/15.438	123877.438	-2.155	0.53±0.000	1.14E+08	2.24E-09	3.20E-08	-8.46
2198/01/15.439	123877.439	-2.155	0.36±0.000	2.37E+08	1.08E-09	1.55E-08	-8.77
2199/01/15.822	124242.822	-2.108	2.96±0.000	8.75E+05	2.16E-07	3.08E-06	-6.48
2199/01/15.847	124242.847	-2.108	2.12±0.000	6.06E+05	3.68E-07	5.26E-06	-6.24
2200/01/16.032	124608.032	-1.783	2.77±0.000	8.16E+05	3.98E-07	5.69E-06	-6.21
17 asteroids, 4 sigma							
2192/01/15.802	121685.802	-2.197	0.93±0.000	2.70E+08	8.35E-10	1.19E-08	-8.87
2195/01/15.538	122781.538	-2.281	3.21±0.003	5.94E+04	2.31E-06	3.31E-05	-5.44
2198/01/15.434	123877.434	-2.155	1.51±0.089	5.78E+06	4.18E-08	5.98E-07	-7.19
2199/01/15.876	124242.876	-2.108	2.92±0.005	1.32E+06	1.26E-07	1.80E-06	-6.71
2199/01/15.843	124242.843	-2.108	2.10±0.003	2.31E+05	9.68E-07	1.38E-05	-5.83
2200/01/15.966	124607.966	-1.783	2.90±0.009	4.45E+05	9.87E-07	1.41E-05	-5.82
17 asteroids, 5 sigma							
2195/01/15.533	122781.533	-2.281	3.19±0.004	3.48E+04	4.91E-06	7.02E-05	-5.11
2198/01/15.440	123877.440	-2.155	0.30±0.000	2.45E+08	1.05E-09	1.50E-08	-8.79
2198/01/15.437	123877.437	-2.155	0.68±0.000	6.04E+07	4.19E-09	5.99E-08	-8.19
2199/01/15.843	124242.843	-2.108	2.10±0.003	2.08E+05	1.08E-06	1.54E-05	-5.78
2199/01/15.841	124242.841	-2.108	2.10±0.010	6.32E+04	4.45E-06	6.35E-05	-5.16
2199/01/15.844	124242.844	-2.108	2.10±0.000	3.25E+05	6.88E-07	9.81E-06	-5.97
2200/01/16.053	124608.053	-1.783	3.31±0.000	9.60E+05	2.37E-07	3.38E-06	-6.44
2200/01/15.999	124607.999	-1.783	2.27±0.003	8.93E+04	4.67E-06	6.69E-05	-5.14

at 0.00468 au and in 2185.044 at 0.0001251 au. After 2150, we computed deep CAs with Venus in 2182.399 at 0.0165 au. At the end of the integration period, *i.e.* after 2182, we found many CAs with the Earth which can lead to many possible impacts as we can see in the next Section.

4 Impact risk

To compute impact risk with the Earth, we computed the VAs of asteroid 2001 BB16 similarly to the method of computing CAs in the previous Section. But now, we compute more VAs to search for impacts with a smaller probability. To do this, we computed the orbital elements of 6001 clones (VAs) and propagated all of them till 2200 and searched for close approaches with the Earth and other planets. We used the first method of computing *LOV* called thereafter *LOV1*, with the uniform sampling of the *LOV* parameter. For example, in the *LOV1* method the step-size sampling of the starting semimajor axis is constant and equal to about $4.497\cdot10^{-11}$ au between each of 6001 VAs mentioned above. The exceptions are clones numbered 1 to 119 and 5883 to 6001, which

lie outside the *LOV* parameterization area. They are not further considered and propagated.

Orbital elements of VAs are computed with different sigma of *LOV*, from 3 to 5. Next, we searched for impacts with the Earth. The results are presented in Table 2.

Table 2, it is so-called Impact Risk Data, as at the JPL Sentry: Earth Impact Monitoring (<https://cneos.jpl.nasa.gov/sentry/>) or the Risk Page of the NEODyS-2 (<http://newton.dm.unipi.it/neody/index.php?pc=4.1>), or Impactor Table, where date denotes potential impacts with the Earth: calendar date, MJD - Modified Julian Day *i.e.* JD-2400000.5, sigma - location along the line of the *LOV* in the sigma space for intervals up to $(-5, 5)$, *RE* and *RE/sigma* - distance and width - minimum distance from the *LOV* to the center of the Earth in the Earth radii, and stretch denotes how much the confidence region is stretched in time of possible impact, impact probability, expected energy of impact in megatons MT (1 MT=4.2E15 J) - it is the product of impact energy and the impact probability, PS - Palermo scale. According to the JPL Center for Near Earth Object Studies and Sentry: Earth Impact Monitoring (<https://cneos.jpl.nasa.gov/sentry/>) impact energy is a kinetic en-

Table 3. The asteroid 2001BB16. The *LOV1* method. The dependence of computed impact risk on number of perturbing massive asteroids (in column AST.) for computed non-gravitational parameter $A_2 = (34.551 \pm 7.384) \cdot 10^{-14}$ au/d². The JPL DE431 Solar System model is used.

AST.	date	MJD	sigma	distance and width	stretch	impact probability	exp. en.	PS
	YYYY/MM/DD			RE	RE/sigma		MT	
number of asteroids (AST.), 3 sigma								
0	2195/01/15.503	122781.503	-2.281	3.28±0.005	1.45E+05	5.09E-07	7.28E-06	-6.09
1	2195/01/15.537	122781.537	-2.281	3.20±0.003	5.58E+04	2.52E-06	3.61E-05	-5.40
2	2195/01/15.504	122781.504	-2.281	3.28±0.005	1.42E+05	5.06E-07	7.25E-06	-6.10
3	2195/01/15.505	122781.505	-2.281	3.27±0.005	1.36E+05	5.09E-07	7.28E-06	-6.09
4	2195/01/15.550	122781.550	-2.281	3.29±0.003	1.02E+05	9.61E-07	1.38E-05	-5.82
5	2195/01/15.549	122781.549	-2.281	3.28±0.003	1.01E+05	1.01E-06	1.45E-05	-5.79
6	2195/01/15.538	122781.538	-2.281	3.21±0.003	5.76E+04	2.41E-06	3.45E-05	-5.42
7	2195/01/15.540	122781.540	-2.281	3.22±0.003	6.93E+04	1.86E-06	2.66E-05	-5.53
8	2195/01/15.533	122781.533	-2.281	3.19±0.004	3.56E+04	4.75E-06	6.80E-05	-5.12
9	2195/01/15.538	122781.538	-2.281	3.21±0.003	6.01E+04	2.28E-06	3.26E-05	-5.44
	2195/01/15.529	122781.529	-2.196	2.91±0.000	2.22E+07	8.02E-09	1.15E-07	-7.90
10	2195/01/15.543	122781.543	-2.281	3.23±0.003	7.83E+04	1.56E-06	2.23E-05	-5.61
	2195/01/15.534	122781.534	-2.196	2.74±0.069	5.68E+06	3.71E-08	5.29E-07	-7.23
11	2195/01/15.532	122781.532	-2.281	3.19±0.004	3.30E+04	5.28E-06	7.55E-05	-5.08
12	2195/01/15.532	122781.532	-2.281	3.19±0.004	3.30E+04	5.27E-06	7.54E-05	-5.08
13	2195/01/15.530	122781.530	-2.281	3.18±0.005	2.63E+04	7.20E-06	1.03E-04	-4.94
14	2195/01/15.538	122781.538	-2.281	3.21±0.003	6.00E+04	2.28E-06	3.27E-05	-5.44
15	2195/01/15.533	122781.533	-2.281	3.19±0.004	3.47E+04	4.93E-06	7.05E-05	-5.11
16	2195/01/15.543	122781.543	-2.281	3.23±0.003	7.88E+04	1.54E-06	2.21E-05	-5.61
17	2195/01/15.544	122781.544	-2.281	3.24±0.003	8.31E+04	1.43E-06	2.05E-05	-5.65

ergy for a computed absolute magnitude and impact velocity.

It is visible from Table 2 that first possible impacts with the Earth are in 2192 and the last in 2200. In all the cases presented, possible impacts occur in 2195. The expected energy of impact is similar to the expected impact energy of another small asteroid (410777) 2009 FD (<https://cneos.jpl.nasa.gov/sentry/>). One of the best-known dangerous asteroids (99942) Apophis has the expected impact energy in the range (10^{-3} to 10^{-8}) MT (f.e. Włodarczyk (2017)).

The consequence of different level of impact energy is depicted by hazard rating according to the Palermo technical impact hazard scale, so called Palermo scale (*PS*). The Palermo Technical Scale (https://cneos.jpl.nasa.gov/sentry/palermo_scale.html) is shown in Chesley *et al.* (2002). Value of $PS < -2$ has no consequence for potential impact risk. $-2 < PS < 0$ gives situations that demand careful monitoring. Values of $PS > 0$ shows situations that require attention. The greatest value of PS in Table 2 for the asteroid 2001 BB16 is -5.11. Hence there is no consequences for the Earth. The algorithm for calculating the PS can be found at https://cneos.jpl.nasa.gov/sentry/palermo_scale.html.

To explain how the Line of Variation is defined, some discussion in the literature is presented for the gravity-only case (see Milani *et al.* (2005a)). When Yarkovsky is involved

the computations are more complicated and one generally needs to map to a suitable scattering encounter to identify the Line of Variation at the initial conditions (see Del Vigna *et al.* (2019)). To do this, we showed in Figure 3 in Section 6, *LOV* results with the Yarkovsky effect.

There is a significant effect of the number of perturbing asteroids (AST.) on impact solutions in all of the headings in Table 3.

5 The *LOV1* and *LOV2* methods

Generally, the analysis that go out to 3, 4, and 5 sigma should yield the same impact probability since the 2195 virtual impactor is at 2.2 sigma. The only thing that changes is how much of the uncertainty region is explored. We report an impact probability of $1.4 \cdot 10^{-6}$, $2.3 \cdot 10^{-6}$ and $4.9 \cdot 10^{-6}$ for the different sigma thresholds. Finally, the number of virtual impactors changes by increasing the exploration space. That would make sense, except that all the virtual impactors are within 3-sigma, i.e., the smallest interval considered, and so the list of virtual impactors should be the same. Actually, since the number of samples is the same, the 3-sigma search should have higher resolution than the

Table 4. The asteroid 2001BB16. The *LOV2* method. The dependence of computed impact risk on *IP* parameter for computed non-gravitational parameter $A2=(34.551\pm7.384)\cdot10^{-14}$ au/d². $\sigma_{max}=3$, $\Delta\sigma_{max}=0.01$. The JPL DE431 Solar System model is used. 17 perturbing asteroids are added.

date	MJD	sigma	distance and width	stretch	impact probability	exp. en.	PS
YYYY/MM/DD			RE	RE/sigma		MT	
<i>IP</i> = $8\cdot10^{-8}$							
2198/01/15.441	123877.441	-2.151	0.29±0.000	2.07E+08	1.26E-09	1.80E-08	-8.71
2198/01/15.440	123877.440	-2.151	0.34±0.000	2.53E+08	1.03E-09	1.47E-08	-8.80
2199/01/15.843	124242.843	-2.104	2.10±0.003	2.50E+05	8.99E-07	1.28E-05	-5.86
2199/01/15.842	124242.842	-2.104	2.10±0.004	1.53E+05	1.48E-06	2.11E-05	-5.64
2200/01/16.016	124608.016	-1.781	2.45±0.000	5.18E+05	7.50E-07	1.07E-05	-5.94
2200/01/16.032	124608.032	-1.781	2.78±0.000	8.21E+05	3.94E-07	5.64E-06	-6.22
<i>IP</i> = $1\cdot10^{-7}$							
2199/01/15.834	124242.834	-2.103	2.21±0.000	6.09E+05	3.77E-07	5.38E-06	-6.23
2200/01/15.982	124607.982	-1.781	2.39±0.008	4.62E+05	8.54E-07	1.22E-05	-5.88

Figure 2. Lyapunov time of 101 VAs of the asteroid 2001 BB16. Horizontal line denotes mean value.

5-sigma one. And yet, the 5-sigma search finds more virtual impactors.

But in the complicated case of the asteroid 2001 BB16 in 2195, the *LOV* is very chaotic (see Figure 3), and it is difficult to compute 'true' positions of clones. Hence, using the OrbFit software, we computed different probability for different sigma.

To consider this, we examined in detail two different methods of calculating the *LOV*.

First method presented above, called *LOV1*, is based on the uniform sampling of the *LOV* parameter, the second one, called *LOV2*, is based on a non-uniformly sampling of the *LOV*. We showed that these different methods of sampling the *LOV* can give different impact solutions.

The second method of computing the impact solutions, *LOV2*, presented in Del Vigna et al. (2019), is based on a non-uniformly sampling of the *LOV*. The *LOV2* construction is based on the following assumptions: constant step in the Impact Probability (*IP*), probability step (completeness limit) - f.e. 10^{-7} , sigma step max (control against too long steps, $\Delta\sigma$) - f.e. 0.01, sigma max (extent on *LOV*, σ) - f.e. 3. We propagated multiple solution or search for close approaches. More detailed in the case of the asteroid 2001 BB16 and for impacts presented in Table 4: $IP=8\cdot10^{-8}$, 3σ , $\Delta\sigma=0.01$, for clones 14 do 5356: differences between the semimajor axis of the successive clones, $da=1.268\cdot10^{-11}$ au for clone with the serial number 2643 to $da=2.6984\cdot10^{-10}$ au for clone with the serial number 5356.

The same analysis for impact orbits in Table 4 for $IP=1\cdot10^{-7}$, 3σ , $\Delta\sigma=0.01$ for clones 14 do 4306: differences between the semimajor axis of the successive clones, $da=1.585\cdot10^{-11}$ au for clone with the serial number 2210 to $da=2.6984\cdot10^{-10}$ au for clone with the serial number 4299. There are almost the same results as for sampling *LOV2* for previous case, and in both cases there are similar impact dates. In the case of the *LOV1* method sampling the difference between semimajor axis of successive clones is constant and equal to $4.497\cdot10^{-11}$ au.

Table 4 lists for the asteroid 2001BB16, the impact risk using the second method of computation of the *LOV*, i.e. *LOV2*.

Table 4 presents the dependence of computed impact risk on *IP* parameter for computed non-gravitational parameter $A2=(34.551\pm7.384)\cdot10^{-14}$ au/d², $\sigma_{max}=3$ and $\Delta\sigma_{max}=0.01$. The JPL DE431 Solar System model is used. 17 perturbing asteroids are added. It is visible that for the chaotic orbit of the asteroid 2001 BB16 we got different results using different *IP* parameter. It is difficult to find

Figure 3. The *LOV1* method. Position of orbital elements of the asteroid 2001 BB16 computed for 5 sigma and for 3000 clones on both side on the *LOV*. These clones are computed with the non-gravitational parameter A_2 for 17 massive asteroids - top panels, and for 0 asteroids - bottom panels. Left panels - phase space of (a, e) on MJD122774.0 (JD2522774.5 = 2195-Jan-8) of 6001 VAs where circle - denotes initial position on MJD58200 (JD2458200.5=2018-March-23), square - nominal orbit on MJD122774.0 (JD2522774.5 = 2195-Jan-8), great star - position of VA #1632, which is close to impact orbit in 2195. Right panels - distance of ascending node of VAs of asteroid 2001 BB16 to the center of the Earth, at that same time as in the left panel, *i.e.* on MJD122774.0 (JD2522774.5 = 2195-Jan-8). Distances selected from subsequent clones (#1624 and #1635, from bottom to top), which are smallest among all VAs are marked by stars.

the possible data of impact risk. Despite many attempts to search for impacts for different IP , σ_{max} and $\Delta\sigma_{max}$ no more impact solutions were found. To find impacts in the case of the *LOV2* method we followed a trial-and-error approach.

Table 4 shows that the impact solutions are different for two different IP parameter.

Summarizing, different methods of sampling the Line of Variation, *i.e.* *LOV1* and *LOV2*, can give different impact solutions but all computed impact risk dates occur at the end of 22nd century.

Here it is worth to pay attention to the application of the *LOV* method, especially with non-uniform steps in the *LOV* parameter. As discussed in Del Vigna et al. (2018), the main consequence of establishing a completeness level IP^* (pre-set impact probability) is that all the VIs with $IP > IP^*$

should be detected by the system, whereas the detection of the VIs with $IP < IP^*$ is probabilistic, in the sense that their detection does depend on the *LOV* sampling chosen. Thus, the results obtained with $IP^* = 1E-7$ and $IP^* = 8E-8$ are not different and identifying the reason in the choice of IP^* is not correct: for instance, the two VIs that have to be the same in the two impactor tables reported in Table 4 actually are compatible; the one that disappears (*i.e.*, the 2198 VIs in the upper table) has a probability that is below the completeness level IP^* .

Moreover, in case of a very deep close approach and a very well-constrained orbit, the linear *LOV* with direction given by the scattering TP metric should be used, not the non-linear *LOV*.

Figure 4. The *LOV1* method. 6001 VAs of the asteroid 2001 BB16 with different non-gravitational parameter A_2 . Circle denotes the position of nominal orbit and star - position of VA#1632 for orbit close to impact in 2195.

6 Lyapunov Time

Next, we computed values of Lyapunov time (LT) for the asteroid 2001 BB16. The method is based on Knežević & Milani (2000) and Milani & Nobili (1988). The *LOV1* method was used. First, we computed 101 Keplerian orbital elements of starting VAs using the OrbFit software. Next, we propagated them using the Orbit9 procedure and filter 5 yr. It denotes that the time between two outputs of orbital elements is 5 yr. We made 40000 outputs at job termination, hence we followed the time evolution of orbital elements and LT 200000 yrs forward. We used the JPL DE431 with perturbations of the planet from Venus to Neptune and Pluto. For Mercury, we used the barycenter correction.

Figure 2 presents values of computed LT for 101 clones (VAs) of the asteroid 2001 BB16. Clones are computed with the use of the OrbFit software v. 5.0.5 and the method of Milani (Milani, 2006) with computed 50 clones of both sides of the *LOV* for 3 sigma, 17 perturbing massive asteroids and for pure gravitational method, *i.e.* without the non-gravitational parameter A_2 . Values of the LT are between 12.6 and 60.9 years. They are almost independent of the serial number of VAs. Mean LT is 25.0 years. This means that all clones are in a similar area of chaos. Computed impact risks does not depend on value of Lyapunov Time of clones. It turned out that all of them have similar short Lyapunov Time, hence the difficulty in calculating probable collisions with the Earth. It appeared that 75% of 175 asteroids with perihelia less than 1.6 au have $LT < 100$ years and 18 asteroids have $LT < 50$ years.

7 Time evolution of orbital elements

Figure 3 presents position of orbital elements of the asteroid 2001 BB16 computed for 5 sigma and for 6001 clones on the *LOV* computed using first method (*LOV1*). On the top panels they are computed for 17 perturbing massive asteroids, on the bottom panels without any asteroids. Left panels show phase space of (a, e) of 6001 VAs on MJD122774.0 (JD2522774.5 = 2195-Jan-8) of 6001 VAs. Circle denotes initial position on MJD58200.0 (JD2458200.5=2018-March-23), square - nominal orbit on MJD122774 (JD2522774.5 = 2195-Jan-8), great star - position of VA #1632, which is close to impact orbit in 2195. Right panels - distance of ascending node of serial number of VAs of asteroid 2001 BB16 to the center of the Earth, at that same time than in the left panel, *i.e.* on MJD122774.0 (JD2522774.5 = 2195-Jan-8). Distances selected from subsequent clones (#1624 and #1635, from bottom to top), which are smallest among all VAs are marked by stars. Moreover, VA#1632 has CA with the Earth on 2195-Jan-11.98832 at 0.00719 au, and VA#1633 has CA with the Earth on 2195-Jan-22.40762 at 0.00716 au. It is visible that in both cases, clones are similarly dispersed except for a few clones, in particular in the right panels near the collision in 2195. Hence we observe the difference in probability of collision with and without massive perturbing asteroids.

Figure 4 presents the non-gravitational parameter A_2 computed for all 6001 VAs for 5 sigma and with 17 massive perturbing asteroids. Circle denotes the position of nominal orbit and star - position of VA#1632 for orbit close to impact in 2195. We can see that A_2 is from about $(0 \text{ to } 70.0) \times 10^{-14} \text{ au/d}^2$. It comes from computed nominal non-gravitational parameter A_2 , which has the value $(34.55 \pm 7.38) \cdot 10^{-14} \text{ au/d}^2$. Hence 1-sigma uncertainty = $7.38 \cdot 10^{-14} \text{ au/d}^2$, and 5-sigma uncertainty is $36.9 \cdot 10^{-14} \text{ au/d}^2$.

For impact orbit in 2195, the non-gravitational parameter $A_2 = 20.0 \cdot 10^{-14} \text{ au/d}^2$. The value of each A_2 for different VAs we can find from Figure 4. Hence we have conditions for the physical parameters of the VA (clone) colliding with the Earth as is predicted in Table 2.

Figures 3-7 shows a complex chaos area for 2001 BB16 asteroids using the *LOV1* and *LOV2* methods.

In Figure 3, we can see position of orbital elements of the asteroid 2001 BB16 computed for 5 sigma and for 3000 clones on both side of the *LOV*. These clones are computed with the non-gravitational parameter A_2 for 17 massive asteroids - top panels, and for 0 asteroids - bottom panels. Left panels - phase space of (a, e) . Right panels - distance of ascending node of serial number of VAs of asteroid 2001 BB16 to the center of the Earth, at that same time as in

Figure 5. The *LOV2* method. Left panels - phase space of (a, e) on MJD122774.0 (JD2522774.5 = 2195-Jan-8). Right panels - distance of ascending node of VAs of asteroid 2001 BB16 relative to the σ parameter, at that same time as in the left panel, *i.e.* on MJD122774.0 (JD2522774.5 = 2195-Jan-8). $\sigma_{max}=5$ and $\Delta\sigma_{max}=0.01$. $IP = 2 \cdot 10^{-6}$ which gives 963 multiple solutions for top panels and $IP = 2 \cdot 10^{-7}$ with 2599 multiple solutions for bottom panels. Compare with Figure 3 resulting from the *LOV1* method.

the left panel, *i.e.* on MJD122774.0 (JD2522774.5 = 2195-Jan-8). Distances selected from subsequent clones (#1624 and #1635, from bottom to top), which are smallest among all VAs are marked by stars.

Figures 4-7 represent computed clones (Virtual Asteroids) using the *LOV2* method. In all cases, you can see that, at least both *LOVs* not much differs, but it is the placement of individual clones in the area of possible collisions near $\sigma = 2$. Hence they are different. Hence the differences in the collision possibilities of these clones with the Earth. To check this we have to perform complicated calculation procedures for various Solar System models, including additional perturbing asteroids, for a different number of clones and the σ parameter, as in the case of the *LOV1* method. And in the case of the *LOV2* using different values of the parameters σ_{max} , $\Delta\sigma_{max}$ and IP .

If both methods give a similar result, it means that we calculated collision parameters, *i.e.* the moment, site, probability and energy more realistically.

Table 5 presents for the *LOV2* method interesting dependence on the used IP parameter. The non-gravitational parameter $A2=(34.551\pm7.384)\cdot10^{-14}$ au/d², the JPL DE431 Solar System model is used and 17 perturbing asteroids are added, as in all presented computations. It is visible that for different IP parameters and for adopted σ with σ_{max} there are different multiple solutions used to propagate them in the future to search for close approaches and for possible impact solutions. Only using specially selected parameters it is possible to find interesting impact solutions. It is a more complex task than in the case of the *LOV1* method.

As presented in Milani *et al.* (2005a), the impact probability is computed by applying a Gaussian approximation on the Target Plane (*TP*), in a suitable neighborhood of the *TP* point corresponding to the minimum distance along the

Figure 6. The same as in Figure 5 but for $\sigma_{\max}=5$ and $IP = 2 \cdot 10^{-8}$. Additionally, $\Delta\sigma_{\max}=0.1$ which gives 21341 multiple solutions for top panels and $\Delta\sigma_{\max}=0.001$ with 25962 multiple solutions for bottom panels. For better visibility, the sigma-axis is limited to the range $(-3, -1)$.

Table 5. The asteroid 2001BB16. The LOV2 method. The dependence on the used IP parameter. The non-gravitational parameter $A2=(34.551\pm 7.384)\cdot 10^{-14}$ au/d², the JPL DE431 Solar System model is used and 17 perturbing asteroids are added.

IP	σ	$\Delta\sigma$	clones	min. distance in a	max. distance in a	note
$8 \cdot 10^{-8}$	3	0.01	14 to 5356	$1.268 \cdot 10^{-11}$ (2643)	$2.6984 \cdot 10^{-10}$ (5356)	impacts in Table 4
$1 \cdot 10^{-7}$	3	0.01	14 to 4306	$1.585 \cdot 10^{-11}$ (2210)	$2.6984 \cdot 10^{-10}$ (4299)	impacts in Table 4
$2 \cdot 10^{-6}$	5	0.01	20 to 982	$2.6980 \cdot 10^{-10}$ (36)	$2.6984 \cdot 10^{-10}$ (962)	see Figure 5
$2 \cdot 10^{-7}$	5	0.01	21 to 2619	$3.171 \cdot 10^{-11}$ (1316)	$2.6985 \cdot 10^{-10}$ (2598)	see Figure 5
$2 \cdot 10^{-8}$	5	0.1	4 to 21344	$3.153 \cdot 10^{-12}$ (10936)	$2.6983 \cdot 10^{-9}$ (21355)	see Figure 6
$2 \cdot 10^{-8}$	5	0.001	201 to 26162	$3.153 \cdot 10^{-12}$ (13247)	$2.701 \cdot 10^{-11}$ (254)	see Figure 6
$1 \cdot 10^{-8}$	5	0.01	21 to 42922	$1.565 \cdot 10^{-12}$ (22166)	$2.698 \cdot 10^{-10}$ (42891)	see Figure 7
$2 \cdot 10^{-8}$	5	0.01	22 to 21646	$3.153 \cdot 10^{-12}$ (11078)	$2.698 \cdot 10^{-10}$ (21638)	see Figure 7

LOV. As a consequence, slightly changing the *LOV* sampling parameters produces differences in the computed *TP* point corresponding to the impacting orbit along the *LOV*, thus causing differences in the computation of the impact probability. Differences of the order of a factor even $\simeq 5$ are very well admissible and do not indicate an intrinsic difference due to the behavior of the *LOV*. By the way, the

chaoticity of the orbits along the *LOV* is due to close approaches and causes the *LOV* trace on a given *TP* to be complicated as long as the *TP* epoch becomes far from the initial time.

Figure 7. The same as in Figure 5 but for $\sigma_{max}=5$ and $\Delta\sigma_{max}=0.01$. Additionally, $IP = 1 \cdot 10^{-8}$ which gives 42902 multiple solutions for top panels and $IP = 2 \cdot 10^{-8}$ with 21625 multiple solutions for bottom panels. For better visibility, the sigma-axis is limited to the range $(-3, -1)$.

8 Summary

We computed impact solutions of the potentially dangerous asteroid 2001 BB16 based on all published observations updated to July 1, 2018. We used the Line of Variation method out to $\sigma_{LOV} = 5$, computing 6001 VAs and propagated their orbits to February 12, 2201.

We computed the non-gravitational parameter $A2 = (34.55 \pm 7.38) \cdot 10^{-14} \text{ au/d}^2$ for nominal orbit of 2001 BB16. For potential impact in 2195 we find $A2 = 20.0 \cdot 10^{-14} \text{ au/d}^2$, hence 2001 BB16 can be prograde rotator. Moreover, we computed Lyapunov Time for 2001 BB16 equals to 25 y.

For very chaotic behavior of the LOV like presented in Figure 3, using the OrbFit software we can get different probability of impact for different number of clones calculated for a given sigma of the LOV.

We showed that different methods of sampling the LOV can give different impact solutions, but all computed dates

of possible impacts of the asteroid 2001 BB16 with the Earth occur at the end of the 22nd century.

Also, we should take into account the impact of non-gravitational effects, including the Yarkovsky effect.

ACK.

We would like to thank the reviewers for their comments and suggestions on the manuscript.

We thank the Space Research Center of the Polish Academy of Sciences in Warsaw for the chance to work on a computer cluster.

References

- Chesley S. R., Chodas P. W., Milani A., Valsecchi G. B., Yeomans D. K., 2002, *Icar*, 159, 423.
- Chesley S. R., Baer J., Monet D. G., 2010, *Icar*, 210, 158-81.
- Del Vigna A., Milani A., Spoto F., Chessa A., Valsecchi G. B., 2019, *Icar*, 321, 647-660.

- Farnocchia D., Chesley S. R., Vokrouhlický D., Milani A., Spoto F., Bottke W. F., 2013a, *Icar*, 224, 1-13.
- Farnocchia D., Chesley S. R., Chodas P. W., Micheli M., Tholen D. J., Milani A., Elliott G. T., Bernardi F., 2013b, *Icar*, 224, 192-200.
- Farnocchia D., Chesley S. R., Chamberlin A. B., Tholen D. J., 2015, *Icar*, 245, 94-111.
- Knežević Z., Milani A., 2000, *CeMDA*, 78, 17-46.
- Milani A., Nobili A. M., 1988, *CeMec*, 43, 1-34.
- Milani A., Chesley S. R., Sansaturio M. E., Tommei G., Valsecchi G. B., 2005a, *Icar*, 173, 362-384.
- Milani A., 2006, *SerAJ*, 172, 1-11.
- Włodarczyk I., 2015, *AcA*, 65, 215-231.
- Włodarczyk I., 2017, *BlgAJ*, 27, 89-100.
- Włodarczyk I., 2020, *BlgAJ*, in press.