Research Article

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Meshfree approach on buckling and free vibration analysis of porous FGM plate with proposed IHSDT resting on the foundation

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Abstract: In the present work, new inverse hyperbolic higher-order shear deformation theory (IHSDT) is proposed and implemented for buckling analysis and free vibration analysis of porous Functionally Graded Material (FGM) plate on the foundation. The proposed theory follows the approximately parabolic distribution of the transverse stresses through the plate thickness and satisfies the conditions of continuity and differentiability. Three different types of porosity distribution considered. Governing differential equations (GDEs) of the plate is developed in the framework of proposed theories by Hamilton's principle. Multiquadrics radial basis function (MQ-RBF) based Meshfree method used for discretizing the GDEs. The result obtained by the present theory is validated with the three-dimensional elastic theory and other available solutions in the literature to ensure the efficacy and accuracy of the proposed theory. Numerical results obtained for buckling and free vibration for porous FGM plate resting on the foundation. Effect of grading index, porosity fraction, porosity distribution, the effect of foundation, and the span to thickness ratio have discussed. The secured results can consider as a benchmark for future studies.

Keywords: Meshfree; Buckling; Free Vibration; IHSDT; Porosity

1 Introduction

Pure metals or ceramics have limited application in structural engineering because of the demand of conflicting properties requirements. Some new magnificent advanced composite plates which are made up of functionally graded materials (FGM) are in much demand because of their unique material properties which vary from the surface of one direction to the surface of the other direction. In FGM, an excellent combination of properties which are different from the individual parent materials and lighter in weight used. FGM was first proposed by Bever and Duwez [1] in 1972. Due to their excellent properties, FGMs widely used in the aerospace industry, aeronautical engineering applications, biomedical, electronic chips, heat-resisting environment, and many structural applications. Some significant advantages of the FGM are the elimination of delamination mode of failure, residual stresses, high interlinear shear stresses, local stress concentration at the interface or reducing thermal stress, which are the characteristic phenomena observed in laminated composite materials. The research on plates started with the analysis of thin plates using Classical Plate Theory (CPT). Then the first-order shear deformation theory (FSDT) was introduced, whose accuracy is determined by the shear correction factor, which may be difficult to calculate on advanced composites. To avoid such troubles, higher-order shear deformation theories (HSDTs) introduced. However, the majority of these theories avoid transverse extensibility by neglecting the stretching effects. HSDTs further classified as polynomial and non-polynomial. Non-polynomial HSDTs are mainly trigonometric, hyperbolic, algebraic exponential, and logarithmic shape function. Levy M [2], was probably the first to develop a trigonometric theory for the analysis of thick isotropic plates. Touratier [3], used the same function and developed a Sinusoidal Shear Deformation Theory for isotropic laminated plates with five unknowns. Soldatos [4], first used a hyperbolic shape function to develop HSDT for composite plates. Karama *et al.*, [5] developed the first five variable exponential function for the

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plate analysis. Recently, some new transverse shear deformation theories have introduced for additional accuracy. Mantari *et al.*, [6] introduced a new transverse shear deformation theory for composite plates to avoid a shear correction factor under the sinusoidal and uniformly distributed loads. Grover *et al.*, [7] proposed a new inverse hyperbolic shear deformation theory for static buckling analysis of laminated composites and sandwich plates to provide an accurate solution which is less from computational error. Mantari & Guedes Soares [8], proposed a new non-polynomial HSDT for investigation of the static response of composite plates. Aicha Bessaim *et al.*, [9] introduced a new hyperbolic HSDT to improve the accuracy of bending and vibration analysis of a sandwich plate with a thin functionally graded sheet. A new hyperbolic transverse shear deformation theory was introduced by El Meiche *et al.*, [10] to improve the effects of buckling and free vibration analysis of thick FG sandwich plates. Thai and Vo [11], introduced a new sinusoidal transverse shear deformation theory for bending, buckling, and vibration analysis of FG plate to improve the accuracy of the analysis. Mahi *et al.*, [12] developed a new hyperbolic shear transverse theory to avoid the shear correction factor which improved the accuracy of bending and free vibration analysis of isotropic, functionally graded, sandwich and laminated composite. Joshan *et al.*, [13] developed four variables inverse hyperbolic shear deformation theory for analysis of laminated composite plates. Mantari, [15] developed an accurate sinusoidal trigonometric theory for the analysis of the FGM plate and shells. Viola *et al.*, [16] introduced the new shear function for the static analysis of laminated double curved shells by using Generalized Differential Quadrature (GDQ) method. Viola *et al.*, [17], investigated free vibration analysis of double-curved laminated shells and panels and comparison among the FSDT and existing HSDT.

There are many computational methods available for solving the partial differential equation (PDE) and analyzing the composite structures. The generation of a mesh in mesh-based computational methods is a difficult task for any structure, so meshfree methods are an alternative to mesh-based numerical methods. A meshfree method does not require a mesh to discretize the PDE. Some meshless methods depend on background meshes that may support numerical quadrature calculations. Meshfree methods can be categorized into two forms for solving the PDE which is known as a weak form and strong form meshfree methods. Regarding weak meshless methods, Nayroles *et al.*, [18] were first to introduce a new diffuse approximation to interpolate through moving least squares into a Galerkin scheme. On the other side, collocation meshfree methods work on the strong form of the PDE. The radial basis func-

tion method was first used by Hardy [19] and proposed a simple shape parameter $c = 0.815h$, where 'h' is the average distance between the closest neighbors. After 20 years, Kansa [20], developed the solution of PDEs. Franke [21] has ranked MQRBF as the best interpolation method based on its accuracy, execution time, and ease of implementation. Cheng [22] investigated subject positive definiteness, error estimate, optimal shape parameter, traditional and effective condition numbers, round-off error, derivatives of interpolator, and the edge effect of multiquadric radial basis functions and its shape parameter. A detailed discussion of RBFs seen in Liu and Gu [23, 24]. While utilizing RBFs, several shape parameters need to be determined for better performance. Chen *et al.*, [25] investigated free vibration of FGM plate using a local natural neighbor interpolation meshless method based on FSDT. Kumar and Singh, [26] investigated various HSDT for the buckling analysis of FGM plate via the RBF method. Ferreira *et al.*, [27] analyzed composite plate by using MQ-RBF based meshfree method. Kumar *et al.* [28] investigated bending response of FGM plate under patch load by using Wendland RBF. Ferreira *et al.*, [29] used RBF based collocation method for the bending analysis of functionally graded plates. Neves *et al.*, [30] investigated Adaptive Methods for Analysis of Composite Plates with RBF. Roque *et al.*, [31] and Ferreira *et al.*, [32] investigated bending and vibration analysis of composite plates by using RBF based layerwise theory. Singh *et al.*, [33] used the meshfree technique for the analysis of laminated composite and sandwich plates using various RBF. Fantuzzi *et al.*, [34] applied the RBF method for the free vibrations of doubly-curved laminated composite shells and panels by using HSDT.

The typical literature for the buckling and vibration analysis of FGM plates are rare as compared to isotropic, laminated and sandwich plates. Zhu and Liew [35], carried out a free vibration analysis of functionally graded plates using FSDT based on local Kriging meshless technique. Ferreira *et al.*, [36] carried out a free vibration analysis of FGM plate by using the multiquadric radial basis function-based collocation method. Xiang and Xing [37] developed a new FSDT having two independent variables for the free vibrations of a rectangular plate. Zhao *et al.* [38] implemented an element free kp-Ritz technique to analyze free vibration of functionally graded plates. FSDT is used to analyze transverse shear strain and, mesh-free kernel functions used to approximate the 2-dimensional displacement fields. Vu *et al.*, [39] introduced four variable FSDT for the static and free vibration analysis of FGM plate. Neves *et al.*, [40] investigated static and free vibration analysis of FGM plates by using quasi-3D hyperbolic shear deformation theory with RBF based meshless technique. Wu

and Chiu [41], carried RMVT-based meshless collocation and element-free Galerkin technique for the quasi-3D free vibration analysis of FGM plates. Thai and Choi, [42] presented a buckling analysis of FGM plate with four variables by applying closed form solutions. Park and Choi, [43] used two variables FSDT models for bending, buckling, and free vibration analyses of isotropic plates. Buckling analysis of plates under uniaxial and biaxial compression with shear loads was carried out by Bui *et al.*, [44] using the efficient meshfree method. Galerkin meshfree formulation was presented by Sadamoto *et al.*, [45] for buckling analysis of stiffened plates. Zhao *et al.*, [46] carried a buckling analysis of FGM plate based on the element-free kp-Ritz method, and the formulations based on FSDT, also including the effect of the imperfection of initial geometry. Moslemi *et al.*, [47] introduced an analytical solution for the buckling analysis of isotropic plate subjected to uniaxial or biaxial uniformly distributed in-plane load. Isogeometric buckling analysis of FGM was carried out by Yu *et al.* [48] based on first-order shear deformation theory without considering the effect of shear locking. Chakraverty and Pradhan, [49] used the Rayleigh–Ritz method to investigate the free vibration of FG plates under various boundary conditions. Talha and Singh [50], carried out bending and vibration analysis of FGM plates by using the FEM method.

FGM plates resting on elastic foundations are one of the most important structural components used in aircraft on runways, footings, mat foundations, and many astronautic and marine structures. Malekzadeh, [51] investigated 3D free vibration of FGM plate resting on foundation, and the material properties vary exponentially through the thickness. Ait Atmane *et al.*, [52] investigated free vibration of FGM plate by using trigonometric HSDT resting on foundation. Sobhy, [53] investigated buckling and free vibration response of FGM sandwich plate resting on foundation. Studies on the buckling and vibrations of porous FGM plates have limited in the literature. Shahsavari *et al.*, [54] investigated free vibration of three types of porosity distribution FGM plate by using quasi-3D hyperbolic theory. Wang *et al.*, [55] investigated non-linear free vibration of FGM plate with porosity effect. Zhao *et al.*, [56] used 3-D exact solution for vibration analysis of three types of porosity distributions FGM plates subject to various boundary conditions. Kiran and Kattimani [57], examined the bending and free vibration of the porous FGM plate by using the finite element method.

In the present work, MQ-RBF based meshfree method implemented for the buckling and free vibration analysis of porous FGM plates with proposed HSDTs on the foundation. The GDE of the FGM plate is developed using the principle of virtual works. MQ-RBF based Meshfree method im-

Figure 1: The geometry of a rectangular FGM plate with porosity in the rectangular coordinate system

plemented for discretizing the GDE. The convergence and validation study of a meshfree method for various problems carried out. It noticed that the developed theory results predict closer to the 3-D elasticity results. Some new results for buckling and free vibration analysis of FGM plate in porous media with on foundation are presented using IHHSST based on five variables ESL theory. The effect of porosity distribution, porosity fraction index, grading index, foundation effect, and span to thickness ratio examined.

2 Theoretical formulation

2.1 Description of the FGP plates

A functionally graded material plate with dimensions $(a \times b \times h)$ in the cartesian coordinate system $(x-y-z)$ shown in Figure 1. The mid-plane of the plate considered as the reference plane. The effective materials properties of the FGM plate are uniform across the thickness direction based on the modified power-law in the Voigt model. Three types of porosity distributions along the thickness direction are as follows:

- SCPD: Symmetric center enhanced porosity distribution, [58]

$$E(z) = \left([E_c - E_m] \left(\frac{2z + h}{2h} \right)^n + E_m \right) \left[1 - P \cos \left(\frac{\pi z}{h} \right) \right] \quad (1)$$

Table 1: The shear strain functions proposed by different researchers are as under:

S.No	Transverse shear strain function ($f(z)$)	Abbreviation	Proposed
1	$z \tan^{-1} h \left(\tanh \left(2\pi \left(\frac{z}{h} \right)^2 \right) \right) - \left(\frac{3\pi z}{2} \right)$	IHSDT	New proposed theory
2	$\left(z - \frac{4z^3}{3h^2} \right)$	HSDT-1	Reddy [59]
3	$\frac{h}{\pi} \sin \left(\frac{\pi z}{h} \right)$	HSDT-2	Touratier, [60] Zenkour, [61]
4	$h \sinh \left(\frac{z}{h} \right) - z \cosh \left(\frac{1}{2} \right)$	HSDT-3	Mantari <i>et al.</i> , [62]
5	$\sin \left(\frac{\pi z}{h} \right)$	HSDT-4	Arya <i>et al.</i> , [63]
6	$z \times e^{-2\left(\frac{z}{h}\right)^2}$	HSDT-5	Karama <i>et al.</i> , [64]
7	$z \alpha^{-2(z/h)^2 / \ln \alpha}$	HSDT-6	Aydogdu, [65]
8	$\sin \left(\frac{\pi z}{h} \right) \times e^{\frac{1}{2} \cos \left(\frac{\pi z}{h} \right)} + \frac{\pi z}{2h}$	HSDT-7	Mantari <i>et al.</i> , [42]
9	$\sinh^{-1} \left(\frac{rz}{h} \right) - z \frac{2r}{h\sqrt{r^2+4}}, r = 3$	HSDT-8	Grover <i>et al.</i> , [66]
10	$h \tan^{-1} (2z/h) - z$	HSDT-9	Thai <i>et al.</i> , [67]
11	$\frac{7}{8} z - \frac{2z^3}{h^2} + \frac{2z^5}{h^4}$	HSDT-10	Nguyen-Xuan <i>et al.</i> , [68]
12	$\frac{h}{2} \tanh \left(\frac{2z}{h} \right) - \frac{4}{3 \cosh^2(1)} \frac{z^3}{h^2}$	HSDT-11	Thai <i>et al.</i> , [69]
13	$\tan \left(\frac{mz}{h} \right) + \left\{ 2z \cosh \left(\frac{1}{2} \right) \right\}$ $m=5$	HSDT-12	Singh and Singh, [70]
14	$\left[z - \frac{11}{5h^2} z^3 + \frac{52}{25h^4} z^5 \right]$	HSDT-13	Nguyen <i>et al.</i> , [71]
15	$\left[z - \frac{11}{5h^2} z^3 + \frac{52}{25h^4} z^5 \right]$	HSDT-14	Nguyen <i>et al.</i> , [71]

- BPD: Bottom enhanced porosity distribution, [58]

$$E(z) = \left[[E_c - E_m] \left(\frac{2z + h}{2h} \right)^n + E_m \right] \left(1 - P \cos \left[\frac{\pi}{2} \left(\frac{z}{h} + 0.5 \right) \right] \right) \quad (2)$$

- TPD: Top enhanced porosity distribution, [58]

$$E(z) = \left[[E_c - E_m] \left(\frac{2z + h}{2h} \right)^n + E_m \right] \left(1 - P \cos \left[\frac{\pi}{2} \left(\frac{z}{h} - 0.5 \right) \right] \right) \quad (3)$$

Where E represents the effective material property (Young's modulus), subscripts m and c represents the metallic and ceramic constituents respectively, P is the porosity fraction ($0 < P < 1$) and $P=0$ means pure FGM plate. 'n' is exponent governing the material properties along the thickness direction known as grading index. The value of 'n' equal to 0 and ∞ represents a fully ceramic and metal plate, respectively.

2.2 Displacement field

The displacement model can be represented in terms of generalized transverse HSDT $f(z)$ and the in-plane displacement u, v and w at any point along the x -, y - and z -axes can be expressed in terms of the mid-plane displacements u_0, v_0, w_0

and rotations ϕ_x and ϕ_y which are five unknown displacement functions as :

$$u = u_0(x, y) - z \frac{\partial w_0(x, y)}{\partial x} + f(z) \phi_x(x, y) \quad (4)$$

$$v = v_0(x, y) - z \frac{\partial w_0(x, y)}{\partial y} + f(z) \phi_y(x, y)$$

$$w = w_0(x, y)$$

The $f(z)$ represents the transverse shear deformation function which is responsible for examining the distribution of the transverse shear strains and stresses along with the thickness. The proposed and existing transverse shear deformation function $f(z)$ listed in Table 1. Researchers are always in search of new theory for better accuracy and stability of results. In the same line, authors of this paper tried to find a new theory which can give a better solution. The function $f(z)$ is chosen in such a way that it satisfies the conditions of continuity and differentiability, such that

$$\int_{-h/2}^{h/2} f(z) dz = 0 \quad f'(\pm h/2) = 0 \quad (5)$$

These conditions are essential for a transverse shear deformation function in order to eliminate the requirement of the shear correction factor. From Figure 2, it can clearly be seen that the newly proposed theory accounts for the parabolic variation of transverse shear strains through the span to a thickness of the plate. Furthermore, the proposed theory

Figure 2: Transverse stress profile of different transverse shear strain function

satisfies the zero traction boundary conditions on the top and bottom surfaces of the plate.

2.3 Strain-displacement relationships

According to the linear, small-strain elasticity theory, the strain-displacement relations expressed as:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} = \frac{\partial u_0}{\partial x} - z \frac{\partial^2 w_0}{\partial x^2} + f(z) \frac{\partial \phi_x}{\partial x} \quad (6)$$

$$\varepsilon_{yy} = \frac{\partial v}{\partial y} = \frac{\partial v_0}{\partial y} - z \frac{\partial^2 w_0}{\partial y^2} + f(z) \frac{\partial \phi_y}{\partial y} \quad (7)$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} - 2z \frac{\partial^2 w_0}{\partial x \partial y} + f(z) \frac{\partial \phi_x}{\partial y} + f(z) \frac{\partial \phi_y}{\partial x} \quad (8)$$

$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = \frac{\partial f(z)}{\partial z} \phi_y \quad (9)$$

$$\gamma_{zx} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \frac{\partial f(z)}{\partial z} \phi_x \quad (10)$$

2.4 Constitutive Stress-Strain Relations

The stress-strain relation using Hook's law concerning the structural axis system (X-Y-Z) for the isotropic FGM plate

with $\sigma_{zz} = 0$ be expressed as Ferreira *et al.*, [29];

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{zx} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & 0 & 0 & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & 0 & 0 & 0 \\ 0 & 0 & \bar{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \bar{Q}_{44} & 0 \\ 0 & 0 & 0 & 0 & \bar{Q}_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} \quad (11)$$

Where $\{\sigma\}$ and $\{\varepsilon\}$ are the stress vector and strain vector respectively and $\bar{Q}_{i,j}$ is transformed reduced stiffness matrix.

$$\bar{Q}_{11} = \bar{Q}_{22} = \frac{E}{(1 - \nu^2)},$$

$$\bar{Q}_{12} = \frac{\nu E}{(1 - \nu^2)}, \quad \bar{Q}_{44} = \bar{Q}_{55} = \bar{Q}_{66} = \frac{E}{2(1 + \nu)}$$

2.5 Governing Differential Equations

Hamilton's principle is used herein to derive the governing differential equations of the FGM plate along with variationally admissible boundary conditions. The principle can be expressed as

$$\int_{t_1}^{t_2} \delta (KE - (UE + VE)) dt = 0 \quad (12)$$

Where δ denotes the variational symbol and ρ is the mass per unit volume.

The strain energy (UE) of the FGM plate is expressed by [72]

$$UE = \frac{1}{2} \int_{-\frac{h}{2}}^{\frac{h}{2}} \int_A (\sigma_{xx} \varepsilon_{xx} + \sigma_{yy} \varepsilon_{yy} + \sigma_{xy} \gamma_{xy} + \sigma_{yz} \gamma_{yz} + \sigma_{xz} \gamma_{xz}) dz dA \quad (13)$$

The kinetic energy (KE) of the FGM plate is expressed by [72]

$$KE = \frac{1}{2} \int_{-\frac{h}{2}}^{\frac{h}{2}} \int_A \rho \left\{ \left(\frac{\partial u}{\partial \tau} \right)^2 + \left(\frac{\partial v}{\partial \tau} \right)^2 + \left(\frac{\partial w}{\partial \tau} \right)^2 \right\} dz dA \quad (14)$$

The potential energy (PE) due to the in-plane mechanical loading of the FGM plate is expressed by

$$VE = \frac{1}{2} \int_A \left\{ N_x^b \left(\frac{\partial w}{\partial x} \right)^2 \right\} dA \quad (15a)$$

$$+N_y^b \left(\frac{\partial w}{\partial y} \right)^2 + 2 N_{xy}^b \left(\frac{\partial w}{\partial x} \right) \left(\frac{\partial w}{\partial y} \right) \Big\} dA$$

The potential energy (VE_f) of the elastic foundation is expressed by [72]

$$VE_f = \frac{1}{2} \left(\int_A K_w w^2 - K_s \left(\left(\frac{\partial w}{\partial x} \right)^2 + \left(\frac{\partial w}{\partial y} \right)^2 \right) dA \right) \quad (15b)$$

Considering displacement-strain relations and stress-strain relations, and applying integration by parts and the fundamental lemma of variational calculus and collecting the coefficients of δu_0 , δv_0 , δw_0 , $\delta \phi_x$, and $\delta \phi_y$. The governing differential equations of the plate obtained as:

$$\delta u_0 : \frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_0 \frac{\partial^2 u_0}{\partial \tau^2} - I_1 \frac{\partial^3 w_0}{\partial x \partial \tau^2} + I_3 \frac{\partial^2 \phi_x}{\partial \tau^2} \quad (16)$$

$$\delta v_0 : \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_{yy}}{\partial y} = I_0 \frac{\partial^2 v_0}{\partial \tau^2} - I_1 \frac{\partial^3 w_0}{\partial y \partial \tau^2} + I_3 \frac{\partial^2 \phi_y}{\partial \tau^2} \quad (17)$$

$$\begin{aligned} \delta w_0 : & \frac{\partial^2 M_{xx}}{\partial x^2} + \frac{\partial^2 M_{yy}}{\partial y^2} + 2 \frac{\partial^2 M_{xy}}{\partial x \partial y} + K_w w_0 \\ & - K_s \left(\frac{\partial^2 w_0}{\partial x^2} + \frac{\partial^2 w_0}{\partial y^2} \right) = N_{xx}^b \frac{\partial^2 w}{\partial x^2} + N_{yy}^b \frac{\partial^2 w}{\partial y^2} \\ & + 2 N_{xy}^b \frac{\partial^2 w}{\partial x \partial y} + I_0 \frac{\partial^2 w_0}{\partial \tau^2} + I_1 \left(\frac{\partial^3 u_0}{\partial x \partial \tau^2} + \frac{\partial^3 v_0}{\partial y \partial \tau^2} \right) \\ & - I_2 \left(\frac{\partial^4 w_0}{\partial x^2 \partial \tau^2} + \frac{\partial^4 w_0}{\partial y^2 \partial \tau^2} \right) \\ & + I_4 \left(\frac{\partial^3 \phi_x}{\partial x \partial \tau^2} + \frac{\partial^3 \phi_y}{\partial y \partial \tau^2} \right) \end{aligned} \quad (18)$$

$$\delta \phi_x : \frac{\partial M_{xx}^f}{\partial x} + \frac{\partial M_{xy}^f}{\partial y} - Q_x^f = I_3 \frac{\partial^2 u_0}{\partial \tau^2} - I_4 \frac{\partial^3 w_0}{\partial x \partial \tau^2} + I_5 \frac{\partial^2 \phi_x}{\partial \tau^2} \quad (19)$$

$$\delta \phi_y : \frac{\partial M_{xy}^f}{\partial x} + \frac{\partial M_{yy}^f}{\partial y} - Q_y^f = I_3 \frac{\partial^2 v_0}{\partial \tau^2} - I_4 \frac{\partial^3 w_0}{\partial y \partial \tau^2} + I_5 \frac{\partial^2 \phi_y}{\partial \tau^2} \quad (20)$$

Where, N_{xx}^b and N_{yy}^b are the in-plane axial forces and $N_{xy}^b = N_{yx}^b$ is the in-plane shear force is listed in Figure 3. In

equ. 18 if $N_{xx}^b = 1$, $N_{yy}^b = 0$, $N_{xy}^b = 0$ is uniaxial loading, $N_{xx}^b = 1$, $N_{yy}^b = 1$, $N_{xy}^b = 0$ is biaxial loading, and $N_{xx}^b = 0$, $N_{yy}^b = 0$, $N_{xy}^b = 1$ is pure shear loading. K_s and K_w are the equivalent shear and Winkler foundation parameters. The axial force resultants, N_{ij} the bending moment resultants M_{ij} , the additional moment resultants related to the transverse shear function M_{ij}^f and the transverse shear force resultants Q_x^f and Q_y^f expressed as:

$$N_{ij}, M_{ij}, M_{ij}^f = \int_{-h/2}^{+h/2} (\sigma_{ij}, z\sigma_{ij}, f(z)\sigma_{ij}) dz \quad (21)$$

$$Q_x^f, Q_y^f = \int_{-h/2}^{+h/2} (\sigma_{xz}, \sigma_{yz}) \left(\frac{\partial f(z)}{\partial z} \right) dz \quad (22)$$

2.6 Governing differential equation in terms of displacement

The governing differential Eqs (16) – (20) are present in terms of displacement components using Eqs 6 and 10 and finally expressed as:

$$\left(A_{11} \frac{\partial^2 u_0}{\partial x^2} + A_{66} \frac{\partial^2 u_0}{\partial y^2} \right) \quad (23)$$

$$\begin{aligned} & + \left(A_{12} \frac{\partial^2 v_0}{\partial x \partial y} + A_{66} \frac{\partial^2 v_0}{\partial x \partial y} \right) \\ & + \left(-B_{11} \frac{\partial^3 w_0}{\partial x^3} - B_{12} \frac{\partial^3 w_0}{\partial x \partial y^2} - 2B_{66} \frac{\partial^3 w_0}{\partial x \partial y^2} \right) \\ & + \left(E_{11} \frac{\partial^2 \phi_x}{\partial x^2} + E_{66} \frac{\partial^2 \phi_x}{\partial y^2} \right) + \left(E_{12} \frac{\partial^2 \phi_y}{\partial x \partial y} + E_{66} \frac{\partial^2 \phi_y}{\partial x \partial y} \right) \\ & = I_0 \frac{\partial^2 u_0}{\partial \tau^2} - I_1 \frac{\partial^3 w_0}{\partial x \partial \tau^2} + I_3 \frac{\partial^2 \phi_x}{\partial \tau^2} \end{aligned}$$

$$\left(A_{66} \frac{\partial^2 u_0}{\partial x \partial y} + A_{12} \frac{\partial^2 u_0}{\partial x \partial y} \right) + \left(A_{66} \frac{\partial^2 v_0}{\partial x^2} + A_{22} \frac{\partial^2 v_0}{\partial y^2} \right) \quad (24)$$

$$\begin{aligned} & + \left(-2B_{66} \frac{\partial^3 w_0}{\partial x^2 \partial y} - B_{12} \frac{\partial^3 w_0}{\partial x^2 \partial y} - B_{22} \frac{\partial^3 w_0}{\partial y^3} \right) \\ & + \left((E_{12} + E_{66}) \frac{\partial^2 \phi_x}{\partial x \partial y} \right) + \left(E_{66} \frac{\partial^2 \phi_y}{\partial x^2} + E_{22} \frac{\partial^2 \phi_y}{\partial y^2} \right) \\ & = I_0 \frac{\partial^2 v_0}{\partial \tau^2} - I_1 \frac{\partial^3 w_0}{\partial y \partial \tau^2} + I_3 \frac{\partial^2 \phi_y}{\partial \tau^2} \end{aligned}$$

$$\left(B_{11} \frac{\partial^3 u_0}{\partial x^3} + (B_{12} + 2B_{66}) \frac{\partial^3 u_0}{\partial x \partial y^2} \right) \quad (25)$$

$$\begin{aligned} & + \left(B_{22} \frac{\partial^3 v_0}{\partial y^3} + (B_{12} + 2B_{66}) \frac{\partial^3 v_0}{\partial x^2 \partial y} \right) \\ & - \left(D_{11} \frac{\partial^4 w_0}{\partial x^4} + (2D_{12} + 4D_{66}) \frac{\partial^4 w_0}{\partial x^2 \partial y^2} + D_{22} \frac{\partial^4 w_0}{\partial y^4} \right) \end{aligned}$$

$$\begin{aligned}
& + \left(F_{11} \frac{\partial^3 \phi_x}{\partial x^3} + (F_{12} + 2F_{66}) \frac{\partial^3 \phi_x}{\partial x \partial y^2} \right) \\
& + \left(F_{22} \frac{\partial^3 \phi_y}{\partial y^3} + (F_{12} + 2F_{66}) \frac{\partial^3 \phi_y}{\partial x^2 \partial y} \right) + K_w w_0 \\
& - K_s \left(\frac{\partial^2 w_0}{\partial x^2} + \frac{\partial^2 w_0}{\partial y^2} \right) = N_{xx}^b \frac{\partial^2 w}{\partial x^2} + N_{yy}^b \frac{\partial^2 w}{\partial y^2} \\
& + 2N_{xy}^b \frac{\partial^2 w}{\partial x \partial y} + I_0 \frac{\partial^2 w_0}{\partial t^2} + I_1 \left(\frac{\partial^3 u_0}{\partial x \partial \tau^2} + \frac{\partial^3 v_0}{\partial y \partial \tau^2} \right) \\
& - I_2 \left(\frac{\partial^4 w_0}{\partial x^2 \partial \tau^2} + \frac{\partial^4 w_0}{\partial y^2 \partial \tau^2} \right) + I_4 \left(\frac{\partial^3 \phi_x}{\partial x \partial \tau^2} + \frac{\partial^3 \phi_y}{\partial y \partial \tau^2} \right) \\
& \left(E_{11} \frac{\partial^2 u_0}{\partial x^2} + E_{66} \frac{\partial^2 u_0}{\partial y^2} \right) + \left(E_{12} \frac{\partial^2 v_0}{\partial x \partial y} + E_{66} \frac{\partial^2 v_0}{\partial x \partial y} \right) \quad (26) \\
& + \left(-F_{11} \frac{\partial^3 w_0}{\partial x^3} - (F_{12} + 2F_{66}) \frac{\partial^3 w_0}{\partial x \partial y^2} \right) \\
& + \left(H_{11} \frac{\partial^2 \phi_x}{\partial x^2} + H_{66} \frac{\partial^2 \phi_x}{\partial y^2} - A_{55} \phi_x \right) \\
& + \left((H_{12} + H_{66}) \frac{\partial^2 \phi_y}{\partial x \partial y} + H_{22} \frac{\partial^2 \phi_y}{\partial y^2} \right) = I_3 \frac{\partial^2 u_0}{\partial \tau^2} \\
& - I_4 \frac{\partial^3 w_0}{\partial x \partial \tau^2} + I_5 \frac{\partial^2 \phi_x}{\partial \tau^2}
\end{aligned}$$

$$\begin{aligned}
& \left(E_{66} \frac{\partial^2 u_0}{\partial x \partial y} + E_{12} \frac{\partial^2 u_0}{\partial x \partial y} \right) \\
& + \left(+E_{66} \frac{\partial^2 v_0}{\partial x^2} + E_{22} \frac{\partial^2 v_0}{\partial y^2} \right) \\
& + \left(-(F_{12} + F_{66}) \frac{\partial^3 w_0}{\partial y \partial x^2} - F_{22} \frac{\partial^3 w_0}{\partial y^3} \right) \\
& + \left(H_{66} \frac{\partial^2 \phi_x}{\partial x \partial y} + H_{12} \frac{\partial^2 \phi_x}{\partial x \partial y} \right) \\
& + \left(H_{66} \frac{\partial^2 \phi_y}{\partial x^2} + H_{22} \frac{\partial^2 \phi_y}{\partial y^2} - A_{44} \phi_y \right) = I_3 \frac{\partial^2 v_0}{\partial \tau^2} \\
& - I_4 \frac{\partial^3 w_0}{\partial y \partial \tau^2} + I_5 \frac{\partial^2 \phi_y}{\partial \tau^2}
\end{aligned} \quad (27)$$

The stiffness coefficients for the FGM plate are taken as:

$$A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij} \quad (28)$$

$$= \int_{-h/2}^{h/2} \left\{ \left(\frac{E(z)}{1-\nu^2} \right) \left(1, z, z^2, f(z), zf(z), f^2(z) \right) \right\} dz$$

$i, j = 1, 2, 6$

$$A_{ij} = \int_{-h/2}^{h/2} \left\{ \left(\frac{E(z)}{2(1+\nu)} \right) \left(\frac{\partial f(z)}{\partial z} \right)^2 \right\} dz \quad i, j = 4, 5 \quad (29)$$

$I_0, I_1, I_2, I_3, I_4, I_5$ are mass inertias which is expressed as

$$I_0, I_1, I_2, I_3, I_4, I_5 \quad (30)$$

$$= \int_{h_1}^{h_2} \rho(z) \left(1, z, z^2, f(z), zf(z), f^2(z) \right) dz$$

2.7 Simply supported boundary condition SS boundary condition

$$x = 0, \quad (31)$$

$$a : N_{xx} = 0, \quad v_0 = 0, \quad w_0 = 0, \quad M_{xx} = 0, \quad \phi_y = 0$$

$$y = 0,$$

$$b : u_0 = 0, \quad N_{yy} = 0, \quad w_0 = 0, \quad \phi_x = 0, \quad M_{yy} = 0$$

3 Solution methodology

The governing equations, along with the appropriate boundary conditions are solved using multi quadratic radial basis functions RBF based meshfree method. For the present analysis, MQ- RBF is taken and which expressed as:

$$\text{Multiquadrics function (MQ-RBF)} \quad g = (r^2 + c^2)^m; \quad (32)$$

$$c = \alpha \cdot \sqrt{\left(\frac{a}{n_x} \right)^2 + \left(\frac{b}{n_y} \right)^2}$$

Where, $r = \|X - X_j\| = \sqrt{(x - x_j)^2 + (y - y_j)^2}$, 'm' and 'c' are shape parameters. n_x and n_y are a number of divisions (number of nodes-1) along with the 'a' and 'b' respectively. α is a constant that governs the value of 'c' for interior and boundary nodes for MQ-RBF. Optimization of the shape parameter has been discussed by Singh and Shukla [87]. For the present investigation, a MATLAB code is developed to acquire the solution. First, the convergence of results was checked by shifting the shape parameter. When convergence is confirmed and stable, the accuracy was likewise checked. Also, given stability and accuracy, the authors proposed the given shape parameter. A 2-D rectangular with any arbitrary boundary having NB boundary nodes and NI domain interior nodes subjected to compressive in-plane forces and shear distributed forces is shown in Figure 3.

3.1 Discretization of Governing Differential Equations

The unknown field variables u_0, v_0, w_0, ϕ_x and ϕ_y appearing in governing differential equations can be interpolated in

Figure 3: An arbitrary 2-D domain with boundary and domain interior nodes in the form of scattered data subjected to compressive in-plane forces and shear distributed forces

the form of the radial distance between nodes using the following expression:

$$u_0(x, y) = \sum_{j=1}^N \alpha_j^{u_0} g(\|X - X_j\|, c) \quad (33)$$

$$v_0(x, y) = \sum_{j=1}^N \alpha_j^{v_0} g(\|X - X_j\|, c) \quad (34)$$

$$w_0(x, y) = \sum_{j=1}^N \alpha_j^{w_0} g(\|X - X_j\|, c) \quad (35)$$

$$\phi_x(x, y) = \sum_{j=1}^N \alpha_j^{\phi_x} g(\|X - X_j\|, c) \quad (36)$$

$$\phi_y(x, y) = \sum_{j=1}^N \alpha_j^{\phi_y} g(\|X - X_j\|, c) \quad (37)$$

Where, N is a total number of nodes, which is equal to the summation of boundary nodes NB and domain interior nodes NI, α_j^u is unknown coefficient, $g(\|X - X_j\|, c)$ is radial basis function, $\|X - X_j\|$ is the distance between the nodes.

The governing differential equations are discretized and finally expressed in compact matrix form as:

$$[K]_L + [K]_{fL} + \lambda [K]_G \{\delta\} = \chi \omega^2 [M] \{\delta\} \quad (38)$$

Here,

For free vibration analysis: $\lambda = 0, \chi = 1$

For buckling analysis: $\lambda = 1, \chi = 0$

Where,

$$\{\delta\} = [\alpha^{u_0} \alpha^{v_0} \alpha^{w_0} \alpha^{\phi_x} \alpha^{\phi_y}]^T \quad (39)$$

$$[K]_L = \begin{bmatrix} [K^L_{1u}]_{(NI,N)} & [K^L_{1v}]_{(NI,N)} & \cdots \\ [K^L_{2u}]_{(NI,N)} & [K^L_{2v}]_{(NI,N)} & \cdots \\ [K^L_{3u}]_{(NI,N)} & [K^L_{3v}]_{(NI,N)} & \cdots \\ [K^L_{4u}]_{(NI,N)} & [K^L_{4v}]_{(NI,N)} & \cdots \\ [K^L_{5u}]_{(NI,N)} & [K^L_{5v}]_{(NI,N)} & \cdots \end{bmatrix} \quad (40)$$

$$\begin{bmatrix} [K^L_{1w}]_{(NI,N)} & [K^L_{1\phi_x}]_{(NI,N)} & [K^L_{1\phi_y}]_{(NI,N)} \\ [K^L_{2w}]_{(NI,N)} & [K^L_{2\phi_x}]_{(NI,N)} & [K^L_{2\phi_y}]_{(NI,N)} \\ [K^L_{3w}]_{(NI,N)} & [K^L_{3\phi_x}]_{(NI,N)} & [K^L_{3\phi_y}]_{(NI,N)} \\ [K^L_{4w}]_{(NI,N)} & [K^L_{4\phi_x}]_{(NI,N)} & [K^L_{4\phi_y}]_{(NI,N)} \\ [K^L_{5w}]_{(NI,N)} & [K^L_{5\phi_x}]_{(NI,N)} & [K^L_{5\phi_y}]_{(NI,N)} \end{bmatrix}_{(5 \times NI, 5 \times N)}$$

$$[K]_G = \begin{bmatrix} [0] & [0] & [0] & [0] & [0] \\ [0] & [0] & [0] & [0] & [0] \\ [0] & [0] & [G^L_{3w_0}] & [0] & [0] \\ [0] & [0] & [0] & [0] & [0] \\ [0] & [0] & [0] & [0] & [0] \end{bmatrix}_{(5 \times NI, 5 \times N)} \quad (41)$$

$$[M] = -\omega^2 \sum_{i=1}^{NI} \sum_{j=1}^N \begin{bmatrix} [I_0 g_{(i,j)}] & [0] & \cdots \\ [0] & [I_0 g_{(i,j)}] & \cdots \\ [I_1 \frac{\partial g_{(i,j)}}{\partial x}] & [I_1 \frac{\partial g_{(i,j)}}{\partial y}] & \cdots \\ [I_3 g_{(i,j)}] & [0] & \cdots \\ [0] & [I_3 g_{(i,j)}] & \cdots \end{bmatrix} \quad (42)$$

Table 2: Mechanical properties of metallic and ceramic materials considered

Types of Functionally graded material		Properties		
		E (GPa)	ρ (kg/m ³)	ν
FGM-1	Metal (Al)	70	2702	0.3
	Ceramic (Al ₂ O ₃)	380	3800	0.3
FGM-2	Metal(Al)	70	2702	0.3
	Ceramic (ZrO ₂)	151	3000	0.3
FGM-3	Metal(Ti-6Al-4V)	105.7	4429	0.298
	Ceramic (aluminum oxide)	320.2	3750	0.26
FGM-4	Stainless steel	201.04	8166	0.3262
	Silicon nitride	348.43	2370	0.2400

Table 3: Convergence and validation study of the critical buckling parameter $\bar{N} = N_x d^2 / \pi^2 (Eh^3 / 12(1 - \nu^2))$ of an isotropic square plate under uniaxial loading (a=b=1)

Theory	'a/h'		Diff%	5	Avg. Diff%
	20	Diff%			
Rao and Srinivas, [73]	3.911	0	-	3.15	-
Uymaz and Aydogdu [74]	3.9499	0.99	1.84	3.3299	5.71
Present result (11×11 nodes)	3.973	1.59	2.22	3.317	5.30
Present result (13×13 nodes)	3.961	1.28	1.87	3.303	4.86
Present result (15×15 nodes)	3.955	1.13	1.68	3.296	4.63
Present result (17×17 nodes)	3.952	1.05	1.58	3.292	4.51

Where Avg. Diff % = Abs ((Present result- 3D result) / 3D result) × 100

$$\begin{bmatrix} -I_0 \frac{\partial g(i,j)}{\partial x} \\ -I_0 \frac{\partial g(i,j)}{\partial y} \\ I_0 g(i,j) - I_2 \frac{\partial^2 g(i,j)}{\partial x^2} - I_2 \frac{\partial^2 g(i,j)}{\partial y^2} \\ -I_4 \frac{\partial g(i,j)}{\partial x} \\ -I_4 \frac{\partial g(i,j)}{\partial y} \end{bmatrix} \begin{bmatrix} I_3 g(i,j) \\ [0] \\ I_4 \frac{\partial^2 g(i,j)}{\partial x^2} \\ I_5 g(i,j) \\ [0] \end{bmatrix} \begin{bmatrix} [0] \\ I_3 g(i,j) \\ I_4 \frac{\partial^2 g(i,j)}{\partial y^2} \\ [0] \\ I_5 g(i,j) \end{bmatrix} \quad (5 \times NI, 5 \times N)$$

4 Numerical results and discussions

To validate the accuracy, stability, and efficiency of the proposed theory with MQ-RBF based meshfree method, a significant number of numerical examples presented and discussed to verify the effectiveness of the proposed IHSDT considered firstly. Based on the convergence study, a 15×15 node used throughout the study. Then, the influence of some critical parameters, including boundary condition, material property distribution, elastic foundation, porosity volume fraction behaviors of the FGM plate discussed in detail. The equations solved by developing a computer program using MATLAB (2015) to obtain the results. Four

types of FG Plates used in this study and their material properties listed in Table 2. The equivalent shear (K_s) and Winkler (K_w) parameters considered throughout the study and which defined as $K_s = \frac{k_s 12(1-\nu^2)}{E_m h^3}$ & $K_w = \frac{k_w 12(1-\nu^2)}{E_m h^3}$

4.1 Convergence and Validation study

In order to check the accuracy, stability and validation purpose of the MQ-RBF method and proposed IHSDT for the analyses of the buckling and free vibration of the FGM plate, the present results compared with several results available in the open literature. The equations solved by developing a computer program using MATLAB (2015) to obtain the results.

For the accuracy and validation purpose of the present method and proposed theory, the first problems carried out for a simply supported isotropic plate. The numerical results for the uniaxial critical buckling parameter with various spans to thickness ratios were examined in Table 3. It can be observed that the present results are a good agreement with the analytical 3-D solution given by Rao and Srinivas, [73] with the deviation is up to 5%.and also good agreement with the 3D buckling solutions using the Ritz method by Uymaz and Aydogdu [74]. The present results

Table 4: Convergence and comparison study of non-dimensional frequency parameter $\bar{\omega} = \omega a^2 / h \sqrt{\rho_c / E_c}$ of five modes for FGM-1 square plate ($a/b = 1$, $a/h=10$, 'n'=1)

Modes	Number of nodes						Jin <i>et al.</i> [75]
	7×7	9×9	11×11	13×13	15×15	17×17	
Mode 1	4.511	4.465	4.446	4.436	4.430	4.427	4.428
Mode 2	10.865	10.720	10.648	10.610	10.588	10.575	10.630
Mode 3	10.865	10.720	10.648	10.610	10.588	10.575	10.630
Mode 4	15.623	15.694	15.763	15.821	15.868	15.907	16.200
Mode 5	15.623	15.694	15.763	15.821	15.868	15.907	16.200

Figure 4: Convergence study of with effect of span to thickness ratio of an FGM-4 plate under biaxial loading ($a=b=1$, $n=0.5$)

Figure 5: Convergence and validation study of non-dimensional frequency parameter of SS FGM-2 plate with different span to thickness ratio ($a=b=1$, $n=1$)

are closer to the Rao and Srinivas, [73] as compared to the Rao and Srinivas, [73].

Figure 4 represents the convergence and validation study of critical buckling parameter of SS FGM plate from thick to thin plate with 'n'=1 considered. It can be observed that convergence of present results with effect of power index is achieved after 13×13 nodes and validated with the 3-D and elasticity solution given by B. Uymaz, M. Aydogdu [74] with the deviation is up to 2%. It observed that present converged results are closer to 3-D results, which is better than 2-D results.

The convergence and validation study of the non-dimensional frequency parameter of five modes for SS FGM-1 square plate presented in Table 4. It can be seen that the present results obtained by proposed theory and method is convergence within 2% at 15×15 nodes and shows good agreement with the 3-D solutions by Jin *et al.*, [75].

Figure 5 shows the normalized fundamental frequency parameter from thick to thin FGM-2 plate. The non-dimensional frequency parameter $\bar{\omega} =$

$\omega h \sqrt{12(1-\nu^2)\rho_c a^2 b^2 / (\pi^4 h^2 E_c)}$ is taken. It can be observed that present results are achieved good convergence after 15×15 nodes and is predict near to the 3-D exact solution of Uymaz and Aydogdu [76] from thick to the thin plate. So, it can conclude that the present method and proposed theory is sufficiently accurate for its acceptability for buckling and free vibration analysis of FGM plate and the nodes distribution is 15×15 nodes can give the results with excellent accuracy and is intensity in its computing time and acceptable for the further analysis.

4.2 Comparison study

In this section, the present method results compared with several method results available in the open literature and proposed theory is compared with the 3D solution [76] and the other five variables HSDT theories carried out with the same method to determine their accuracy and applicabil-

Table 5: Convergence study of $\bar{N} = N_x a^2 / \pi^2 (Eh^3 / 12(1 - \nu^2))$ with effect of grading index of SS FGM-4 under uniaxial loading ($a=b=1$, $a/h=10$)

Theory	Grading index 'n'					Avg. Diff %
	0	0.5	1	5	10	
3D solution [77]	3.8099	3.2299	2.9899	2.6699	2.5699	
IHSDT	3.8161	3.2292	3.0083	2.6883	2.5751	0.34
HSDT-1	3.8164	3.2293	3.0087	2.6886	2.5756	0.35
HSDT-2	3.8167	3.2294	3.0088	2.6883	2.5755	0.35
HSDT-3	3.8402	3.2483	3.0278	2.7144	2.5991	1.09
HSDT-4	3.8167	3.2296	3.0088	2.6883	2.5755	0.35
HSDT-5	3.8208	3.2328	3.0121	2.6932	2.5796	0.47
HSDT-6	3.818	3.2308	3.0099	2.6892	2.5766	0.38
HSDT-7	3.8406	3.2485	3.0282	2.714	2.6005	1.10
HSDT-8	3.9082	3.304	3.0083	2.7764	2.6599	2.60
HSDT-9	3.8385	3.2467	3.0264	2.7124	2.5986	1.04

Where Avg. Diff % = Abs ((Present result- 3D result) / 3D result) $\times 100$

Table 6: Comparison of non-dimensional frequency parameter with 3-D solution and other HSDTs of SS FGM-2 plate with different span to thickness ratio ($a=b=1$, $a/h=10$)

Theories	Grading index					Average Diff %
	0	0.1	1	5	10	
3D Solution [76]	1.9339	1.8788	1.6583	1.5491	1.5066	
IHSDT	1.9307	1.8775	1.6587	1.5494	1.5079	0.073
HSDT-1	1.9307	1.8774	1.6588	1.5494	1.5079	0.075
HSDT-2	1.9308	1.8776	1.6589	1.5494	1.508	0.075
HSDT-3	1.9373	1.8836	1.6642	1.5576	1.5154	0.384
HSDT-4	1.932	1.8788	1.6599	1.5505	1.509	0.089
HSDT-5	1.932	1.8786	1.6597	1.551	1.5094	0.100
HSDT-6	1.9312	1.878	1.6591	1.5497	1.5084	0.078
HSDT-7	1.9422	1.8883	1.6685	1.5616	1.5199	0.648
HSDT-8	1.9561	1.9012	1.6799	1.5764	1.5341	1.446
HSDT-9	1.9368	1.8831	1.6638	1.557	1.5154	0.361
HSDT-10	1.9319	1.8787	1.6597	1.5502	1.509	0.085
HSDT-11	1.9311	1.8778	1.659	1.5499	1.5085	0.084
HSDT-12	1.9392	1.8854	1.6658	1.5596	1.5173	0.493
HSDT-13	1.9309	1.8769	1.6589	1.5494	1.5086	0.089
HSDT-14	1.9317	1.8785	1.6595	1.55	1.5088	0.081

ity. For checking the accuracy of the proposed theory, the numerical results for the uniaxial critical buckling parameter and non-dimensional frequency parameter are examined in Table 5 and Table 6, respectively. A SS FGM square plate is taken for investigation with different grading index. It can be observed from Table 5 and Table 6 that the proposed IHSDT is very close to the 3-D exact solution results and also give low average Diff % for the results obtained by other existing HSDT theories solved by the present method. So for further analysis, IHSDT is considered for investigation.

Comparison study of present results with the other HSDT and FSDT result carried out with the different methods is listed in Table 7. Fundamental frequency parameter of FGM-1 plate for different values of span to thickness ratio ($a/h = 5, 10$, and 20) is presented, with power index 'n' = $0, 0.5, 1, 4$, and 10 . It should be noted that the results reported by Hosseini-Hashemi *et al.*, [78] has carried out an exact solution, Nguyen-Xuan *et al.*, [79], Nguyen-Xuan *et al.*, [80], Hosseini-Hashemi *et al.*, [81], Zhao *et al.*, [82] and Tran *et al.*, [83] carried out FSDT with different values of shear correction factors in their study work and the results

Table 7: Comparison of fundamental frequency parameter $\bar{\omega} = \omega h \sqrt{\rho_c/E_c}$ for FGM-1 square plates ($a/b = 1$)

a/h	Theory	Grading index 'n'				
		0	0.5	1	4	10
5	Ref. [78]	0.2112	0.1805	0.1631	0.1397	0.1324
	Ref. [79]	0.2114	0.1805	0.163	0.1394	0.1323
	Ref. [80]	0.2127	0.1816	0.164	0.1403	0.1331
	Ref. [84]	0.2112	0.1806	0.1628	0.1375	0.13
	Ref. [81]	0.2112	0.1806	0.165	0.1371	0.1304
	Ref. [82]	0.2055	0.1757	0.1587	0.1356	0.1284
	Ref. [83]	0.214	0.1826	0.1649	0.1414	0.1343
	Ref. [83]	0.2113	0.1807	0.1631	0.1377	0.13
	IHSDT	0.2096	0.1796	0.1622	0.1369	0.129
10	Ref. [78]	0.0577	0.049	0.0442	0.0382	0.0366
	Ref. [79]	0.0578	0.0491	0.0443	0.0383	0.0366
	Ref. [80]	0.0582	0.0494	0.0445	0.0385	0.0368
	Ref. [84]	0.0577	0.049	0.0442	0.038	0.0364
	Ref. [81]	0.0578	0.0492	0.0443	0.0381	0.0364
	Ref. [82]	0.0567	0.0482	0.0435	0.0376	0.0359
	Ref. [83]	0.0579	0.0492	0.0443	0.0384	0.0367
	Ref. [83]	0.0577	0.049	0.0442	0.038	0.0363
	IHSDT	0.0577	0.04905	0.0443	0.03781	0.0364
20	Ref. [78]	0.0148	0.0125	0.0113	0.0098	0.0094
	Ref. [79]	0.0148	0.0126	0.0113	0.0098	0.0094
	Ref. [80]	0.0149	0.0127	0.0114	0.0099	0.0095
	Ref. [84]	0.0148	0.0125	0.0113	0.0098	0.0094
	Ref. [81]	0.0148	0.0128	0.0115	0.0101	0.0096
	Ref. [82]	0.0146	0.0124	0.0112	0.0097	0.0093
	Ref. [83]	0.0148	0.0126	0.0113	0.0098	0.0094
	Ref. [83]	0.0148	0.0125	0.0113	0.0098	0.0094
	IHSDT	0.01479	0.0125	0.0113	0.0098	0.0094

reported by Benachour *et al.*, [84], Tran *et al.*, [83] has carried out HSDT. It can be observed that present results are in good agreement with the other method result.

To verify the accuracy of the present method and theory for FGM plate on elastic foundation, buckling load parameter, and frequency parameter of the plates compared with those reported in the literature. Table 8 and Table 9 represent a comparison study on the effect of the elastic foundation for buckling load parameter and frequency parameter obtained by the present theory of SS FGM-1 plate, respectively. It can be noticed that the results of the present theory are in good agreement with those generated by TSDT, Thai and Kim, [85] and Thai and Choi, [86] for buckling load parameter and frequency parameter respectively. So, it can be concluded that IHSDT and method is not only accurate and stable but also efficient in critical buckling load parameter and frequency parameter of plates on elastic foundation.

4.3 Parameter studies

After verifying the accuracy and stability of the present method and proposed theory, parameter studies are carried out to examine the effects of grading index 'n', porosity distribution, porosity index span to thickness ratio, and foundation parameter on the buckling load and frequency parameter. After convergence and comparison studies of proposed theory and method, a 15×15 nodes and the proposed IHSDT is considered for throughout the parameter studies considering the computational cost and accuracy of the studies.

Table 10 and Table 11 show the influences of porosity distribution and porosity index on critical buckling load parameter and critical natural frequency parameter of square SS porous FGM plate with various 'n' = (0,1,2,5,8, and 10) respectively. The material property for Table 10 and Table 11 are FGM-4 and FGM-3 for a thick plate with

Table 8: Comparison for the non-dimensional critical buckling load parameter $\bar{N}_{cr} = N_{cr}a^2/Eh^3$ for the uniaxial SS FGM-1 plate on foundation ($a=b=1$, FGM 1, $P=0$)

(Kw,Ks)	a/h	Method	Grading index 'n'					
			0	0.5	1	2	5	10
(0,0)	5	Ref., [85]	16.021	10.625	8.224	6.343	5.053	4.480
		Present method	16.173	10.736	8.327	6.438	5.129	4.541
	10	Ref., [85]	18.579	12.123	9.339	7.263	6.035	5.452
		Present method	18.663	12.207	9.438	7.368	6.113	5.506
	20	Ref., [85]	19.353	12.567	9.667	7.537	6.344	5.766
		Present method	19.403	12.644	9.778	7.664	6.437	5.821
(10 ² ,10)	5	Ref., [85]	18.780	13.385	10.984	9.102	7.345	6.549
		Present method	18.937	13.498	11.088	9.197	7.446	6.629
	10	Ref., [85]	21.338	14.882	12.099	10.022	8.794	8.212
		Present method	21.427	14.970	12.201	10.130	8.875	8.268
	20	Ref., [85]	22.112	15.326	12.427	10.297	9.104	8.526
		Present method	22.166	15.407	12.541	10.426	9.199	8.584
(10 ³ ,10 ²)	5	Ref., [85]	33.421	27.094	24.153	21.686	18.784	17.643
		Present method	33.634	27.248	24.289	21.801	18.880	17.713
	10	Ref., [85]	40.648	31.461	27.432	24.347	22.36	21.452
		Present method	40.796	31.587	27.567	24.482	22.463	21.525
	20	Ref., [85]	43.388	33.049	28.611	25.325	23.432	22.525
		Present method	43.476	33.152	28.742	25.468	23.539	22.592

Figure 6: Effects of porosity index with various 'a/h' on critical buckling load parameter under biaxial loading of SS porous FGM-4 plate (SCPD, 'n'=2, a=b) **Figure 7:** Effects of porosity index with various 'a/h' on critical natural frequency parameter of SS porous FGM-3 plate (SCPD, 'n'=1, a=b)

$a/h = 10$, respectively. It can be observed from both the tables that grading index moves from the ceramic layer to the metal layer, the critical buckling load parameter and critical natural frequency parameter start decreasing. The increase in 'n' transforms the pure ceramic into a combination of ceramic and metals phases. The combination increases the stiffness of the pure and porous FGM plate. Fur-

ther, at $n=0$, the plate is completely ceramics and obtained the highest critical buckling load parameter and critical natural frequency parameter. Three types of porosity distribution compared according to the porosity location. It can be observed that porosity denser in centers in SCPD predicts low effects as compared to porosity denser in top and bottom in STPD and SBPD distributions respectively.

Table 9: Comparison of non-dimensional frequency parameter $\bar{\omega} = (\omega \sqrt{\rho_m/E_m})/h$ of the SS FGM-1 plate ($a=b=1$, FGM 1, $P=0$)

Kw,	Ks	a/h	Method	Grading index 'n'					
				0	0.5	1	2	5	10
0	0	5	Ref. [86]	10.376	8.876	8.012	7.231	6.668	6.388
			Present	10.295	8.819	7.967	7.192	6.616	6.327
		10	Ref. [86]	11.335	9.630	8.682	7.876	7.403	7.145
			Present	11.329	9.636	8.704	7.910	7.429	7.158
		20	Ref. [86]	11.631	9.859	8.886	8.075	7.639	7.392
			Present	11.623	9.870	8.920	8.128	7.681	7.412
0	100	5	Ref. [86]	15.187	14.643	14.382	14.236	14.305	14.376
			Present	15.036	14.507	14.246	14.091	14.332	14.405
		10	Ref. [86]	15.973	15.259	14.940	14.780	14.869	14.916
			Present	15.948	15.238	14.926	14.769	14.850	14.889
		20	Ref. [86]	16.226	15.455	15.118	14.955	15.061	15.105
			Present	16.206	15.445	15.120	14.966	15.063	15.095
100	0	5	Ref. [86]	10.672	9.256	8.452	7.741	7.253	7.018
			Present	10.587	9.193	8.400	7.693	7.190	6.944
		10	Ref. [86]	11.615	9.992	9.104	8.364	7.952	7.729
			Present	11.608	9.997	9.123	8.395	7.974	7.739
		20	Ref. [86]	11.906	10.216	9.303	8.558	8.179	7.964
			Present	11.899	10.227	9.335	8.608	8.218	7.983
100	100	5	Ref. [86]	15.390	14.876	14.631	14.500	14.584	14.664
			Present	15.238	14.737	14.492	14.127	14.352	14.425
		10	Ref. [86]	16.173	15.490	15.189	15.046	15.150	15.205
			Present	16.148	15.469	15.174	15.034	15.130	15.177
		20	Ref. [86]	16.425	15.685	15.366	15.221	15.341	15.393
			Present	16.405	15.676	15.369	15.232	15.344	15.383

Table 10: Effects of porosity distribution with various 'n' on critical buckling load parameter $\bar{N}_{cr} = N_{cr}a^2/E_ch^3$ of SS porous FGM-4 plate uniaxial loading ($a/h=10, a=b$)

Porosity distribution	Porosity index 'P'	Grading index 'n'					
		0	1	2	5	8	10
Pure FGM	0	5.772	4.550	4.300	4.066	3.949	3.895
	0.1	5.547	4.371	4.133	3.911	3.800	3.748
	0.2	5.321	4.190	3.965	3.755	3.649	3.599
	0.3	5.094	4.007	3.795	3.597	3.497	3.450
STPD	0.1	5.429	4.277	4.042	3.824	3.717	3.666
	0.2	5.074	3.995	3.774	3.574	3.475	3.428
	0.3	4.705	3.701	3.495	3.312	3.223	3.180
SBPD	0.1	5.429	4.282	4.048	3.826	3.715	3.663
	0.2	5.074	4.004	3.787	3.577	3.472	3.423
	0.3	4.705	3.716	3.516	3.319	3.219	3.173

It also noticed that by increasing the value of porosity index, critical buckling load parameter, and critical natural frequency parameter decreases.

Figure 6 and Figure 7 represent the influences of span to thickness ratio on critical buckling load parameter

and critical natural frequency parameter with $n=1$ for SS porous FGM plate. SCPD porosity distribution taken. It can be seen from both the tables that critical buckling load parameter and critical natural frequency parameter increase from thick to the thin plate for the porous FGM and the

Table 11: Effects of porosity distribution with various 'n' on critical natural frequency parameter $\omega = \left(\omega \sqrt{\rho_m / E_m} \right) / h$ of SS porous FGM-3 plate ($a/h=10, a=b$)

Porosity distribution	Porosity index 'P'	Grading index 'n'					
		0	1	2	5	8	10
Pure FGM	0	7.628	5.741	5.286	4.930	4.777	4.704
SCPD	0.1	7.599	5.713	5.264	4.915	4.764	4.693
	0.2	7.568	5.682	5.240	4.897	4.750	4.679
	0.3	7.533	5.647	5.212	4.877	4.732	4.663
	0.3	7.533	5.647	5.212	4.877	4.732	4.663
STPD	0.1	7.518	5.658	5.207	4.857	4.709	4.639
	0.2	7.391	5.561	5.114	4.771	4.629	4.561
	0.3	7.240	5.446	5.005	4.669	4.533	4.468
SBPD	0.1	7.518	5.657	5.213	4.863	4.710	4.637
	0.2	7.391	5.561	5.129	4.784	4.632	4.559
	0.3	7.240	5.449	5.031	4.692	4.539	4.467

Table 12: Effects of foundation and porosity index on critical buckling load parameter and critical natural frequency parameter of SS porous FGM-3 plate (SBPD, 'a/h'=10, a=b)

Ks	Kw	'P'	Grading index 'n'									
			$\bar{N}_{cr} = N_{cr} a^2 / E_c h^3$					$\omega = \left(\omega \sqrt{\rho_m / E_m} \right) / h$				
			0	1	2	5	10	0	1	2	5	10
0	0	0	10.19	6.30	5.49	4.90	4.51	7.63	5.74	5.29	4.93	4.70
		0.1	9.58	5.93	5.17	4.62	4.25	7.60	5.71	5.26	4.91	4.69
		0.3	8.30	5.14	4.50	4.02	3.68	7.53	5.65	5.21	4.88	4.66
0	10	0	10.28	6.39	5.58	4.99	4.61	7.66	5.78	5.33	4.98	4.75
		0.1	9.67	6.02	5.26	4.71	4.34	7.55	5.70	5.26	4.91	4.69
		0.3	8.40	5.24	4.60	4.11	3.78	7.28	5.50	5.08	4.75	4.52
0	100	0	11.12	7.23	6.42	5.83	5.44	7.97	6.15	5.72	5.38	5.17
		0.1	10.51	6.86	6.10	5.55	5.18	7.87	6.09	5.66	5.33	5.12
		0.3	9.23	5.43	5.43	6.07	4.61	7.64	5.92	5.53	5.21	5.00
10	0	0	12.02	8.13	7.32	6.73	6.34	8.28	6.52	6.10	5.78	5.58
		0.1	11.41	7.75	7.00	6.45	6.08	8.20	6.47	6.07	5.75	5.55
		0.3	10.13	6.97	6.33	5.85	5.51	8.00	6.34	5.97	5.66	5.46
100	0	0	26.24	20.62	19.39	18.44	17.87	12.75	11.34	11.00	10.73	10.58
		0.1	25.35	20.07	18.92	18.02	17.48	12.82	11.44	11.11	10.83	10.68
		0.3	23.49	18.92	17.94	17.16	16.67	12.96	11.63	11.32	11.06	10.91
100	100	0	26.47	20.85	19.62	18.67	18.10	12.96	11.56	11.22	10.94	10.79
		0.1	25.59	20.30	19.15	18.26	17.71	13.04	11.66	11.32	11.05	10.90
		0.3	23.72	19.15	18.17	17.39	16.90	13.18	11.86	11.55	11.28	13.18

entire porosity index follow the same trend. The effect of thick to the thin plate is negligible after $a/h=50$. Table 12 represents the influences of foundation parameters with a porosity index on the critical buckling load parameter and critical natural frequency parameter of SS porous FGM-3 plate. SBPD porosity distribution and span to thickness ratio considered. It can see that by increasing the value of Winkler and Pasternak parameters cause to increase in the

buckling load and natural frequency. The table shows also, Pasternak parameter of the foundation has a more significant effect than Winkler parameter on the buckling load and fundamental frequency of plate.

Table 13 represents the effect of porosity index on the non-dimensional buckling load parameter for eight modes. SCPD porosity distribution is taken with $n=1$ and $a/h=10$. It can be seen that by increasing the porosity index, non-

Table 13: Non-dimensional buckling load parameter $\bar{N} = Na^2/E_m h^3$ of eight modes for SS porous FGM-1 plate on foundation ($a=b=1$, FGM 1, $n=1$, $a/h=10$, SCPD)

Modes	Uniaxial loading			Biaxial loading			Pure shear loading		
	0	0.1	0.3	0	0.1	0.3	0	0.1	0.3
Mode 1	9.438	9.027	7.681	4.719	4.434	3.840	19.949	18.725	16.183
Mode 2	13.805	13.172	11.207	11.050	10.375	8.971	19.949	18.725	16.183
Mode 3	22.148	21.062	17.906	11.050	10.375	8.971	23.778	22.309	19.265
Mode 4	31.719	30.054	25.515	16.787	15.749	13.597	23.778	22.309	19.265
Mode 5	33.556	31.950	27.185	19.916	18.674	16.104	41.738	39.060	33.546
Mode 6	35.880	34.063	28.955	19.959	18.713	16.135	41.738	39.060	33.546
Mode 7	41.191	38.893	32.971	24.849	23.284	20.052	43.814	40.990	35.183
Mode 8	42.417	40.138	34.073	24.849	23.284	20.052	43.814	40.990	35.183

Table 14: Non-dimensional critical buckling load parameter $\bar{N}_{cr} = N_{cr} a^2/E_m h^3$ for different type of in-plane loading SS porous FGM-1 plate on foundation ($a=b=1$, FGM 2, $a/h=10$, SCPD)

n	P	Uniaxial	Uniaxial + Shear	Diff%	Biaxial	Biaxial + Shear	Diff%
0	0	7.416	6.238	15.88	3.708	3.522	5.00
	0.1	7.126	5.987	15.98	3.563	3.383	5.04
	0.3	6.541	5.478	16.25	3.270	3.102	5.14
1	0	5.204	4.382	15.80	2.602	2.473	4.97
	0.1	4.995	4.201	15.90	2.497	2.372	5.00
	0.3	4.572	3.833	16.15	2.286	2.169	5.10
2	0	4.755	3.996	15.97	2.377	2.258	5.03
	0.1	4.569	3.834	16.07	2.284	2.168	5.07
	0.3	4.191	3.506	16.35	2.095	1.987	5.18
5	0	4.384	3.672	16.22	2.192	2.079	5.13
	0.1	4.217	3.528	16.34	2.109	1.999	5.17
	0.3	3.880	3.234	16.65	1.940	1.837	5.29

Table 15: Effects of porosity index with various ' a/h ' on natural frequency parameter of four modes of SS porous FGM-2 plate (SCPD, ' $n'=1, a=b$)

Modes	$a/h=5$			$a/h=10$			$a/h=20$		
	P=0	P=0.1	P=0.3	P=0	P=0.1	P=0.3	P=0	P=0.1	P=0.3
Mode 1	6.05	6.01	5.90	6.64	6.61	6.55	6.81	6.79	6.73
Mode 2	10.88	10.70	10.29	15.82	15.73	15.50	16.83	16.76	16.61
Mode 3	10.88	10.70	10.29	15.84	15.74	15.51	16.83	16.76	16.61
Mode 4	12.74	12.59	12.21	22.03	21.66	20.83	26.68	26.56	26.29

dimensional buckling load parameter starts decreasing in all loading condition. By increasing the mode numbers, non-dimensional buckling load parameter also increase in all three loading conditions. It also noticed that pure shear loading is more dominating than other loadings. Table 14 represents the effect of loading with and without the shear effect of SS porous FGM-2 plate. SCPD porosity distribution is taken with $a/h=10$. It can be seen that Diff % of uniaxial loading with and without shear on the non-dimensional

critical buckling load parameter is more dominating as compare to the Diff % of biaxial loading with and without shear, this indicates that shear loading is more dominating with uniaxial loading as compared to the biaxial loading. Non-dimensional critical buckling load parameter starts decreasing with increasing porosity index in all the in-plane loadings. Table 15 represents the effects of porosity index on natural frequency parameter of four modes of SS porous FGM-2 plate. It can be seen that natural frequency

parameter starts decreasing by increasing the porosity index in all modes and by increasing the span to thickness ratios the natural frequency parameter increasing.

5 Concluding remarks

In this work, a new five variables inverse hyperbolic higher-order shear deformation theory (IHHSDT) developed for the buckling and free vibration of porous FGM plates resting on two-parameter elastic foundations. The new non-polynomial IHHSDT with five variables developed and satisfied zero transverse shear stress boundary condition at the top and bottom surfaces of the plate. Therefore, a shear correction factor is not required. The accuracy and efficacy of the present theory verified by comparing it with many existing results in the literature. It noticed that the developed theory is closer to the three-dimensional results and better than the results of the existing theories. The results obtained by the proposed theory and the present method can be summarized as follows:

- It has been shown that the present MQ-RBF based meshfree method can accurately predict buckling load and natural frequencies of FGM plate with and without resting on an elastic foundation.
- The buckling loads and natural frequencies of plate decrease with the increase of power index.
- The buckling loads and natural frequencies increase from thick to a thin plate
- The buckling loads and natural frequencies of plate decrease with the increase of porosity index.
- The shear loading is more dominating with uniaxial loading as compared to the biaxial loading for buckling loads
- Porosity distribution located at the center (SCPD) predict buckling loads and natural frequencies closer to the pure FGM plate as compared to the STPD and SBPD
- The buckling loads and natural frequencies increase by increasing the value of Winkler and Pasternak parameters.
- Pasternak elastic foundation has a more significant rule as compared to Winkler elastic foundation in increasing the buckling load and natural frequency.

The newly developed theory and the current method can recommended as more correct and efficient for analyzing the buckling and free vibration behavior of porous FGM plate. The reported results will serve as a benchmark numerical solution to validate other solution methodolo-

gies and theory will be useful to the research community working in this field and in FGM applications.

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