Editorial

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Presentation of the Special Issue on Recent Advances in PDE: Theory, Computations and Applications

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Abstract: This is an introduction to the first eight articles in this volume that contains the special issue on *Recent Advances in PDE: Theory, Computations and Applications*. These peer-reviewed articles address recent developments in the areas of convection-diffusion-reaction problems, stabilizability of control systems with application to Oseen problems, obstacle problems, multigrid methods for quad-curl problems and discontinuous Petrov–Galerkin methods for spectral approximations. Some of the contributors of these articles were plenary speakers of the conference organized in honor of the numerical analyst Professor Amiya Kumar Pani. The conference was organized to acknowledge his outstanding contribution for the growth of applied mathematics in India.

Keywords: Finite Difference, Finite Element, Oseen Problem, Convection-Diffusion-Reaction, Stabilizability, Obstacle Problems, Multigrid, Quad-Curl Problems, Discontinuous Petrov–Galerkin Methods

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Dedicated to Amiya K. Pani on the occasion of his 60th birthday and part of the special issue “Recent Advances in PDE: Theory, Computations and Applications” in his honour.

The interest in the mathematics of the finite element methods (FEM) started in the late 1970s in India with the visit of P. C. Das (IIT Kanpur) to Trieste, Italy and interactions of P. K. Bhattacharyya (IIT Delhi) with pioneers in FEM like P. Ciarlet, O. Pironneau and M. Bernadou during mutual visits. The lecture series organized in IISc-TIFR programme in the 1970s triggered the FEM contributions from India. The graduate students of Das and Bhattacharyya started working on FEM in the 1980s.

The Indo–French workshop held at the IISc-TIFR Centre inspired Professor Amiya Kumar Pani, who was then a graduate student of P. C. Das, to work in the area of FEM. Professor Amiya Kumar Pani, currently employed as an Institute Chair Professor in the Department of Mathematics, Indian Institute of Technology Bombay is a dynamic personality who has been working relentlessly for the development of numerical analysis in such a large scale in India; initially through his work and then with his graduate students and collaborators. The survey article [6] gives more details on Indian contributions to the FEM. Professor Amiya Kumar Pani is undoubtedly a specialist of the theory and analysis of partial differential equations and their numerical solutions by finite element methods applied to linear and nonlinear stationary and evolution problems. He has over 100 refereed publications in internationally leading journals and has an outstanding international and national network that has lead to several successful national and international conferences in addition to the cascade of summer schools to educate the next generation of Indian mathematicians in the field of computational partial differential equations. The DST project *National Programme on Differential*

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Equations: Theory, Computation and Applications from 2012 to 2017 based out of IIT Bombay with principal investigator as Professor Amiya Kumar Pani involved training of students at undergraduate, postgraduate, and graduate students in different parts of the country by national and international experts. The activities through this project played a major role in bringing together researchers of the country who motivated students to take up applied mathematics as a research and career option.

In honor of Professor Amiya Kumar Pani on the occasion of his 60th birthday, a conference *Recent Advances in PDE: Theory, Computations and Applications* was organized from June 8–10, 2017 at the IIT Bombay to acknowledge his outstanding contribution for the growth of applied mathematics in India. The conference brought together national and international experts in the area and paved a way for discussion on theoretical as well as computational aspects of PDEs with emphasis on challenging scientific and industrial applications.

This highly selective special issue component in CMAM contains eight original peer-reviewed research contributions [1–5, 7–9] with some of the plenary speakers of the conference as authors/co-authors. Based on the problems considered, the contributions are classified into five categories below, namely convection-diffusion-reaction problems, stabilizability of control systems with application to Oseen problems, obstacle problems, multigrid methods for quad-curl problems and discontinuous Petrov–Galerkin methods for spectral approximations.

Convection-Diffusion-Reaction Problems. The articles [2, 4, 8] include applications of finite difference and finite element methods to convection-diffusion problems and Oseen problem. The study of the linear Oseen equations are important from an application perspective as they appear as an auxiliary problem while solving the Navier–Stokes equations. The paper [8] discusses a second-order accurate finite difference method for a spatially periodic convection-diffusion problem. The time stepping method is based on the Strang splitting of the spatially semidiscrete solution, in which the diffusion part is approximated by the Crank–Nicolson method and the convection part, by the explicit forward Euler approximation on a shorter time interval. The contribution [2] deals with a priori error analysis for a stabilized nonconforming method for the stationary Oseen problem. The well-posed discrete weak formulation combines standard Galerkin method, edge patch-wise local projection stabilization and weakly imposed boundary condition using Nitsche’s technique. The paper [4] establishes quasi-optimal convergence rates of adaptive mixed FEM by using the lowest-order Raviart–Thomas elements, for stationary convection-diffusion-reaction problems.

Stabilizability of Control Systems. A characterization of stability of control systems for which the underlying semigroup is analytic, and the resolvent of its generator is compact is discussed in [7] from a theoretical perspective. When the stabilizability condition is satisfied, the system is also stabilizable by finite-dimensional controls. The application of this result to the stabilizability of the Oseen equations with mixed boundary conditions is discussed.

Obstacle Problems. While the paper [9] deals with positivity preserving gradient approximation with linear finite elements in a general setting with an application to elliptic obstacle problems, the paper [1] discusses a mixed finite element method for an obstacle problem with the p -Laplace differential operator for $p \in (1, \infty)$, where the obstacle condition is imposed via a Lagrange multiplier. In the discrete setting, the Lagrange multiplier basis forms a biorthogonal system with the standard finite element basis so that the variational inequality is realized in the pointwise form. A priori and a posteriori error estimates are discussed.

Multigrid Methods for Quad-Curl Problems. The paper [3] investigates multigrid methods for a quad-curl problem on graded meshes using a Hodge decomposition based approach. The exact solution is approximated by solving standard second-order elliptic problems and optimal error estimates are obtained on graded meshes. The uniform convergence of the multigrid algorithm for the resulting discrete problem is established.

FEAST for Reaction-Diffusion Operator. The paper [5] discusses FEAST for numerical approximations of the eigenspaces of reaction-diffusion equations. The resolvent of the reaction-diffusion operator is approximated by the discontinuous Petrov–Galerkin method and the discretization error is analyzed. The technique is applied to a practical problem of computation of fiber modes of an optical fiber.

The conference was one of the well-attended conferences organized in India and had 81 participants, out of which 56 gave contributed talks distributed in about three parallel sessions across three days. There were 22 plenary and invited speakers that included ten from abroad and twelve from India.

Given the spectrum of the challenging areas covered in this special issue, it is expected to evoke interest in many researchers, who work in the theoretical and numerical aspects of PDEs.

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