Erratum to Tropical mirror symmetry for elliptic curves

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By *Janko Böhm* at Kaiserslautern, *Kathrin Bringmann* at Köln, *Arne Buchholz* at Saarbrücken
and *Hannah Markwig* at Tübingen

We thank Elise Goujard and Martin Möller for pointing out and closing a gap in Theorem 3.2 of our work [1]:

The statement of the theorem is not accurate. The Feynman integrals in question are indeed quasimodular forms, however, they are not necessarily homogeneous. The following is the accurate formulation of the statement:

Theorem 3.2. *For all Feynman graphs Γ and orders Ω as in Definition 2.5, the function $I_{\Gamma,\Omega}$ is a quasimodular form of mixed weight. The highest appearing weight is $6g - 6$.*

In our work [1], the proof of Theorem 3.2 had relied on Proposition 3.3 whose proof contains a gap in line 7 on page 235.

Theorem 3.2 in its accurate form as stated above follows from [3, Corollary 8.4]. In [3], Goujard and Möller studied quasimodularity questions for graph sums like our Feynman integrals in a more general context. They also provide an example of a Feynman integral which is a quasimodular function of mixed weight in Section 5.4.

Interestingly, if we sum all our Feynman integrals – i.e., the sum over all eligible graphs Γ and all orders Ω , $\sum_{\Gamma} \frac{1}{|\text{Aut } \Gamma|} \sum_{\Omega} I_{\Gamma,\Omega}$, which, by [1, mirror symmetry Theorem 2.6] (see, for instance, [2, Theorem 3]) equals the generating function of covers of an elliptic curve – we get a function which is quasimodular of pure weight $6g - 6$ (see [4]): the lower order contributions cancel in the sum. In [1, Example 3.5], we have computed a sum

$$I_{\Gamma} = \sum_{\Omega} I_{\Gamma,\Omega}$$

of all Feynman integrals for one fixed graph Γ . It turns out that in this example, the lower order terms already cancel and I_{Γ} is quasimodular of pure weight $6g - 6$. It is an interesting open question whether the cancellation always happens already at the level of a single graph, or whether in general we have to sum over all graphs to observe the cancellation. It is computationally challenging to produce more examples.

References

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- [4] *M. Kaneko and D. Zagier*, A generalized Jacobi theta function and quasimodular forms, in: *The moduli space of curves*, *Progr. Math.* **129**, Birkhäuser, Boston (1995), 149–163.

Janko Böhm, Fachbereich Mathematik, Universität Kaiserslautern,
Postfach 3049, 67653 Kaiserslautern, Germany
e-mail: boehm@mathematik.uni-kl.de

Kathrin Bringmann, Mathematisches Institut, Universität zu Köln,
Weyertal 86-90, 50931 Köln, Germany
e-mail: kbringma@math.uni-koeln.de

Arne Buchholz, Cluster of Excellence M²CI, Fachrichtung Mathematik,
Universität des Saarlandes, Postfach 151150, 66041 Saarbrücken, Germany
e-mail: buchholz@math.uni-sb.de

Hannah Markwig, Fachbereich Mathematik, Eberhard Karls Universität Tübingen,
72074 Tübingen, Germany
e-mail: hannah@math.uni-tuebingen.de

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