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SYSTEMS OF ORDINARY DIFFERENTIAL-FUNCTIONAL INEQUALITIES

1. Introduction

Theorems presented in this paper concern the following systems of inequalities

$$\text{(1)}\qquad \frac{du^i}{dt} \le f^i(t,u,u(\cdot)), \qquad \frac{dv^i}{dt} > f^i(t,v,v(\cdot))$$

and

$$\text{(2)}\qquad \frac{du^i}{dt} \le f^i(t,u,u(\cdot)), \qquad \frac{dv^i}{dt} \ge f^i(t,v,v(\cdot)), \quad \text{for } i \in \{1,\dots,m\},\ t\in I.$$

The methods of our proofs are inspired by [4], [1] and [2]. From the two theorems concerning the inequality $u(t)\le v(t)$, $t\in I$, follow the uniqueness theorems of the solution to the problem

$$\text{(3)}\qquad \frac{du^i}{dt} = f^i(t,u,u(\cdot)), \quad i=1,\dots,m,$$

$$\text{(4)}\qquad u^i(t_0) = \mathring{u}_i$$

and the theorem on the m-th order ordinary differential equations.

2. Definitions and notations

Let $I=(0,\infty)$,

$I_\rho = [\rho,\infty)$, $\rho>0$.

Definition 1. Let $u\colon I\longmapsto R^m$ be a continuous function in I. We say that u satisfies the strong (weak) initial condition F with the constant h , if for every sequence $\{t_\nu\}\in I$, where t_ν is strictly decreasing to 0, we have

$$\limsup_{\nu\mapsto\infty} u^i(t_\nu) < h \quad (\le h), \qquad i = 1,\dots,m.$$

Notation 1. Let u satisfy the strong or weak initial condition F. We introduce : $M_u(t) = \max_i \sup_{\tau\in(0,t]} u^i(\tau)$.

Notation 2. By f^i, $i=1,\dots,m$ we denote functions defined on the set of arguments (t,u,z), where $t\in I$, $u\in R^m$, $z: I\mapsto R^m$.

3. Two auxiliary lemmas

Lemma 1. If $w:I\mapsto R^m$ is a continuous function in I satisfying the strong initial condition F with the constant h, then

(5) $\exists\, t^*\in I,\ \forall\, t\in(0,t^*),\ \ w(t)<h.$

Proof. Let us suppose, that (5) is not true. Then, there would exist a sequence $t_\nu\in I$ and an index j such that t_ν would be strictly decreasing to 0 and $w^j(t_\nu) \ge h$. Hence it follows, that $\limsup_{\nu\mapsto\infty} w^j(t_\nu) \ge h$ what contradicts the assumption, that w satisfies the strong initial condition F.

Lemma 2. Let $w:I\mapsto R$ belong to the class $C^1(I)$. We assume, that $\sup_I w(t) = H<\infty$. If w satisfies the weak initial condition F with the constant $h<H$, then

$$\forall\varepsilon>0\ \ \exists\, \tilde t \in I:\ w'(\tilde t) > -\varepsilon,\ H - \varepsilon < w(\tilde t) \le H.$$

Proof. First we will prove, that there exists $\rho_1> 0$, such that $\sup_{(0,\rho_1)} w(t) = H_1$, $H_1< H$. Suppose, that there does not exist such $\rho_1>0$, hence it results that for every $\rho>0$ $\sup_{(0,\rho)} w(t)=H$. We can construct the sequence $\{t_\nu\}\in I$ such that t_ν is strictly decreasing to 0 and besides this $\limsup_{\nu\mapsto\infty} w(t_\nu) =H$, what contradicts the assumed weak initial condition F. For this ρ_1 we put $0<\rho_0< \frac{\rho_1}{3}$, and we consider the interval $\tilde I=I_{\rho_1-2\rho_0}$. Obviously $\tilde I\subset I$, w is of the class $C^1(\tilde I)$ and

$w(t) \le H_1 < H$ for every $t \in \tilde{I} \setminus I_{\rho_1}$. We take an auxiliary function

$$\Phi(t) = \begin{cases} \exp \dfrac{-(t-\bar{t})^2}{\rho_0^2-(t-\bar{t})^2} & \text{for } |t-\bar{t}|<\rho_0 \\ 0 & \text{for } |t-\bar{t}| \ge \rho_0 \end{cases} \tag{6}$$

for arbitrary, fixed $\bar{t}$, and we introduce

$$K = \sup_{t \in R} \Phi'(t). \tag{7}$$

We put $\eta>0$ such that

$$\eta < \min \left(\frac{1}{2}(H-H_1), \frac{\varepsilon}{K+1} \right) \tag{8}$$

where $\varepsilon>0$ is an arbitrary constant.

By the definition of the least upper bound it follows, that there exists $\bar{t} \in \tilde{I}$ such that $w(\bar{t})>H-\eta>\frac{1}{2}(H+H_1)>H_1$. Since $\bar{t}>\rho_1$, so $\bar{t}-\rho_0>\rho_1-2\rho_0$. We put the above $\bar{t}$ in (6) and we construct a new function $\bar{w}(t) = w(t)+\eta\Phi(t)$ for $t \in \tilde{I}$. We have $\bar{w}(\bar{t}) = w(\bar{t})+\eta > H$, $\bar{w}(\bar{t}-\rho_0) = w(\bar{t}-\rho_0) \le H$, hence there exists a point $\tilde{t} \in (\bar{t}-\rho_0, \bar{t}]$ in which $\bar{w}$ attains maximum greater than H. At the point $\tilde{t}$ obviously we have $\bar{w}'(\tilde{t}) \ge 0, \bar{w}(\tilde{t}) > H$. In virtue of (7) and (8) we obtain $w(\tilde{t}) > H-\eta > H-\varepsilon$, $w'(\tilde{t}) \ge -\eta\Phi'(\tilde{t}) \ge$ $\ge -\eta K > -\varepsilon$ what completes the proof.

4. Strong inequalities

Assumption A_1. Let $u,v: I \longmapsto R^m$ belong to the class $C^1(I)$. We assume, that for every index i the inequalities (1) hold in I.

Theorem 1. Let us assume, that

1°. For every fixed i the function $f^i(t,u,z)$ is increasing with respect to $u^1,\dots,u^{i-1},u^{i+1},\dots,u^m,z$.

2°. The functions u and v are of the class $C^1(I)$ and satisfy the Assumption A_1.

3°. The difference u-v satisfies the strong initial condition

F with the constant h=0.

Under these assumptions we have $u(t)\le v(t)$, for $t\in I$.

The method of proof is analogous to that used in the paper [4].

Proof. Since u-v satisfies the strong initial condition F, therefore, by Lemma 1, the set $A=\{t^*\in(0,T): u(t)<v(t)$ for $t\in(0,t^*)\}$ is non-void. We denote by τ its least upper bound. The desired statement is obviously equivalent to the equality $\tau=T$. Let us suppose, that the contrary is true, i.e. $\tau<T$. By the continuity of u-v we have $u(t)\le v(t)$ for $0<t\le\tau$. By the continuity and by the definition of τ, there is an index j such that $u^j(\tau)=v^j(\tau)$. The function $u^j(t)-v^j(t)$ attains maximum in $(0,\tau]$, for $t=\tau$, therefore

$$\frac{d}{dt}(u^j-v^j)(\tau)\ge 0. \tag{9}$$

On the other hand, using successively Assumption A_1 and condition 1^o we obtain

$$\frac{du^j}{dt}(\tau) - \frac{dv^j}{dt}(\tau) < f^j(\tau,u,u(\cdot))-f^j(\tau,v,v(\cdot)) < 0$$

what contradicts (9). This completes the proof of Theorem 1.

Remark 1. If f^i, for i=1,...,m do not depend on the last argument, then the V^+ condition (introduced by J. Szarski in the monograph [3]) is sufficient for our consideration.

Example 1. Let us consider the functions

$$f(t,w) = -\frac{t}{w},\quad u(t) = \frac{1}{e^t-1},\quad v(t) = \frac{1}{t} \quad \text{for } t\in(0,\infty),\ w\in R.$$

Simple computation shows that the functions f,u and v satisfy all the assumptions of Theorem 1 in $I=(0,\infty)$. Hence we have $\frac{1}{e^t-1} < \frac{1}{t}$ for $t\in(0,\infty)$. (Notice that the difference of the functions e^t-1-t satisfies the weak initial condition F with the constant h=0, whereas the difference of the reciplocals $\frac{1}{e^t-1} - \frac{1}{t}$ satisfies the strong initial condition F with h=0).

5. Weak inequalities

In order to simplify the formulation of our theorem we introduce:

Assumption A_2. Let $u,v: I \longmapsto R^m$ belong to the class $C^1(I)$. We denote $E_i = \{ t \in I: u^i(t) > v^i(t) \}$. We assume, that for every index i the inequalities (2) hold in E_i.

Assumption A_3. There exists $L>0$, such that for arbitrary pair of functions $z,\bar{z}: I \longmapsto R^m$, $z,\bar{z} \in C(I)$, such that $z-\bar{z}$ satisfies the weak initial condition F, we have

$$\operatorname{sgn}(z^i-\bar{z}^i)(t)\left[f^i(t,z,z(\cdot))-f^i(t,\bar{z},\bar{z}(\cdot))\right] \le$$

$$\le L\left(\max_j (z^j-\bar{z}^j)(t)+M_{z-\bar{z}}(t)\right)$$

for $t \in E_i$, $i=1,\dots,m$.

Theorem 2. Let the Assumption A_3 hold.

Assume that

1^o. u and v are of the class $C^1(I)$ and satisfy the Assumption A_2.

2^o. u-v is bounded from above in I and satisfies the weak initial condition F with the constant h=0.

Under these assumptions we have $u \le v$ in I.

The proof of the Theorem 2 is similar to that given in the paper [1].

Proof. First we will prove, that inequality $u \le v$ holds true in $(0,T_0]$, where $0<T_0 \le \frac{1}{8L+1}$. Let us suppose, that the contrary is true, then there exists a non-empty subset $E_i \subset (0,T_0]$, $E_i = \{ t: u^i(t) > v^i(t) \}$ for certain indices i. We put

$$H = \max_i \sup_{(0,T_0]} (u^i(t)-v^i(t)).$$

Obviously $H>0$, and there exists an index i_o such that

$$H = \sup_{(0,T_0]} (u^{i_o}(t)-v^{i_o}(t)).$$

We take an auxiliary function

$$w(t) = u^{i_o}(t) - v^{i_o}(t) - \frac{\lambda}{T_0}t, \qquad t\in(0,T_0]$$

where λ is an arbitrary number from the interval

(10) $$\frac{H}{4} < \lambda < \frac{H}{3}.$$

Obviously we have

1^o $w(t) < H$ in $(0,T_0]$.

2^o w satisfies the weak initial condition F with the constant $h=0$.

3^o if we denote $\omega = \sup_{(0,T_0]} w(t)$ then

$$H \geq \omega \geq \sup_{(0,T_0]} u^{i_o}(t) - v^{i_o}(t) - \lambda = H-\lambda > 0.$$

The function w satisfies all the assumptions of Lemma 2, hence for the above λ there exists $\bar{t}\in(0,T_0]$ in which

(11) $$w'(\bar{t}) > -\lambda, \quad w(\bar{t}) > \omega-\lambda \geq H-2\lambda > 0.$$

On the other hand, applying Assumptions A_2 and A_3 we obtain

$$w'(\bar{t}) + \frac{\lambda}{T_0} = (u^{i_o} - v^{i_o})'(\bar{t}) \leq f^{i_o}(\bar{t},u,u(\cdot)) - f^{i_o}(\bar{t},v,v(\cdot)) \leq$$

$$\leq L\left(\max_j (u^j - v^j)(\bar{t}) + M_{u-v}(\bar{t})\right).$$

Since $M_{u-v}(\bar{t}) = \max_i \sup_{(0,\bar{t}]} (u^i - v^i) = H$, therefore

$$w'(\bar{t}) \leq -\frac{\lambda}{T_0} + 2LH.$$

From the inequality (10), it results $H < 4\lambda$. For T_0, which we have choosen at the beginning, we have $w'(\bar{t}) \leq -\lambda$, what contradicts the first one of the inequalities (11). This completes the proof of $u \leq v$ in $(0,T_0]$. Assuming that for certain integer k, the inequality $u \leq v$ holds in the interval $((k-1)T_0, kT_0]$ and repeating the above reasoning we can prove, that this inequality holds also in $(kT_0,(k+1)T_0]$.

Example 2. Let us consider the functions $f(t,w)=1$, $u(t)=\sin t$, $v(t)=t$, for $t\in(0,\infty)$, $w\in R$. The functions f, u, and v satisfy all the assumptions of the Theorem 2, therefore inequality $\sin t \le t$ holds true for $t \ge 0$.

The next corollary is an immediate consequence of Theorem 2.

Theorem 3. Uniqueness criterion.

Let the right hand sides of the system (3) satisfy the Assumption A_3. If the function $u: I \longmapsto R^m$ belonging to $C^1(I)$ is a solution to (3)-(4), bounded in I then u is the unique solution.

Proof is typical (comp. [1], the proof of Theorem 2) therefore we omit it.

6. Arbitrary unbounded solutions

Let $I_1 = [t_0, \infty)$.
Keeping in virtue the Assumption A_2 and the definition of the set E_i (with I replaced by I_1) given in § 5 we introduce now.

Definition 2. We say that the function $u: I_1 \longrightarrow R^m$ belongs to the one sided class $\tilde{M}(H_0)$ (two sided class $M(H_0)$) if there exist a constant $M>0$ and a function H_0: $I_1 \longrightarrow R$, $H_0(t)>0$ in I_1, such that: $u^i(t) \le MH_0(t)$ ($|u^i(t)| \le MH_0(t)$), $t \in I_1$, $i=1,\dots,m$.

Assumption B_1. For every index i there exist functions
c^i: $I_1 \longmapsto R_+$
K^i: $I_1 \times C(I_1) \longmapsto R$
such that for every $z, \bar{z} : I_1 \longmapsto R^m$, $z, \bar{z} \in C(I_1)$ and for every $P \in C(I_1)$ for which $K^i(t, P(\cdot))$ is defined in I_1, the following inequality

$$\left[f^i(t, Pz, P(\cdot)z(\cdot)) - f^i(t, P\bar{z}, P(\cdot)\bar{z}(\cdot))\right] \operatorname{sgn}(z-\bar{z})(t) \le$$
$$\le c^i(t) \sum_l P^l(z^l - \bar{z}^l)(t) + K^i(t, P(\cdot))\, M_{z-\bar{z}}(t)$$

holds in the set E_i.

Assumption B_2. $\exists\, h>0\ \forall \tilde{t} \ge t_0\ \exists\, H: [\tilde{t}, \tilde{t}+h] \longmapsto R^m$, $H(t)>0$ for every $t \in [\tilde{t}, \tilde{t}+h]$, $H \in C^1([\tilde{t}, \tilde{t}+h])$.

We assume that $K^i(t, H(\cdot))$ is defined in $[\tilde{t}, \tilde{t}+h]$, $\frac{H_0}{H^i}$ is bonded from above in I_1 and moreover that

(12) $\frac{d}{dt}H^i(t) > c^i(t)\sum_{l=1}^{m} H^l(t)+K^i(t,H(\cdot))$ in $(\tilde{t},\tilde{t}+h)$, $i=1,\dots,m$.

Theorem 4. Let $u,v:I_1 \longmapsto R^m$, $u-v\in\tilde{M}(H_0)$, $u,v\in C^1(I_1)$. We assume, that for these functions both the Assumptions A_2 and the Assumption B_1 hold, and that there exists a function H, for which all the conditions of the Assumption B_2 hold. If $u(t_0)\le v(t_0)$, then $u(t)\le v(t)$ for $t\in I_1$.

Proof. First we put $\tilde{t}=t_0$ and we introduce new functions

$$\tilde{u}^i(t) = u^i(t)[H^i(t)]^{-1} \quad , \quad \tilde{v}^i(t) = v^i(t)[H^i(t)]^{-1}$$

The functions $\tilde{u}$ and $\tilde{v}$ belong to the class $C^1(I_1)$. We prove first that $\tilde{u}\le\tilde{v}$ in $[t_0,t_0+h]$, where h is that from the Assumption B_2. Let us suppose, that the contrary is true, then

$$\max_l \left\{ \sup_{[t_0,t_0+h]} (\tilde{u}^l(t)-\tilde{v}^l(t)) \right\} = p>0.$$

By continuity of the functions $\tilde{u}$ and $\tilde{v}$ there exist: a point $\bar{t}\in(t_0,t_0+h]$ and an index j such that

(13) $$\tilde{u}^j(\bar{t})-\tilde{v}^j(\bar{t}) = \max_l\left\{ \sup_{[t_0,t_0+h]} (\tilde{u}^l(t)-\tilde{v}^l(t)) \right\} = p>0.$$

The function $\tilde{u}^j-\tilde{v}^j$ attains maximum in $(t_0,\bar{t}]$ for $t=\bar{t}$, therefore we have

(14) $$\frac{d}{dt}(\tilde{u}^j-\tilde{v}^j)(\bar{t}) \ge 0.$$

On the other hand, the point $\bar{t}\in E_j$ and we have

(15) $$\left(H^j\frac{d}{dt}(\tilde{u}^j-\tilde{v}^j)+(\tilde{u}^j-\tilde{v}^j)\frac{d}{dt}H^j\right)(\bar{t}) = \frac{d}{dt}(u^j-v^j)(\bar{t}) \le$$
$$\le f^j(\bar{t},\tilde{u}H,\tilde{u}(\cdot)H(\cdot)) - f^j(\bar{t},\tilde{v}H,\tilde{v}(\cdot)H(\cdot)).$$

To the right side of the last inequality we will apply the Assumption B_1. Then

(16) $$f^j(\bar{t},\tilde{u}H,\tilde{u}(\cdot)H(\cdot)) - f^j(\bar{t},\tilde{v}H,\tilde{v}(\cdot)H(\cdot)) \le$$
$$\le c^j(\bar{t})\sum_{l=1}^{m}(\tilde{u}^l-\tilde{v}^l)(\bar{t})H^l(\bar{t}) + K^j(\bar{t},H(\cdot))\, M_{\tilde{u}-\tilde{v}}(\bar{t}).$$

We notice that $M_{\tilde{u}-\tilde{v}}(\bar{t}) = p$.

From (15) and (16) , and by the Assumption B_2 we obtain

$$\frac{d}{dt}(\tilde{u}^j-\tilde{v}^j)(\bar{t})H^j(\bar{t}) \le p\left[c^j(\bar{t})\sum_{l=1}^{m}H^l(\bar{t}) + K^j(\bar{t},H(\cdot)) - \frac{d}{dt}H^j(\bar{t})\right]<0$$

hence

$$\frac{d}{dt}(\tilde{u}^j-\tilde{v}^j)(\bar{t}) < 0$$

what contradicts the inequality (14). This completes the proof in $[t_0,t_0+h]$. Putting Successively $\bar{t} = t_0+h, t_0+2h,\dots,$ by inductive reasoning we obtain that $u(t) \le v(t)$ in I_1.

Remark 2. As a conclusion of the Theorem 4 we obtain the .ollowing uniqueness criterion for the solution of the problem (3)-(4).

Theorem 5. Let $u,v : I_1 \longmapsto R^m$ belong to the class $C^1(I_1)$ and both of them are solutions of the problem (3)-(4). Let $u-v\in M(H_0)$. If the Assumptions B_1 and B_2 hold, then the solutions u and v are identical in I_1.

The simple proof we omit.

7. Examples of the construction of the function H

Let us suppose, that all the functions f^i do not depend on the last argument. We assume, that the functions $c^i(t)$, defined in the Assumption B_1, satisfy, for every index i, the following condition

(17) $$c^i(t) = c(t) \le L, \quad \text{where } L>0$$

We introduce the following function

$$H^i(t) = H(t) = \exp(Lm+1)t, \quad i=1,\dots,m.$$

Now the system (12) is reduced to the single inequality

$$c(t)\sum_{l=1}^{m}H^l(t) - \frac{d}{dt}H(t) \le LmH(t)-(Lm+1)H(t) = -H(t) < 0 \quad \text{in } I_1.$$

Hence we can formulate the following.

Theorem 6. We assume that f^i, for i=1,...,m do not depend on the last argument. Let $u,v:I_1 \longmapsto R^m$, $u-v\in\tilde{M}(H_0)$, $u,v\in C^1(I_1)$, for which both the Assumption A_2 and the Assumption B_1, with coefficients satisfying (17) hold. If $\frac{H_0(t)}{\exp(Lm+1)t}$ is bounded from above in I_1 and $u(t_0)\le v(t_0)$ then $u(t)\le v(t)$, for $t\in I_1$.

Proof. The function $H(t) = \exp(Lm+1)t$ satisfies all the conditions of Assumption B_2, therefore all the assumptions of Theorem 4 hold. Hence $u(t)\le v(t)$ in I_1.

Let $t_0=0$. In a similar way we obtain.

Theorem 7. Let us suppose, that

$$f^i(t,u,u(\cdot)) = \tilde{f}^i(t,u) + \int_0^t u^i(s)ds \ , \quad \text{for } i=1,\dots,m.$$

We assume, that the functions $\tilde{f}^i$ fulfil the Assumption B_1 with coefficients satisfying (17). Let $u,v:I_1 \longmapsto R^m$, $u,v\in C^1(I_1)$ satisfy the Assumption A_2 and $u-v\in\tilde{M}(H_0)$. If $\frac{H_0(t)}{\mathrm{sh}\alpha t}$, $\alpha > \frac{Lm+\sqrt{L^2m^2+4}}{2}$ is bounded from above in I_1 and $u(0)\le v(0)$ then $u(t)\le v(t)$ for $t\in I_1$.

Proof. Simple computation shows that the functions f^i satify the Assumption B_1 with $c^i(t) = c(t) = L$, $K^i(t,P(\cdot)) = \int_0^t P^i(s)ds$ and the function $H^i(t) = H(t) = \mathrm{sh}\alpha t$, where $\alpha > \frac{Lm+\sqrt{L^2m^2+4}}{2}$ satisfies all the conditions of Assumption B_2. Therefore, all the assumptions of Theorem 4 hold and hence we have $u(t)\le v(t)$, $t\in I_1$.

8. Theorem on m-th order ordinary differential equations

Consider an ordinary differential equation of order $m\ge 2$.

$$u^{(m)}(t) = f(t,u(t),u'(t),\dots,u^{(m-1)}(t)) \tag{18}$$

with the right-hand member $f(t,u_0,\dots,u_{m-1})$ defined in the domain D of the space $(t,u_0,u_1,\dots,u_{m-1})$. Let $(t_0,u_0) = (t_0,\mathring{u}_0,\mathring{u}_1,\dots,\mathring{u}_{m-1})$. and we introduce Cauchy initial conditions

(19) $$u^{(j)}(t_0) = \mathring{u}_j \qquad (j=0,\ldots,m-1).$$

It is a well-known fact that the Cauchy problem for equation (18) with initial conditions (19) is equivalent to the Cauchy problem for the system of first order differential equations

(20) $$\begin{aligned} \frac{du_i}{dt} &= u_{i+1} \qquad (i=0,1,\ldots,m-2) \\ \frac{du_{m-1}}{dt} &= f(t,u_0,u_1,\ldots,u_{m-1}) \end{aligned}$$

with initial values

(21) $$u_j(t_0) = \mathring{u}_j \qquad (j=0,\ldots,m-1).$$

Theorem 8. Let the right-hand member of equation (18) satisfy the following condition:
There exists $L>0$, such that for arbitrary pair of functions $z,\bar{z}:I\longmapsto R^m$, such that $(t,z(t))\in D$ and $(t,\bar{z}(t))\in D$ we have

$$\Big[f(t,z(t))-f(t,\bar{z}(t))\Big]\operatorname{sgn}(z^m-\bar{z}^m)(t)\le L\max_j(z^j-\bar{z}^j)(t) \quad \text{for } t\in E_m.$$

If the function u is a solution to (18)-(19), bounded in I then u is the unique solution.

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