Research Article

Chen Jianrong* and Li Junfeng

Analysis of Orbit Accuracy for Non-cooperative Earth-Orbiting Objects

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Abstract: The atmospheric resistance and solar radiation pressure are the main sources of non-gravitational perturbations in orbit determination and prediction of non-cooperative earth-orbiting objects. The calculation of both perturbation accelerations involves the satellite's surface-mass ratio. Another factor affecting the accuracy of orbit determination and prediction is the sparse observation. In this paper, we examine the sensitivity of orbit prediction accuracy and orbit decay rate relative to surface-mass ratio. The influence of the space-based angle data to orbit determination was also analyzed. The results show that the error of orbit prediction is multiplied by the change of surface-mass ratio. The orbital error determined by using 7-minutes single-station ground-based radar data and 15-minutes double-space-based angle measurements is 63.77 m lower than that only determined by using single-station ground-based radar data. The space-based angle information greatly improves the observation geometry of the non-cooperative target and improves the precision of orbit determination. The position error of the inclined geosynchronous satellite determined by 3-days ground-based radar data is 323.49 m and the prediction error of 1-7 days is less than 1.0 km. Using 3-days ground-based radar data and single spaced-based angle measurement to determine the orbit of the medium-orbit satellite with an altitude of about 1000 km, the orbital accuracy is 14.66 m and the prediction error of 1-7 days is less than 50.00 m. To determine the orbit of the 280 km height satellite with, 1.5-days ground-based radar data and single space-based angle data, the position error is 270.37 m and the prediction error of 1-day is 1.70 km.

Keywords: earth-orbiting objects, surface-mass ratio, orbit determination, accuracy, non-cooperative

1 Introduction

The orbital information about Earth-orbiting objects, of which the vast majority is space debris (Scheeres 1998), is not only a critical piece of information on the space and defense security, but also the prerequisite of many space operations to be efficiently carried out, such as the space collision conjunction assessments, space surveillance and debris removal, and so on. According to statistics on January 2019 by European Space Agency's Space Debris Office at European Space Operations Center, Darmstadt, Germany, there are approximately 22,300 debris objects within the Space Surveillance Networks' catalogue, about 8% of satellites among these debris objects still functioning. Most of the objects are larger than 10 cm in diameter. According to

the National Aeronautics and Space Administration (NASA) statistics, more than 500,000 space debris are larger than 1 cm in diameter (Vasilyev 1998). All the debris pose realistic threats to the current space applications (Zotkin and Tsikulin 1966; Chyba *et al.* 1993). Space collisions are reportedly becoming more frequent, highlighted by the collision between Iridium 33 and Cosmos 2251 in February 2009 (Li *et al.* 2015). On November 5, 2014, a small asteroid impacted the Earth in the Xilin Gol League in Inner Mongolia (Mille *et al.* 2013). This motivated the space situation awareness (SSA) community to search for the accurate orbit determination (OD) and prediction (OP) for non-cooperative Earth-orbiting objects (NCEO), which are necessary for the on-time and reliable space conjunction warning.

The NCEO's OD and OP are essentially the same as the satellite's OD and OP (Brown *et al.* 2013; Zhao 2014; Liu *et al.* 2017), but there are a few different features in terms of the practical applications. The satellite's OD accuracy can reach centimeter level, and short-term OP meter level (Gao and Wu 2014; Bombardelli and Bau 2012; Schweickart 2006; Lubin *et al.* 2014), while the NCEO's OD accuracy is usually at dozens of meters, and the OP accuracy at several hundred

Corresponding Author: Chen Jianrong: School of Aerospace, Tsinghua University, Beijing 100084, China; State Key Laboratory of Astronautic Dynamics, Xi'an 710043, China; Email: chenjianrong9@163.com

Li Junfeng: School of Aerospace, Tsinghua University, Beijing 100084, China

or thousands of meters, or even worse. This is because of sparse and limited observation data (Wie 2005; Dachwald and Wie 2017; Gong *et al.* 2011) of NCEO, low observation accuracy and unknown ballistic coefficient (NASA 2007), etc.

A NCEO has no signal receiving and transmitting devices, so that observation data are mainly obtained by radar, optical and Laser Ranging. These data don't have high precision (Wie 2005), such as optical angle measuring accuracy for 2 arc sec ~5 arc sec, laser ranging (Debris Laser Ranging, DLR) precision for 1 m ~2 m (Rubincam 1995; Vokrouhlicky and Milani 2000) and lower radar precision. At present, only a few observation stations in the world have the capability of DLR observation. Due to observing conditions, the fragments of the observation data obtained is relatively sparse (Dachwald and Wie 2017; Gong *et al.* 2011). The sparse observation and the ballistic coefficients or surface-mass ratios (SMR) for NCEO seriously restrict the accuracy of orbit determination and prediction (NASA 2007).

With the development of space technology, the shortcoming of ground-based observation (GBO) system is becoming more and more prominent, then space-based observation system was proposed. The space-based observation (SBO) platform has four characteristics. The first characteristic is that there is a wide observation range and large scale. The second is that the observation area is not restricted by political region. The third is that the angle has the characteristic of overlooking. The last characteristic is that targets such as space debris are not affected by the atmosphere. Thus the SBO system (Stokes *et al.* 1998; Sharma *et al.* 2002; Scott *et al.* 2006; Flohrer *et al.* 2006) can effectively make up for the GBO system shortcomings, improve monitoring efficiency, and form an effective complementary relationship with the established GBO system. At present, GBO (He and Yao 2017; Ji and Zeng 2018; Li *et al.* 2018) is widely used. Angle measurement is usually easier to be obtained than ranging, especially in SBO system. SBO information has been deeply studied in improving orbit determination accuracy (Li *et al.* 2009; Wen *et al.* 2010; Milani *et al.* 2004, 2005, 2008). Sun and Li (2015) studied the constrained least square method of the initial orbit of space targets based on the space-based optical measurement. Li (2016) analyzed the OD accuracies of near-earth celestial bodies by using space-based optical measurement. Wang *et al.* (2013) took the sun-synchronous orbit as the space-based monitoring orbit, simulated the observation data of space-based camera, and analyzed the orbital accuracy of space targets with different orbital heights. When the space-based observation platforms are in 1100km, 3000km and 10000km height respectively, Wu Junzhong analyzed the factors influencing

the accuracy of space target orbit determination based on space-based angular measurement information. Li *et al.* (2018) analyzed the OD and OP accuracies using short-arc tracking data measured by a sun-synchronous satellite for 200 debris objects. Zhang *et al.* (2018) simulated the angle measurement of the single low-orbit monitor satellite tracking high-orbit non-cooperative space targets, and analyzed the accuracy of the orbit determination based on ultra-short arc sparse optical tracking data.

The space targets mainly distribute over the low-orbit region with the altitude of 400-2000km and the geostationary orbit region with the altitude of 36000km. If the space-based monitoring satellite is a geostationary satellite (GEO), what about the OD and OP accuracies of space target? In this paper, a GEO satellite is taken as the monitoring satellite to analyze the orbit determination accuracy of non-cooperative space targets using the ground-based radar observation and space-based angle measurement. We also explore the influence of a NCEO's SMR on the error of the orbit prediction.

2 Coordinate System

In this article, we used the Earth-centered, Earth-fixed coordinate system (ECF), the Earth-centered inertial coordinate system (ECI), the topocentric system (TGS), and the sensor coordinate system (UEN). The definitions of ECF, ECI and TGS are given in (Byron *et al.* 2004). In the UEN coordinate system (see Figure 1), the center position o' of the sensor S is the origin of the coordinate system, the U axis is an extension of the line connecting the center of the Earth o to point o' , the N axis points to the north, and the E axis points to the east.

The transformation matrix from the UEN to the ECF is given by Eq. (1).

$$Q = \begin{pmatrix} \cos \lambda \cos \varphi & -\sin \lambda & -\cos \lambda \sin \varphi \\ \sin \lambda \cos \varphi & \cos \lambda & -\sin \lambda \sin \varphi \\ \sin \varphi & 0 & \cos \varphi \end{pmatrix} \quad (1)$$

Here λ and φ are respectively the longitude and latitude of the satellite bearing the sensor.

3 Model for Space-based Angle Data

The model for space-based angle data uses the UEN coordinate system. The relationship between the UEN and the

Figure 1. Relationship between UEN and ECF

ECF, and the positions of the sensor S and the low orbit target M are shown in Figure 1.

In Figure 1, s is the distance from the satellite to the center of the Earth. In the ECF, the vector $\mathbf{s}$ for the satellite position is

$$\mathbf{s} = \begin{pmatrix} s_x \\ s_y \\ s_z \end{pmatrix} = \begin{pmatrix} s \cos \varphi \cos \lambda \\ s \cos \varphi \sin \lambda \\ s \sin \varphi \end{pmatrix} \quad (2)$$

the following. The orbit-1 (ORB-1) is a large elliptical orbit, the orbit-2 (ORB-2) is a medium orbit, and the orbit-3 (ORB-3) is a low orbit. The infrared sensor is installed on GEO with a field of view of $\pm 30^\circ$. The angular data are the observation data of the NCEO acquired by the synchronous early warning satellites. Table 1 shows the number of GEO satellite orbit and the number of NCEO orbits. The orbit Epoch is 2018-01-08 00:00:0.0 (UTC). In the Table 1, a represents the orbital semi-major axis, e is orbital eccentricity, i is orbital inclination angle, Ω means the right ascension of orbital ascending node, ω is argument of orbital perigee and M is mean anomaly.

Table 1. The orbits used in simulation

Elements	GEO	ORB-1	ORB-2	ORB-3
$a(\text{km})$	42166.0	42163.5	7335.3	6656.2
e	0.00000822	0.001717	0.0005225	0.002404
$i(^{\circ})$	0.136	55.127	99.381	42.751
$\Omega(^{\circ})$	184.556	333.292	21.429	10.884
$\omega(^{\circ})$	234.279	182.493	310.694	110.597
$M(^{\circ})$	173.319	45.375	145.912	177.513

The three ground stations are at Jiamusi, Kashi, and Sanya. The data types are range R_s , azimuth angle A_s and elevation angle E_s . The errors in the ground data show a normal distribution with mean values of 10 m and 0.02° . The space-based angle data have a random error of 0.001° with a normal distribution. For ORB-1 satellite there are no space-based observation data due to limited orbital altitude and sensor field of view angle. The atmospheric density model is the NRLMSISE-00 model with an atmospheric drag coefficient C_D of 2.1, the solar radiation pressure coefficient S_{rp} is 2.0, and the gravitational field model is JGM3.

6 Analysis and Discussion

The perturbation accelerations influenced by SMR are due to the solar radiation pressure and the atmospheric drag (Byron *et al.* 2004). In this paper, the ECOM solar radiation pressure model (Beutler *et al.* 1994) and DTM94 model of the atmosphere (Picone *et al.* 2002) are accounted. The reference orbit for comparison is the trajectory of the satellite without dynamic error.

6.1 Influence of the surface-mass ratio on the forecast of an orbit for NCEO

When considering the motion of a NCEO, the dissipative forces associated with the SMR are solar radiation pressure and atmospheric resistance. At various values of the SMR (0.1 SMR, 10 SMR, or 100 SMR), the differences between the 180-day forecast of ORB-1 and ORB-2 and the orbit with an SMR are compared in Figure 2 and Figure 3.

For ORB-1, as showed in Figure 2, when the SMR changed by a factor of 0.1, the average position error was 16.74 km. When the SMR changed by a factor of 10, the position error became 167.27 km. When the SMR changed by a factor of 100, the position error increased drastically to an average of 1833.23 km.

Figure 2. Effects of surface-mass ratio changes on ORB-1 forecast error

For a NCEO on ORB-2, the average position error was 27.69 km when the SMR changed by a factor of 0.1. The position error became 280.25 km when the SMR changed by a factor of 10. When the SMR changed by a factor of 100, the position error sharply increased and the average error was 3344.03 km (See Figure 3).

These results showed that for large elliptical and medium altitude orbits, when comparing the 180-days orbit forecast with orbits of a given SMR, the orbit position error increased by many fold when the SMR increased or decreased by many fold.

For low-altitude orbits at a height of 340 km, the influence of the SMR on orbit altitude are showed in Figure 4. Due to atmospheric resistance, the changes of the SMR had a direct effect on the decay speed of the NCEO's orbit height. Figure 4 shows the decay speed of the orbital height. For the given SMR, it took 99 days for the orbit height to decay to

Figure 3. Effects of surface-mass ratio changes on ORB-2 forecast error

100 km. When the SMR was increased by 10 fold, the orbit of the NCEO decreased to 100 km in 9.5 days. When the SMR increased by 100 fold, it took only 23 hours to decrease to 100 km. When the SMR was reduced to 0.1 times of SMR, the NCEO flew for 180 days with hardly any decaying of altitude. Therefore, for low altitude orbits, when the SMR was increased by many fold, the decay time down to an altitude of 100 km also decreased by many fold; however, when the SMR decreased by many fold, the orbital decay rate significantly slowed down.

Figure 4. Effects of surface-mass ratio changes on ORB-3 satellite height forecast error

6.2 Analysis of NCEO orbit accuracy

By using ORB-3 as an example, we analyzed the influence of space-based angle data on the accuracy of orbit determination assuming that there are no errors in the force model. Figure 5 shows the orbital error after solving for the coefficients of atmospheric resistance using 1.5 days of ground-based radar data and space-based angle data respectively. Figure 5 shows that, without the influence of the error in the force model, the root mean square (RMS) of the orbital error is 7.68 m using only 1.5 days of ground-based radar data to determine ORB-3. When a combination of ground-based radar data and space-based angle data were used for orbit determination, the RMS error of the orbit was 2.08 m. This showed that the effect of space-based angle data had an obvious effect of about 5.60 m on the error of ORB-3 at an altitude of 340 km.

Figure 5. Effects of space-based angle data on the accuracy of orbit determination, assuming no errors in dynamic model

Considering the dynamic errors, how much does the space-based angle data affect the orbital accuracy of the non-cooperative target? In Table 2 and Table 3, 'Data Type' means the type of the data used to determine the orbit. 'GBSSRD' is abbreviated from Ground-Based Single-Station Radar Data. 'GBSSRD+SBSSAD' means the combination of GBSSRD and the abbreviation for Space-Based Single-Station Angle Data. In the 'GBSSRD+SBDSAD', the 'SBDSAD' is abbreviated from Space-Based Double-Stations Angle Data. The second space-based satellite was located at longitude 124° east.

Taking the ORB-3 as an example, the OD accuracy is analyzed using short-arc space-based and ground-based data with the dynamic errors and measurement errors. There are 15 minutes data of double space-based platforms and 7 minutes measurements of single ground-based radar station.

Table 2. The residuals of the short arc data used to determine the orbit

Data type	Data Residuals				
	$R_s(\text{m})$	$A_s(^{\circ})$	$E_s(^{\circ})$	$\beta(^{\circ})$	$\alpha(^{\circ})$
GBSSRD	2.75	29.19	35.33	—	—
GBSSRD+	2.75	30.15	35.57	3.13	1.30
SBSSAD	—	—	—	—	—
GBSSRD+	2.75	31.66	35.15	2.17	1.48
SBDSAD	—	—	—	—	—
SBSSAD	—	—	—	61.96	14.23
SBDSAD	—	—	—	43.91	13.34

Table 2 shows the residuals of the short arc data used to determine the orbit. It can be seen that the residuals have the same magnitudes as the preset measurement errors, except in the case of orbit determination using only space-based angle data. Only using the space-based angle information to determine the orbit, the residual errors are one order larger than the preset measurement errors.

Table 3. The accuracy of orbit determination using short arc data

Data type	Position Error(m)			
	Total	Radial	Normal	Tracking
GBSSRD	150.42	28.61	148.43	109.96
GBSSRD+	110.47	16.83	73.19	92.52
SBSSAD	—	—	—	—
GBSSRD+	86.65	36.74	33.66	83.11
SBDSAD	—	—	—	—
SBSSAD	13308.58	11075.21	−6982.59	−2942.10
SBDSAD	6591.37	−338.09	−2490.92	−2741.23

Table 3 gives the position difference between the OD results and the reference trajectory. The position error of the orbit determined by the 15-minutes SBSSAD is about 13.30 km, while the position error of the orbit calculated by the 15-minutes SBDSAD is reduced to 6.59 km. The position error is about 0.15 km of the orbit determined by 7-minutes GBSSRD, and the normal position error is significantly larger than the errors in radial direction and tracking direction. Using the GBSSRD+SBSSAD to determine the orbit, the orbital accuracy is 0.11 km. Compared with the orbital accuracy of GBSSRD, the accuracy of the orbit determination using GBSSRD+SBDSAD significantly reduced to 0.086 km. Adding Space-based angle data to orbit determination, it improved the observation geometry and the OD accuracy, especially in the normal direction and the tracking direction.

Therefore, we adopt the following orbit strategy when analyzing the accuracy of the orbit determination.

For ORB-1, solving the position and state parameters of a NCEO using three days of ground-based radar data, with the C_D fixed at 2.0 and the S_{rp} set to 1.8.

For ORB-2, solving the position and state parameters of a NCEO and also the C_D using three days of ground-based radar data and space-based angle data, with the S_{rp} set to 1.4.

For ORB-3, solving the position and state parameters of a NCEO and also the C_D using 1.5 days of ground-based radar data and space-based angle data, with the S_{rp} set to 1.4.

When determining the orbit, we employed the DTM94 atmospheric density model and the GRIM5 gravitational field model. At the same time, we took into account the influence of the perturbation of the gravitational effects of the Sun and the Moon, the effects of solid tide, ocean tide, and atmospheric tide, relativistic effects, the effects of solar radiation and infrared radiation of the Earth, as well as Earth radiation pressure. In this paper we shall investigate the orbit accuracy based on the orbital errors obtained from comparing overlapping arcs and from comparing the forecast orbit with the standard orbit.

6.3 Comparison of Overlapping Arcs

ORB-1 and ORB-2 were determined using three days of data that had one day of overlap. ORB-3 was determined using 1.5 days of data that had 0.5 days of overlap. Figure 6 shows the position errors obtained from a comparison of the overlapping arc segments of the three types of orbits. For ORB-1, under the combined influence of gravitational field, error in the solar radiation pressure model, and measurement error, the orbital error of the overlapping arc is about 323.49 m, which is much larger than the orbital error of 31.84 m with only the measurement error (see Figure 7). For ORB-2 the orbital error of the overlapping arc is about 14.66 m, and the orbital error containing only the measurement error is 6.46 m (see Figure 8). For ORB-3 the orbital error with only the measurement error is 2.08 m (see Figure 5); however combining the errors of the gravitational field model and the atmospheric drag model with the measurement error produces an orbital error 270.37 m, which is two orders of magnitude greater than with only the measurement error. A comparison of the orbital errors of ORB-1 and ORB-3 shows that errors of the force model are the main source of orbital errors.

Table 4. Orbital errors in 1-day, 3-days, and 7-days predictions of three type orbits

Orbit type	Prediction time/day	Position error (m)			
		Total	Radial	Tracking	Normal
ORB-1	1	457.73	43.83	435.37	134.34
	3	600.15	57.12	538.41	258.91
	7	922.46	86.56	759.83	515.85
ORB-2	1	19.13	1.87	16.48	9.53
	3	26.26	2.59	18.89	18.06
	7	42.23	4.01	36.42	20.98
ORB-3	1	1701.66	109.82	1696.89	64.56
	3	5180.35	175.89	5174.45	173.56
	7	99097.89	1795.25	99035.12	3035.42

Figure 6. Comparison of overlapping arcs of three types of orbits based on orbit determination strategy **Figure 7.** Position error of ORB-1 determined using 3 days of ground-based radar data, assuming no errors in dynamic model

6.4 Analysis of Orbit Prediction Errors

By using the orbits determined in Section 6.3, predictions were made for one day, three days, and seven days respectively. The prediction results were then compared with the

Figure 8. Position error of ORB-2 determined using 3 days of ground-based radar data, assuming no errors in dynamic model

standard orbits; the results of the comparison are shown in Table 4. The comparison shows that, for ORB-1, the orbital position errors of forecasts for 1 ~7 days were within 1 km. For ORB-2, the position errors of forecasts for 1 ~7 days were less than 50.00 m. For ORB-3, the prediction made for one day had a position error of 1.70 km, the prediction made for three days had a position error of 5.18 km, and the prediction made for seven days had a position error reaching 99.09 km.

7 Conclusion

In this paper, we explored the effects of SMR on the accuracy of predicting orbits and on the decay of orbit height for NCEOs orbiting in large elliptical orbits, medium altitude orbits, and low altitude orbits around the Earth. Using ground-based radar data and space-based angle data, we analyzed the accuracy for determining and predicting the three types of orbits of NCEOs.

1. For large elliptical and medium altitude orbits, the 180-days forecast results are compared to the orbits of a given SMR. When the SMR increased or decreased many fold, the errors of the orbit positions also increased by many fold. For low orbits at a height of 340 km when the SMR increased by many fold, the time for the orbit height to decrease to 100 km shortened drastically. However, when the SMR decreased by many fold, the decay time of the orbit height became longer. For low orbits at 340 km, when the SMR increased by many fold, the time for the orbit to decay to a height of 100 km also decreased by many fold, but when the SMR decreased by many fold, the decay rate of the orbit slowed down considerably.
2. Under the conditions of zero errors in the force model and for given measurement errors, the accuracy for the three types of orbits is better than 10.0 m.
3. Under the combined effect of errors in the force model and measurement errors, the error for determining the large elliptical ORB-1 position using three days of ground-based radar data was 323.49 m and the error for 1 ~7 days forecast of ORB-1 was less than 1 km. The error for determining ORB-2 positions using three days of ground-based radar data and space-based angle data was 14.66 m and the 1 ~7 days forecast error for ORB-2 positions was less than 50.00 m. The error for determining ORB-3 positions using 1.5 days of combined ground-based radar data and space-based angle data was 270.37 m and the position error was 1.70 km for 1-day forecast, 5.18 km for 3-days forecast, and 99.09 km for 7-days forecast.
4. The orbital error determined by using 7-minutes GBSSRD and 15-minutes SBDSAD is 63.77 m lower than that only determined by using GBSSRD. The space-based angle information greatly improves the observation geometry of the non-cooperative target and improves the precision of orbit determination.

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