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Degree of (L, M) -fuzzy semi-precontinuous and (L, M) -fuzzy semi-preirresolute functions

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Abstract: The aim of this paper is to present the degree of semi-preopenness, semi-precontinuity, and semi-preirresoluteness for functions in (L, M) -fuzzy pretopology with the help of implication operation and (L, M) -fuzzy semi-preopen operator introduced by [Ghareeb A., *L*-fuzzy semi-preopen operator in *L*-fuzzy topological spaces, Neural Comput. & Appl., 2012, 21, 87–92]. Further, we generalize the properties of semi-preopenness, semi-precontinuity and semi-preirresoluteness to (L, M) -fuzzy pretopological setting relying on graded concepts. Also, we discuss their relationships with the corresponding degrees of semi-precompactness, semi-preconnectedness and semi-preseparation axioms.

Keywords: (L, M) -fuzzy pretopology, (L, M) -fuzzy semi-preopen operator, (L, M) -fuzzy semi-precontinuous function, (L, M) -fuzzy semi-preirresolute function

MSC: 03E72, 54A40, 54C20

1 Introduction

Study of fuzzy topological spaces was pioneered by Chang [2] in 1968. Chang's fuzzy topology is a crisp subset of the family of I^X (the collection of all fuzzy sets defined on a non-empty set X) that fulfills the same conditions as in topology. This notion was extended to the L -fuzzy setting by Goguen [3], which currently holds the name " L -topological spaces". However, in a completely different direction, Höhle [4] presented the notion of a fuzzy topology being viewed as an L -subset of a powerset 2^X . Ying [5] studied Höhle's topology from a logical point of view and gave it the name "fuzzifying topology". Šostak [6] and Kubiak [7] independently extended Höhle's fuzzy topology to I -subsets of I^X . Subsequently, Ghanim et al. [8] introduced the degree of supra openness of a fuzzy set on X which is now called I -fuzzy pretopological spaces.

The development of fuzzy topology was accompanied with many topological properties that were endowed with degrees, like connectedness, separability, compactness, and filter convergence. In 1988, Šostak [9, 10] introduced the degree of connectedness via the level I -topological spaces. Yue and Fang [11] introduced the connectivity for the whole L -fuzzy topological space. In [12], Pang defined the openness, closeness, and continuity degrees of functions in L -fuzzifying topology and generalized some results in general topology to L -fuzzifying topology. Recently, Liang and Shi [13] introduced the degrees to which a function is continuous, open or closed by using implication operation, and extended most of their elementary properties in topological spaces to (L, M) -fuzzy topological spaces by means of graded concepts. In 2009, Shi [14] presented

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separatedness and connectedness degrees of L -subset based on L -fuzzy closure operator. Recently, Shi [15] used L -fuzzy semiopen and L -fuzzy preopen operators in L -fuzzy pretopological spaces as a tool to measure the semiopenness and preopenness degrees of an L -subset, respectively. Based on Shi's operator, the notion of semicompactness is introduced and characterized in [16, 17]. Moreover, the preconnectedness degree presented by Ghareeb [18], was defined using L -fuzzy preopen operator. In 2012, Ghareeb [1] used L -fuzzy preopen operator to present a new operator to measure the degree of semi-preopenness of an L -subset. The notions of semi-precompactness degree, semi-preseparatedness degree and semi-preconnectedness degree of an L -subset are discussed in L -fuzzy pretopological spaces via L -fuzzy semi-preopen operator and implication operator in a way that doesn't depend on the basis lattice structure [19]. Recently, Ghareeb and Al-Omeri [20] introduced new degrees for functions in (L, M) -fuzzy topological spaces based on (L, M) -fuzzy semiopen and (L, M) -fuzzy preopen operators.

The main purpose of this paper is to present the degree of semi-preopenness, semi-precontinuity and semi-preirresoluteness for functions in (L, M) -fuzzy pretopological spaces relied on (L, M) -fuzzy semi-preopen operator [1] and implication operation. The characteristic properties of the new notions will be discussed through graded concepts. Furthermore, we study the relationship with semi-precompactness, semi-preconnectedness, and some semipre-separation axioms degree in (L, M) -fuzzy pretopological spaces.

2 Preliminaries

Throughout this paper, unless otherwise stated, both L and M refer to completely distributive De Morgan algebras and X is a nonempty set. The smallest and greatest members in L and M are denoted by $0_L, 1_L$ and $0_M, 1_M$, respectively. For any $a, b \in L$, we say that the element a is wedge below b [21], in symbols $a \triangleleft b$, if for all $\mathcal{A} \subseteq L$, $\bigvee \mathcal{A} \geq b$ it follows that $c \geq a$ for $c \in \mathcal{A}$. The complete lattice L is said to be a completely distributive lattice iff $b = \bigvee_{a \triangleleft b} a$ for all $b \in L$. The element $b \in L$ is called co-prime if $a \leq b \vee c$ it follows that $a \leq b$ or $a \leq c$.

The set of all non-zero co-prime members in L is denoted by $P(L)$. By L^X , we denote the family of all L -subsets defined on X . The smallest and the greatest L -subsets in L^X are denoted by 0_{L^X} and 1_{L^X} , respectively. Further, $2^{\mathcal{B}}$ refers to the family of all finite sub-families of $\mathcal{B}$ of L^X . It is clear that L^X forms a completely distributive De Morgan algebra. Moreover, $\{x_a\}_{a \in P(L)}$ is a family of non-zero co-primes in L^X . The implication operation $\mapsto: M \times M \rightarrow M$ is defined on any completely distributive De Morgan algebra M by $a \mapsto b = \bigvee_{a \wedge c \leq b} c$. Moreover, the operation " $\leftrightarrow$ " is defined also on M based on " $\mapsto$ " by

$$a \leftrightarrow b = (a \mapsto b) \wedge (b \mapsto a).$$

An L -fuzzy inclusion [22, 23] on X is a mapping $\widetilde{\subset}: L^X \times L^X \rightarrow L$ defined by the equality

$$\widetilde{\subset}(G_1, G_2) = \bigwedge_{x \in X} \left\{ G_1'(x) \vee G_2(x) \right\}.$$

It is customary to denote the L -fuzzy inclusion $\widetilde{\subset}(G_1, G_2)$ by the symbol $[G_1 \widetilde{\subset} G_2]$.

Lemma 2.1. [24] For any completely distributive lattice $(M, \vee, \wedge)$ and an implication operation $\mapsto$ defined on M , we have:

- (1) $(a \mapsto b) \geq c \iff a \wedge c \leq b$.
- (2) $a \leq b \iff a \mapsto b = 1_M$.
- (3) $a \mapsto (b \mapsto c) = (a \wedge b) \mapsto c$.
- (4) $(c \mapsto a) \wedge (a \mapsto b) \leq c \mapsto b$.
- (5) $c \mapsto a \leq (a \mapsto b) \mapsto (c \mapsto b)$.
- (6) $a \mapsto \bigwedge_{i \in I} a_i = \bigwedge_{i \in I} (a \mapsto a_i)$, hence $a \mapsto b \leq a \mapsto c$ whenever $b \leq c$.

$$(7) \bigvee_{i \in I} a_i \mapsto b = \bigwedge_{i \in I} (a_i \mapsto b), \text{ hence } a \mapsto c \geq b \mapsto c \text{ whenever } a \leq b.$$

Where $a, b, c \in M$, $\{a_i\}_{i \in I}$, and $\{b_i\}_{i \in I} \subseteq M$.

Lemma 2.2. [25] Let $f : X \rightarrow Y$ be a mapping. Then for each $\mathcal{H} \subseteq L^Y$, we have

$$\bigwedge_{y \in Y} \left\{ f_L^{\rightarrow}(G)'(y) \vee \bigvee_{H \in \mathcal{H}} H(y) \right\} = \bigwedge_{x \in X} \left\{ G'(x) \vee \bigvee_{H \in \mathcal{H}} f_L^{\leftarrow}(H)(x) \right\}.$$

Definition 2.3. For any non-empty set X , a function $\mathcal{T} : L^X \rightarrow M$ which satisfies the following statements:

$$\begin{aligned} (\mathcal{P}1) \quad & \mathcal{T}(\underline{1}_L) = \mathcal{T}(\underline{0}_L) = 1_M, \\ (\mathcal{P}2) \quad & \mathcal{T}\left(\bigvee_{i \in I} G_i\right) \geq \bigwedge_{i \in I} \mathcal{T}(G_i) \text{ for all } \{G_i\}_{i \in I} \subseteq L^X, \end{aligned}$$

is called an (L, M) -fuzzy pretopology on X [8]. The pair $(X, \mathcal{T})$ is called (L, M) -fuzzy pretopological space $((L, M)$ -fpts, for short). An (L, M) -fuzzy pretopology $\mathcal{T}$ is said to be (L, M) -fuzzy topology $((L, M)$ -ft, for short) on X [6, 7, 24, 26] if it satisfies the following additional statement:

$$(\mathcal{O}) \quad \mathcal{T}(G_1 \wedge G_2) \geq \mathcal{T}(G_1) \wedge \mathcal{T}(G_2) \text{ for all } G_1, G_2 \in L^X.$$

The pair $(X, \mathcal{T})$ is called (L, M) -fuzzy topological space $((L, M)$ -fts, for short). The gradation of openness and closeness of an L -subset G is given by $\mathcal{T}(G)$ and $\mathcal{T}^*(G)$ respectively, where $\mathcal{T}^*(G) = \mathcal{T}(G')$. A mapping $f : (X, \mathcal{T}_1) \rightarrow (Y, \mathcal{T}_2)$ is called an (L, M) -fuzzy continuous iff $\mathcal{T}_1(f_L^{\leftarrow}(H)) \geq \mathcal{T}_2(H)$ for all $H \in L^Y$.

Definition 2.4. [27] Let $(X, \mathcal{T})$ be an (L, M) -pfts and $G \in L^X$. An (L, M) -fuzzy preopen operator $\mathcal{P} : L^X \rightarrow M$ is given by

$$\mathcal{P}(G) = \bigwedge_{x_\alpha \triangleleft G} \bigvee_{x_\alpha \triangleleft G_1} \left\{ \mathcal{T}(G_1) \wedge \bigwedge_{y_\alpha \triangleleft G_1} \bigwedge_{y_\alpha \triangleleft G_2 \geq G} (\mathcal{T}(G_2'))' \right\}.$$

The value $\mathcal{P}(G)$ interprets the degree to which G is preopen and $\mathcal{P}^*(G) = \mathcal{P}(G')$ interprets the degree to which G is a preclosed L -subset.

Definition 2.5. [1] Let $(X, \mathcal{T})$ be an (L, M) -pfts and $G \in L^X$. An (L, M) -fuzzy semi-preopen operator $\mathcal{T}_{sp} : L^X \rightarrow M$ is given by

$$\mathcal{T}_{sp}(G) = \bigvee_{G_1 \leq G} \left\{ \mathcal{P}(G_1) \wedge \bigwedge_{x_\alpha \triangleleft G} \bigwedge_{x_\alpha \triangleleft G_2 \geq G_1} (\mathcal{T}(G_2'))' \right\},$$

where $\mathcal{T}_{sp}(G)$ and $\mathcal{T}_{sp}(G')$ can be regarded as the semi-preopenness and the semi-precloseness degree of an L -subset G , respectively.

Lemma 2.6. [19] For any (L, M) -pfts $(X, \mathcal{T})$, an (L, M) -fuzzy semi-preopen operator $\mathcal{T}_{sp}$ verifies the following statements:

$$\begin{aligned} (1) \quad & \mathcal{T}_{sp}(\underline{0}_{L^X}) = \mathcal{T}_{sp}(\underline{1}_{L^X}) = 1_M, \\ (2) \quad & \mathcal{T}_{sp}\left(\bigvee_{i \in I} G_i\right) \geq \bigwedge_{i \in I} \mathcal{T}_{sp}(G_i) \text{ for any } \{G_i\}_{i \in I} \subseteq L^X. \end{aligned}$$

Theorem 2.7. [28] Let $(X, \mathcal{T})$ be an (L, M) -pfts and $\mathcal{P}$ be the corresponding (L, M) -fuzzy preopen operator. Then $\mathcal{T}(G) \leq \mathcal{P}(G)$ for any $G \in L^X$.

Corollary 2.8. For any (L, M) -pfts $(X, \mathcal{T})$, if $\mathcal{T}_{sp}$ is the corresponding (L, M) -fuzzy semi-preopen operator, then $\mathcal{T}(G) \leq \mathcal{T}_{sp}(G)$ for any $G \in L^X$.

Definition 2.9. [1] Let $(X, \mathcal{T}_1)$ and $(Y, \mathcal{T}_2)$ be two (L, M) -pfts. The mapping $f : (X, \mathcal{T}_1) \rightarrow (Y, \mathcal{T}_2)$ is said to be:

- (1) (L, M) -fuzzy semi-precontinuous mapping iff $\mathcal{T}_2(H) \leq (\mathcal{T}_1)_{sp}(f_L^{\leftarrow}(H))$ for any $H \in L^Y$.
- (2) (L, M) -fuzzy semi-preirresolute mapping iff $(\mathcal{T}_2)_{sp}(H) \leq (\mathcal{T}_1)_{sp}(f_L^{\leftarrow}(H))$ for any $H \in L^Y$.

Corollary 2.10. Let $f : (X, \mathcal{T}_1) \rightarrow (Y, \mathcal{T}_2)$ be a mapping between two (L, M) -pfts $(X, \mathcal{T}_1)$ and $(Y, \mathcal{T}_2)$. Then:

- (1) f is (L, M) -fuzzy semi-precontinuous iff $\mathcal{T}_2^*(H) \leq (\mathcal{T}_{sp})_1^*(f_L^{\leftarrow}(H))$ for all $H \in L^Y$.
- (2) f is (L, M) -fuzzy semi-preirresolute iff $(\mathcal{T}_{sp})_2^*(H) \leq (\mathcal{T}_{sp})_1^*(f_L^{\leftarrow}(H))$ for all $H \in L^Y$.

Theorem 2.11. Let $f : (X, \mathcal{T}_1) \rightarrow (Y, \mathcal{T}_2)$ be a mapping between two (L, M) -pfts $(X, \mathcal{T}_1)$ and $(Y, \mathcal{T}_2)$. Then:

- (1) If f is (L, M) -fuzzy continuous, then f is (L, M) -fuzzy semi-precontinuous.
- (2) If f is (L, M) -fuzzy semi-preirresolute, then f is (L, M) -fuzzy semi-precontinuous.

Proof. (1) If f is (L, M) -fuzzy continuous mapping, then $\mathcal{T}_2(H) \leq \mathcal{T}_1(f_L^{\leftarrow}(H))$ for any $H \in L^Y$. By using Corollary 2.8, we have

$$\mathcal{T}_2(H) \leq \mathcal{T}_1(f_L^{\leftarrow}(H)) \leq (\mathcal{T}_{sp})_1(f_L^{\leftarrow}(H)),$$

for any $H \in L^Y$. Hence, f is an (L, M) -fuzzy semi-precontinuous.

(2) If f is (L, M) -fuzzy semi-preirresolute mapping, then $(\mathcal{T}_{sp})_2(H) \leq (\mathcal{T}_{sp})_1(f_L^{\leftarrow}(H))$ for all $H \in L^Y$. Based on Corollary 2.8, we get

$$\mathcal{T}_2(H) \leq (\mathcal{T}_{sp})_2(H) \leq (\mathcal{T}_{sp})_1(f_L^{\leftarrow}(H)),$$

for any $H \in L^Y$. Hence f is an (L, M) -fuzzy semi-precontinuous. □

Definition 2.12. A mapping $f : (X, \mathcal{T}) \rightarrow (Y, \mathcal{T}_2)$ between two (L, M) -pfts's $(X, \mathcal{T}_1)$ and $(Y, \mathcal{T}_2)$ is said to be an (L, M) -fuzzy semi-preopen mapping iff $\mathcal{T}_1(G) \leq (\mathcal{T}_{sp})_2(f_L^{\rightarrow}(G))$ for all $G \in L^X$.

Definition 2.13. [1] An (L, M) -fuzzy semi-preclosure operator on X is a function $spCl : L^X \rightarrow M^{P(L^X)}$ satisfying the following statements:

- (1) $spCl(G)(x_a) = \bigwedge_{b \triangleleft a} spCl(G)(x_b)$ for all $x_a \in P(L^X)$.
- (2) $spCl(\underline{0}_{L^X})(x_a) = 0_M$ for all $x_a \in P(L^X)$.
- (3) $spCl(G)(x_a) = 1_M$ for all $x_a \leq G$.
- (4) for any $b \in M_0$, $(spCl(\bigvee (spCl(G))_{[b]}))_{[b]} \subset (spCl(G))_{[b]}$.

The value $spCl(G)(x_a)$ represents the degree to which x_a belongs to the semi-preclosure of an L -subset G .

Theorem 2.14. [1] If $\mathcal{T}_{sp}$ is an (L, M) -fuzzy semi-preopen operator on X and $spCl$ is an (L, M) -fuzzy semi-preclosure operator, then we have

$$spCl(G)(x_a) = \bigwedge_{x_a \not\leq G_1 \leq G} (\mathcal{T}_{sp}(G'_1))',$$

for all $x_a \in P(L^X)$ and $G \in L^X$.

Definition 2.15. An (L, M) -fuzzy quasi semi-preneighborhood system on X is a family $spQ = \{spQ_{x_a} \mid x_a \in P(L^X)\}$ of mappings $\{spQ_{x_a} : L^X \rightarrow M\}$ which satisfies the following axioms:

- (1) $spQ_{x_a}(\underline{0}_{L^X}) = 0_M$.
- (2) $spQ_{x_a}(G) \neq 0_M$, then $x_a \not\leq G'$.
- (3) $spQ_{x_a}(G_1 \wedge G_2) \leq spQ_{x_a}(G_1) \wedge spQ_{x_a}(G_2)$.

$$(4) \quad spQ_{x_a}(G) = \bigvee_{x_a \leq G_1 \leq G'} \bigwedge_{y_b \leq G'_1} spQ_{y_b}(G_1).$$

From [27] and based on Lemma 2.6, we can easily prove Theorems 2.16 and 2.19.

Theorem 2.16. Let $(X, \mathcal{T})$ be an (L, M) -pfts, $\mathcal{T}_{sp}$ be the corresponding (L, M) -fuzzy semi-preopen operator, and $spQ^{\mathcal{T}_{sp}} = \{spQ_{x_a} \mid x_a \in P(L^X)\}$ be the (L, M) -fuzzy quasi semi-preneighborhood system induced by $\mathcal{T}_{sp}$. If the mapping $spC^{\mathcal{T}_{sp}} : L^X \rightarrow M^{P(L^X)}$ is defined by

$$spC^{\mathcal{T}_{sp}}(G)(x_a) = \left(spQ_{x_a}(G') \right)',$$

then $spC^{\mathcal{T}_{sp}}$ is an (L, M) -fuzzy semi-preclosure operator on X .

Definition 2.17. An (L, M) -fuzzy semi-preneighborhood system on X is the family $spN = \{spN_{x_a} \mid x_a \in P(L^X)\}$ of mappings $\{spN_{x_a} : L^X \rightarrow M\}$ which satisfy the following statements:

- (1) $spN_{x_a}(\underline{0}_{L^X}) = 0_M$.
- (2) $spN_{x_a}(G) \neq 0_M$, then $x_a \leq G$.
- (3) $spN_{x_a}(G_1 \wedge G_2) \leq spN_{x_a}(G_1) \wedge spN_{x_a}(G_2)$.
- (4) $spN_{x_a}(G) = \bigvee_{x_a \leq G_1 \leq G} \bigwedge_{y_b \leq G_1} spN_{y_b}(G_1)$.

Definition 2.18. An (L, M) -fuzzy semi-preinterior operator on X is a mapping $spI : L^X \rightarrow M^{P(L^X)}$ which satisfy the following statements:

- (1) $spI(G)(x_a) = \bigwedge_{b \leq a} spI(G)(x_b)$ for all $x_a \in P(L^X)$.
- (2) $spI(\underline{1}_{L^X})(x_a) = 1_M$ for all $x_a \in P(L^X)$.
- (3) $spI(G)(x_a) = 0_M$ for all $x_a \not\leq G$.
- (4) For any $b \in M_0$, $(spI(G))^{(b)} \subseteq (spI(\bigvee (spI(G))^{(b)}))^{(b)}$.

The value $spI(G)(x_a)$ represents the degree to which x_a belongs to the semi-preinterior of G .

Theorem 2.19. Let $(X, \mathcal{T})$ be an (L, M) -pfts and $spN^{\mathcal{T}} = \{spN_{x_a}^{\mathcal{T}} \mid x_a \in P(L^X)\}$ be the corresponding (L, M) -fuzzy semi-preneighborhood system. If the functions $spI^{\mathcal{T}_{sp}} : L^X \rightarrow M^{P(L^X)}$ is defined by

$$spI^{\mathcal{T}_{sp}}(G)(x_a) = spN_{x_a}^{\mathcal{T}_{sp}}(G),$$

then $spI^{\mathcal{T}_{sp}}$ is an (L, M) -fuzzy semi-preinterior operator on X .

Definition 2.20. [19] For an (L, M) -pfts $(X, \mathcal{T})$ and $G \in L^X$, let

$$spCon(G) = \bigwedge_{\substack{G_1, G_2 \in L^X \setminus \{\underline{0}_{L^X}\}, \\ G = G_1 \vee G_2}} \left\{ \bigvee_{x_a \leq G_1} spCl(G_2)(x_a) \vee \bigvee_{y_b \leq G_2} spCl(G_1)(y_b) \right\}.$$

Then $spCon(G)$ is called the degree of fuzzy semi-preconnectedness of an L -subset G .

Theorem 2.21. [19] Let $(X, \mathcal{T})$ be an (L, M) -pfts and $G \in L^X$. Then

$$spCon(G) = \bigwedge_{\substack{G \wedge G_1 \neq \underline{0}_{L^X}, G \wedge G_2 \neq \underline{0}_{L^X}, \\ G \wedge G_1 \wedge G_2 \neq \underline{0}_{L^X}, G \leq G_1 \vee G_2}} \left\{ (\mathcal{T}_{sp}(G'_1))' \vee (\mathcal{T}_{sp}(G'_2))' \right\}.$$

In [1], Ghareeb introduced the notion of semi-precompactness in (L, M) -fuzzy pretopology based on the (L, M) -fuzzy semi-preopen operator. In the following definitions, we define the degree

of semi-precompactness, semipre- T_1 , and semipre- T_2 in (L, M) -fuzzy pretopology by using the implication operation.

Definition 2.22. Let $(X, \mathcal{T})$ be an (L, M) -pfts, $L = M$, and

$$\begin{aligned} \mathbf{spcom}_{\mathcal{T}_{sp}}(G) &= \bigwedge_{\mathcal{A} \subseteq L^X} \left\{ \mathcal{T}_{sp}(\mathcal{A}) \mapsto \left(\left[G \widetilde{\subset} \bigvee \mathcal{A} \right] \mapsto \bigvee_{\mathcal{B} \in 2^{(\mathcal{A})}} \left[G \widetilde{\subset} \bigvee \mathcal{B} \right] \right) \right\} \\ &= \bigwedge_{\mathcal{A} \subseteq L^X} \left\{ \bigwedge_{G_1 \in \mathcal{A}} \mathcal{T}_{sp}(G_1) \mapsto \left\{ \bigwedge_{x \in X} \left(G' \vee \bigvee_{G_1 \in \mathcal{A}} G_1 \right)(x) \mapsto \bigvee_{\mathcal{B} \in 2^{(\mathcal{A})}} \bigwedge_{x \in X} \left(G' \vee \bigvee_{G_1 \in \mathcal{B}} G_1 \right)(x) \right\} \right\}. \end{aligned}$$

for each $G \in L^X$. Then $\mathbf{spcom}_{\mathcal{T}_{sp}}(G)$ is called the semi-precompactness degree of G with respect to $\mathcal{T}_{sp}$.

In this paper, we extend **Semipre- T_1** and **Semipre- T_2** in our sense.

Definition 2.23. If $(X, \mathcal{T})$ be an (L, M) -pfts, then

- (1) The degree semipre- $T_1(X, \mathcal{T})$ to which $\mathcal{T}$ is semipre- T_1 is given by:

$$\text{semipre-}T_1(X, \mathcal{T}) = \bigwedge_{\alpha_1 \neq \alpha_2} \bigvee_{\alpha_1 \neq G \geq \alpha_2} \mathcal{T}_{sp}(G').$$

- (2) The degree semipre- $T_2(X, \mathcal{T})$ to which $\mathcal{T}$ is semipre- T_2 is given by:

$$\text{semipre-}T_2(X, \mathcal{T}) = \bigwedge_{\alpha_1 \neq \alpha_2} \bigvee \left\{ \mathcal{T}_{sp}(G'_1) \wedge \mathcal{T}_{sp}(G_2) \mid \alpha_1 \neq G_1 \geq G_2 \geq \alpha_2 \right\}.$$

3 Semi-preopenness, semi-precontinuity and semi-preirresoluteness degree of mappings based on (L, M) -fuzzy semi-preopen operator

We introduce semi-preopenness, semi-precontinuity, and semi-preirresoluteness degree of mappings in (L, M) -fuzzy pretopology. Moreover, we characterize it by using (L, M) -fuzzy quasi semi-preneighborhood systems, (L, M) -fuzzy semi-preneighborhood systems, (L, M) -fuzzy semi-preinterior operators and (L, M) -fuzzy semi-preclosure operators.

Definition 3.1. If $f : (X, \mathcal{T}_1) \rightarrow (Y, \mathcal{T}_2)$ is a mapping between two (L, M) -fpts $(X, \mathcal{T}_1)$ and $(Y, \mathcal{T}_2)$, then:

- (1) The degree $\mathbf{SPc}(f)$ to which f is semi-precontinuous is given by

$$\mathbf{SPc}(f) = \bigwedge_{H \in L^Y} \left\{ \mathcal{T}_2(H) \mapsto (\mathcal{T}_{sp})_1(f_L^{\leftarrow}(H)) \right\}.$$

- (2) The degree $\mathbf{SPo}(f)$ to which f is semi-preopen is given by

$$\mathbf{SPo}(f) = \bigwedge_{G \in L^X} \left\{ (\mathcal{T}_{sp})_1(G) \mapsto (\mathcal{T}_{sp})_2(f_L^{\rightarrow}(G)) \right\}.$$

- (3) The degree $\mathbf{SPi}(f)$ to which f is semi-preirresolute is given by

$$\mathbf{SPi}(f) = \bigwedge_{H \in L^Y} \left\{ (\mathcal{T}_{sp})_2(H) \mapsto (\mathcal{T}_{sp})_1(f_L^{\leftarrow}(H)) \right\}.$$

Definition 3.2. If $f : (X, \mathcal{T}_1) \rightarrow (Y, \mathcal{T}_2)$ is a bijective mapping, then the degree **Semipre-Hom**(f) to which f is semi-prehomomorphism, is defined by

$$\mathbf{Semipre-Hom}(f) = \mathbf{SPi}(f) \wedge \mathbf{SPo}(f),$$

and

$$\mathbf{Semipre-Hom}(f) = \mathbf{SPc}(f) \wedge \mathbf{SPo}(f).$$

Remark 3.3. By Lemma 2.1 (2), if $\mathbf{SPc}(f) = 1_M$, we have $(\mathcal{T}_{sp})_1(f_L^{\leftarrow}(H)) \geq \mathcal{T}_2(H)$ for any $H \in L^Y$. This is just the definition of semi-precontinuous function between two (L, M) -fpts. Similarly for the cases $\mathbf{SPo}(f) = 1_M$ and $\mathbf{SPi}(f) = 1_M$. If $id : (X, \mathcal{T}_1) \rightarrow (X, \mathcal{T}_1)$ is the identity function, then $\mathbf{SPi}(id) = \mathbf{SPo}(id) = \mathbf{Semipre-Hom}(id) = 1_M$. Moreover, if $L = \{0_L, 1_L\}$, then we get the same degrees but between M -fuzzifying pretopological spaces.

The following corollaries are direct results from Definition 3.4 and Corollary 2.10.

Corollary 3.4. If $f : (X, \mathcal{T}_1) \rightarrow (Y, \mathcal{T}_2)$ is a mapping between two (L, M) -fpts's $(X, \mathcal{T}_1)$ and $(Y, \mathcal{T}_2)$, then:

(1) The degree $\mathbf{SPc}(f)$ to which f is semi-precontinuous, is given by

$$\mathbf{SPc}(f) = \bigwedge_{H \in L^Y} \left\{ \mathcal{T}_2^*(H) \mapsto (\mathcal{T}_{sp})_1^*(f_L^{\leftarrow}(H)) \right\}.$$

(2) The degree $\mathbf{SPi}(f)$ to which f is semi-preirresolute, is given by

$$\mathbf{SPi}(f) = \bigwedge_{H \in L^Y} \left\{ (\mathcal{T}_{sp})_2^*(H) \mapsto (\mathcal{T}_{sp})_1^*(f_L^{\leftarrow}(H)) \right\}.$$

Definition 3.5. If $f : (X, \mathcal{T}_1) \rightarrow (Y, \mathcal{T}_2)$ is a mapping between two (L, M) -fpts's $(X, \mathcal{T}_1)$ and $(Y, \mathcal{T}_2)$, then the degree $\mathbf{SPc}(f)$ to which f is semi-preclosed, is defined by

$$\mathbf{SPc}(f) = \bigwedge_{H \in L^Y} \left\{ \mathcal{T}_1^*(H) \mapsto (\mathcal{T}_{sp})_2^*(f_L^{\leftarrow}(H)) \right\}.$$

Theorem 3.6. Let $(X, \mathcal{T}_1)$, $(Y, \mathcal{T}_2)$ and $(Z, \mathcal{T}_3)$ be (L, M) -fpts's, $f : (X, \mathcal{T}_1) \rightarrow (Y, \mathcal{T}_2)$ and $g : (Y, \mathcal{T}_2) \rightarrow (Z, \mathcal{T}_3)$ be two mappings. Then the following properties are satisfied:

- (1) $\mathbf{SPi}(f) \wedge \mathbf{SPi}(g) \leq \mathbf{SPi}(g \circ f)$.
- (2) $\mathbf{SPo}(f) \wedge \mathbf{SPo}(g) \leq \mathbf{SPi}(g \circ f)$.
- (3) $\mathbf{SPcl}(f) \wedge \mathbf{SPi}(g) \leq \mathbf{SPcl}(g \circ f)$.

Proof. Proving (1) is enough since the other results can be proved similarly. By Lemma 2.1 and Definition 3.4, we have

$$\begin{aligned} \mathbf{SPi}(f) \wedge \mathbf{SPi}(g) &= \bigwedge_{H \in L^Y} \left\{ (\mathcal{T}_{sp})_2(H) \mapsto (\mathcal{T}_{sp})_1(f_L^{\leftarrow}(H)) \right\} \wedge \bigwedge_{W \in L^Z} \left\{ (\mathcal{T}_{sp})_3(W) \mapsto (\mathcal{T}_{sp})_2(g_L^{\leftarrow}(W)) \right\} \\ &\leq \bigwedge_{W \in L^Z} \left\{ (\mathcal{T}_{sp})_2(g_L^{\leftarrow}(W)) \mapsto (\mathcal{T}_{sp})_1(f_L^{\leftarrow}(g_L^{\leftarrow}(W))) \right\} \wedge \bigwedge_{W \in L^Z} \left\{ (\mathcal{T}_{sp})_3(W) \mapsto (\mathcal{T}_{sp})_2(g_L^{\leftarrow}(W)) \right\} \\ &= \bigwedge_{W \in L^Z} \left\{ \left((\mathcal{T}_{sp})_2(g_L^{\leftarrow}(W)) \mapsto (\mathcal{T}_{sp})_1((g_L \circ f_L)^{\leftarrow}(W)) \right) \wedge \left((\mathcal{T}_{sp})_3(W) \mapsto (\mathcal{T}_{sp})_2(g_L^{\leftarrow}(W)) \right) \right\} \\ &\leq \bigwedge_{W \in L^Z} \left((\mathcal{T}_{sp})_3(g_L^{\leftarrow}(W)) \mapsto (\mathcal{T}_{sp})_1((g_L \circ f_L)^{\leftarrow}(W)) \right) \\ &= \mathbf{SPi}(g \circ f). \end{aligned}$$

Thus we completed the proof. $\square$

The following corollaries are direct results from Definition 3.2 and Theorem 3.6.

Corollary 3.7. Let $f : (X, \mathcal{T}_1) \rightarrow (Y, \mathcal{T}_2)$ and $g : (Y, \mathcal{T}_2) \rightarrow (Z, \mathcal{T}_3)$ be two mappings between three (L, M) -fpts's $(X, \mathcal{T}_1)$, $(Y, \mathcal{T}_2)$, and $(Z, \mathcal{T}_3)$. If f and g are two bijective mappings, then the following statement holds:

$$\text{Semipre-Hom}(f) \wedge \text{Semipre-Hom}(g) \leq \text{Semipre-Hom}(f \circ g).$$

Theorem 3.8. Let $f : (X, \mathcal{T}_1) \rightarrow (Y, \mathcal{T}_2)$ and $g : (Y, \mathcal{T}_2) \rightarrow (Z, \mathcal{T}_3)$ be two (L, M) -fpts's on X and Y . If f is a surjective function, then the following statements hold:

- (1) $\text{SPo}(g \circ f) \wedge \text{SPi}(f) \leq \text{SPo}(g)$.
- (2) $\text{SPcl}(g \circ f) \wedge \text{SPi}(f) \leq \text{SPcl}(g)$.

Proof. Proving (1) is enough since the other statements are similarly proved. Since f is surjective mapping, we obtain $(g_L \circ f_L)^{\rightarrow}(f_L^{\leftarrow}(H)) = g_L^{\rightarrow}(H)$ for any $H \in L^Y$. Based on Lemma 2.1 (4), we have

$$\begin{aligned} \text{SPo}(g \circ f) \wedge \text{SPi}(f) &= \bigwedge_{G \in L^X} \left\{ (\mathcal{T}_{sp})_1(G) \mapsto (\mathcal{T}_{sp})_3((g_L \circ f_L)^{\rightarrow}(G)) \right\} \wedge \bigwedge_{H \in L^Y} \left\{ (\mathcal{T}_{sp})_2(H) \mapsto (\mathcal{T}_{sp})_1(f_L^{\leftarrow}(H)) \right\} \\ &\leq \bigwedge_{H \in L^Y} \left\{ (\mathcal{T}_{sp})_1(f_L^{\leftarrow}(H)) \mapsto (\mathcal{T}_{sp})_3((g_L \circ f_L)^{\rightarrow}(f_L^{\leftarrow}(H))) \right\} \wedge \bigwedge_{H \in L^Y} \left\{ (\mathcal{T}_{sp})_2(H) \mapsto (\mathcal{T}_{sp})_1(f_L^{\leftarrow}(H)) \right\} \\ &= \bigwedge_{H \in L^Y} \left\{ \left((\mathcal{T}_{sp})_1(f_L^{\leftarrow}(H)) \mapsto (\mathcal{T}_{sp})_3(g_L^{\rightarrow}(H)) \right) \wedge \left((\mathcal{T}_{sp})_2(H) \mapsto (\mathcal{T}_{sp})_1(f_L^{\leftarrow}(H)) \right) \right\} \\ &\leq \bigwedge_{W \in L^Y} \left\{ (\mathcal{T}_{sp})_2(H) \mapsto (\mathcal{T}_{sp})_3(g_L^{\rightarrow}(W)) \right\} = \text{SPo}(g). \end{aligned}$$

□

Similarly, it can be verified that:

Theorem 3.9. Let $f : (X, \mathcal{T}_1) \rightarrow (Y, \mathcal{T}_2)$ and $g : (Y, \mathcal{T}_2) \rightarrow (Z, \mathcal{T}_3)$ be two (L, M) -fpts's on X , Y and Z . If f is a surjective mapping, then the following statements hold:

- (1) $\text{SPo}(g \circ f) \wedge \text{SPi}(g) \leq \text{SPo}(f)$.
- (2) $\text{SPcl}(g \circ f) \wedge \text{SPi}(g) \leq \text{SPcl}(f)$.

Theorem 3.10. Let $f : (X, \mathcal{T}_1) \rightarrow (Y, \mathcal{T}_2)$ be a bijective mapping between (L, M) -fpts's on X and Y , then

- (1) $\text{SPi}(f) = \bigwedge_{G \in L^X} \left\{ (\mathcal{T}_{sp})_2(f_L^{\rightarrow}(G)) \mapsto (\mathcal{T}_{sp})_1(G) \right\}$.
- (2) $\text{SPo}(f) = \bigwedge_{H \in L^Y} \left\{ (\mathcal{T}_{sp})_1(f_L^{\leftarrow}(H)) \mapsto (\mathcal{T}_{sp})_2(H) \right\}$.
- (3) $\text{SPi}(f^{-1}) = \text{SPo}(f) = \text{SPcl}(f)$.

Proof. Proving (1) and (3) is enough since condition (2) is proved similarly to (1).

(1) Since $f : X \rightarrow Y$ is a bijective mapping, we have $f_L^{\leftarrow}(f_L^{\rightarrow}(G)) = G$, for all $G \in L^X$, and $f_L^{\rightarrow}(f_L^{\leftarrow}(H)) = H$, for all $H \in L^Y$. The following inequalities are obtained

$$\begin{aligned} \bigwedge_{G \in L^X} \left\{ (\mathcal{T}_{sp})_2(f_L^{\leftarrow}(G)) \mapsto (\mathcal{T}_{sp})_1(G) \right\} &= \bigwedge_{G \in L^X} \left\{ (\mathcal{T}_{sp})_2(f_L^{\leftarrow}(G)) \mapsto (\mathcal{T}_{sp})_1(f_L^{\leftarrow}(f_L^{\rightarrow}(G))) \right\} \\ &\geq \bigwedge_{H \in L^Y} \left\{ (\mathcal{T}_{sp})_2(H) \mapsto (\mathcal{T}_{sp})_1(f_L^{\leftarrow}(H)) \right\} \\ &= \bigwedge_{H \in L^Y} \left\{ (\mathcal{T}_{sp})_2(f_L^{\leftarrow}(f_L^{\rightarrow}(H))) \mapsto (\mathcal{T}_{sp})_1(f_L^{\leftarrow}(H)) \right\} \end{aligned}$$

$$\geq \bigwedge_{G \in L^X} \left\{ (\mathcal{T}_{sp})_2(f_L^{\rightarrow}(G)) \mapsto (\mathcal{T}_{sp})_1(G) \right\}.$$

Hence

$$\mathbf{SPi}(f) = \bigwedge_{H \in L^Y} \left\{ (\mathcal{T}_{sp})_2(f_L^{\leftarrow}(H)) \mapsto (\mathcal{T}_{sp})_1(H) \right\} = \bigwedge_{G \in L^X} \left\{ (\mathcal{T}_{sp})_2(f_L^{\leftarrow}(G)) \mapsto (\mathcal{T}_{sp})_1(G) \right\}.$$

(3) Since $f : X \rightarrow Y$ is a bijective mapping, we have $(f^{-1})_L^{\leftarrow}(G) = f_L^{\rightarrow}(G)$ and $f_L^{\rightarrow}(G') = f_L^{\leftarrow}(G')'$ for any $G \in L^X$. Then

$$\begin{aligned} \mathbf{SPi}(f^{-1}) &= \bigwedge_{G \in L^X} \left\{ (\mathcal{T}_{sp})_1(G) \mapsto (\mathcal{T}_{sp})_2((f^{-1})_L^{\leftarrow}(G)) \right\} \\ &= \bigwedge_{G \in L^X} \left\{ (\mathcal{T}_{sp})_1(G) \mapsto (\mathcal{T}_{sp})_2(f_L^{\rightarrow}(G)) \right\} \\ &= \mathbf{SPo}(f). \end{aligned}$$

and

$$\begin{aligned} \mathbf{SPo}(f^{-1}) &= \bigwedge_{G \in L^X} \left\{ (\mathcal{T}_{sp})_1(G) \mapsto (\mathcal{T}_{sp})_2(f_L^{\rightarrow}(G)) \right\} \\ &= \bigwedge_{G \in L^X} \left\{ (\mathcal{T}_{sp})_1(G') \mapsto (\mathcal{T}_{sp})_2(f_L^{\rightarrow}(G')) \right\} \\ &= \bigwedge_{G \in L^X} \left\{ (\mathcal{T}_{sp})_1(G') \mapsto (\mathcal{T}_{sp})_2(f_L^{\rightarrow}(G'))' \right\} \\ &= \mathbf{SPcl}(f). \end{aligned}$$

□

Corollary 3.11. Let $f : (X, \mathcal{T}_1) \rightarrow (Y, \mathcal{T}_2)$ be a bijective mapping between (L, M) -fpts's X and Y , then

- (1) $\mathbf{Semipre-Hom}(f) = \mathbf{SPi}(f) \wedge \mathbf{SPi}(f^{-1}) = \mathbf{SPi}(f) \wedge \mathbf{SPcl}(f).$
- (2) $\mathbf{Semipre-Hom}(f) = \bigwedge_{G \in L^X} \left\{ (\mathcal{T}_{sp})_2(f_L^{\rightarrow}(G)) \leftrightarrow (\mathcal{T}_{sp})_1(G) \right\}.$
- (3) $\mathbf{Semipre-Hom}(f) = \bigwedge_{H \in L^Y} \left\{ (\mathcal{T}_{sp})_1(f_L^{\leftarrow}(H)) \leftrightarrow (\mathcal{T}_{sp})_2(H) \right\}.$

The following corollaries and theorems characterize the degree of semi-preirresolutness, semi-preopenness by (L, M) -fuzzy quasi semi-preneighborhood systems, (L, M) -fuzzy semi-preclosure operators, and (L, M) -fuzzy semi-preinterior operators.

Corollary 3.12. Let $f : (X, \mathcal{T}_1) \rightarrow (Y, \mathcal{T}_2)$ be a mapping between (L, M) -fpts's X and Y , then

- (1) $\mathbf{SPi}(f) = \bigwedge_{H \in L^Y} \bigwedge_{x_a \in P(L^X)} \left\{ spQ_{f(x)_a}^{(\mathcal{T}_{sp})_2}(H) \mapsto spQ_{x_a}^{(\mathcal{T}_{sp})_1}(f_L^{\leftarrow}(H)) \right\}.$
- (2) $\mathbf{SPi}(f) = \bigwedge_{H \in L^Y} \bigwedge_{x_a \in P(L^X)} \left\{ spN_{f(x)_a}^{(\mathcal{T}_{sp})_2}(H)(f(x)_a) \mapsto spN_{x_a}^{(\mathcal{T}_{sp})_1}(f_L^{\leftarrow}(H)) \right\}.$
- (3) $\mathbf{SPi}(f) = \bigwedge_{H \in L^Y} \bigwedge_{x_a \in P(L^X)} \left\{ spI^{(\mathcal{T}_{sp})_2}(H)(f(x)_a) \mapsto spI^{(\mathcal{T}_{sp})_1}(f_L^{\leftarrow}(H))(x_a) \right\}.$
- (4) $\mathbf{SPi}(f) = \bigwedge_{H \in L^Y} \bigwedge_{x_a \in P(L^X)} \left\{ spCl^{(\mathcal{T}_{sp})_2}(H)(f(x)_a)' \mapsto spCl^{(\mathcal{T}_{sp})_1}(f_L^{\leftarrow}(H))(x_a)' \right\}.$

Proof. (1) Since $spQ_{x_a}^{(\mathcal{T}_{sp})_1}(G) = \bigvee_{x_a \not\leq G_1' > G'} \mathcal{T}_{sp}(G_1)$ for all $G \in L^X$, and for any $x_a \in P(L^X)$ and $H_1, H \in L^Y$, we have $f(x)_a \not\leq H_1' \geq H' \Rightarrow x_a \not\leq f_L^{\leftarrow}(H_1)' \geq f_L^{\leftarrow}(H)'$. Then

$$\begin{aligned} & \bigwedge_{H \in L^Y} \bigwedge_{x_a \in P(L^X)} \left\{ spQ_{f(x)_a}^{(\mathcal{T}_{sp})_2}(H) \mapsto spQ_{x_a}^{(\mathcal{T}_{sp})_1}(f_L^{\leftarrow}(H)) \right\} \\ &= \bigwedge_{H \in L^Y} \bigwedge_{x_a \in P(L^X)} \left\{ \bigvee_{f(x)_a \not\leq H_1' \geq H'} (\mathcal{T}_{sp})_2(H_1) \mapsto \bigvee_{x_a \not\leq H_1' \geq f_L^{\leftarrow}(H)'} (\mathcal{T}_{sp})_1(H_1) \right\} \\ &\geq \bigwedge_{H \in L^Y} \bigwedge_{x_a \in P(L^X)} \left\{ \bigvee_{f(x)_a \not\leq H_1' \geq H'} (\mathcal{T}_{sp})_2(H_1) \mapsto \bigvee_{x_a \not\leq f_L^{\leftarrow}(H_2)' \geq f_L^{\leftarrow}(H)'} (\mathcal{T}_{sp})_1(f_L^{\leftarrow}(H_2)) \right\} \\ &\geq \bigwedge_{H \in L^Y} \bigwedge_{x_a \in P(L^X)} \bigwedge_{f(x)_a \not\leq H_1' \geq H'} \left\{ (\mathcal{T}_{sp})_2(H_1) \mapsto (\mathcal{T}_{sp})_1(f_L^{\leftarrow}(H_1)) \right\} \\ &\geq \bigwedge_{H \in L^Y} \left\{ (\mathcal{T}_{sp})_2(H) \mapsto (\mathcal{T}_{sp})_1(f_L^{\leftarrow}(H)) \right\} = \mathbf{SPi}(f). \end{aligned}$$

Conversely, since

$$\mathcal{T}_{sp_1}(G) = \bigwedge_{x_a \not\leq G'} spQ_{x_a}^{(\mathcal{T}_{sp})_1}(G)$$

for each $G \in L^X$, and for each $x_a \in P(L^X)$, $H \in L^Y$, and $x_a \not\leq f_L^{\leftarrow}(H)' \Rightarrow f(x)_a \not\leq H'$. The following is valid

$$\begin{aligned} \mathbf{SPi}(f) &= \bigwedge_{H \in L^Y} \left\{ (\mathcal{T}_{sp})_2(H) \mapsto (\mathcal{T}_{sp})_1(f_L^{\leftarrow}(H)) \right\} \\ &= \bigwedge_{H \in L^Y} \left\{ \bigwedge_{y_\mu \not\leq H'} spQ_{y_\mu}^{(\mathcal{T}_{sp})_2}(H) \mapsto \bigwedge_{x_a \not\leq f_L^{\leftarrow}(H)'} spQ_{x_a}^{(\mathcal{T}_{sp})_1}(f_L^{\leftarrow}(H)) \right\} \\ &\geq \bigwedge_{H \in L^Y} \left\{ \bigwedge_{f(x)_a \not\leq H'} spQ_{f(x)_a}^{(\mathcal{T}_{sp})_2}(H) \mapsto \bigwedge_{x_a \not\leq f_L^{\leftarrow}(H)'} spQ_{x_a}^{(\mathcal{T}_{sp})_1}(f_L^{\leftarrow}(H)) \right\} \\ &\geq \bigwedge_{H \in L^Y} \bigwedge_{x_a \not\leq f_L^{\leftarrow}(H)'} \left(spQ_{f(x)_a}^{(\mathcal{T}_{sp})_2}(H) \mapsto spQ_{x_a}^{(\mathcal{T}_{sp})_1}(f_L^{\leftarrow}(H)) \right) \\ &\geq \bigwedge_{H \in L^Y} \bigwedge_{x_a \in P(L^X)} \left\{ spQ_{f(x)_a}^{(\mathcal{T}_{sp})_2}(H) \mapsto spQ_{x_a}^{(\mathcal{T}_{sp})_1}(f_L^{\leftarrow}(H)) \right\}. \end{aligned}$$

Thus we complete the proof of (1). By Theorem 2.16 and Theorem 2.19, we can prove (2), (3) and (4). $\square$

Theorem 3.13. Let $f : (X, \mathcal{T}_1) \rightarrow (Y, \mathcal{T}_2)$ be a mapping between (L, M) -fpts's X and Y , then

- (1) $\mathbf{SPo}(f) = \bigwedge_{G \in L^X} \bigwedge_{x_a \in P(L^X)} \left\{ spQ_{x_a}^{(\mathcal{T}_{sp})_1}(G) \mapsto spQ_{f(x)_a}^{(\mathcal{T}_{sp})_2}(f_L^{\rightarrow}(G)) \right\}.$
- (2) $\mathbf{SPo}(f) = \bigwedge_{G \in L^X} \bigwedge_{x_a \in P(L^X)} \left\{ spN_{f(x)_a}^{(\mathcal{T}_{sp})_1}(G)(x_a) \mapsto spN_{f(x)_a}^{(\mathcal{T}_{sp})_2}(f_L^{\rightarrow}(G)) \right\}.$
- (3) $\mathbf{SPo}(f) = \bigwedge_{G \in L^X} \bigwedge_{x_a \in P(L^X)} \left\{ spI^{(\mathcal{T}_{sp})_1}(G)(x_a) \mapsto spI^{(\mathcal{T}_{sp})_2}(f_L^{\rightarrow}(G))(x_a) \right\}.$
- (4) $\mathbf{SPo}(f) = \bigwedge_{G \in L^X} \bigwedge_{x_a \in P(L^X)} \left\{ spCI^{(\mathcal{T}_{sp})_1}(G')(x_a)' \mapsto spCI^{(\mathcal{T}_{sp})_2}(f_L^{\rightarrow}(G'))(f(x)_a)' \right\}.$
- (5) $\mathbf{SPo}(f) = \bigwedge_{H \in L^Y} \bigwedge_{x_a \in P(L^X)} \left\{ spQ_{x_a}^{(\mathcal{T}_{sp})_1}(f_L^{\leftarrow}(H)) \mapsto spQ_{f(x)_a}^{(\mathcal{T}_{sp})_2}(H) \right\}.$
- (6) $\mathbf{SPo}(f) = \bigwedge_{H \in L^Y} \bigwedge_{x_a \in P(L^X)} \left\{ spN_{f(x)_a}^{(\mathcal{T}_{sp})_1}(f_L^{\leftarrow}(H))(x_a) \mapsto spN_{f(x)_a}^{(\mathcal{T}_{sp})_2}(H) \right\}.$
- (7) $\mathbf{SPo}(f) = \bigwedge_{H \in L^Y} \bigwedge_{x_a \in P(L^X)} \left\{ spI^{(\mathcal{T}_{sp})_1}(f_L^{\leftarrow}(H))(x_a) \mapsto spI^{(\mathcal{T}_{sp})_2}(H)(x_a) \right\}.$

$$(8) \text{ SPo}(f) = \bigwedge_{H \in L^Y} \bigwedge_{x_a \in P(L^X)} \left\{ spCl^{(\mathcal{T}_{sp})_1}(f_L^{\leftarrow}(H))(x_a)' \mapsto spCl^{(\mathcal{T}_{sp})_2}(H')(f(x)_a)' \right\}.$$

Proof. Proving (5) is enough since the other conditions are similarly proved.

$$\begin{aligned} & \bigwedge_{H \in L^Y} \bigwedge_{x_a \in P(L^X)} \left\{ spQ_{x_a}^{(\mathcal{T}_{sp})_1}(f_L^{\leftarrow}(H)) \mapsto spQ_{f(x)_a}^{(\mathcal{T}_{sp})_2}(H) \right\} \\ &= \bigwedge_{H \in L^Y} \bigwedge_{x_a \in P(L^X)} \left\{ \bigvee_{x_a \not\leq H_1' \geq f_L^{\leftarrow}(H)'} (\mathcal{T}_{sp})_1(H_1) \mapsto \bigvee_{f(x)_a \not\leq H_1' \geq H'} (\mathcal{T}_{sp})_2(H_1) \right\} \\ &\geq \bigwedge_{H \in L^Y} \bigwedge_{x_a \in P(L^X)} \left\{ \bigvee_{x_a \not\leq H_1' \geq f_L^{\leftarrow}(H)'} (\mathcal{T}_{sp})_1(H_1) \mapsto \bigvee_{f(x)_a \not\leq f_L^{\rightarrow}(H_2)'} (\mathcal{T}_{sp})_2(f_L^{\rightarrow}(H_2)) \right\} \\ &\geq \bigwedge_{H \in L^Y} \bigwedge_{x_a \in P(L^X)} \left\{ \bigvee_{x_a \not\leq H_1' \geq f_L^{\leftarrow}(H)'} (\mathcal{T}_{sp})_1(H_1) \mapsto \bigvee_{f(x)_a \not\leq H_2' \geq f^{\leftarrow}(H)'} (\mathcal{T}_{sp})_2(f_L^{\rightarrow}(H_2)) \right\} \\ &\geq \bigwedge_{H \in L^Y} \bigwedge_{x_a \in P(L^X)} \bigwedge_{x_a \not\leq H_1' \geq f_L^{\leftarrow}(H)'} \left\{ (\mathcal{T}_{sp})_1(H_1) \mapsto \bigvee_{f(x)_a \not\leq H_2' \geq f^{\leftarrow}(H)'} (\mathcal{T}_{sp})_2(f_L^{\rightarrow}(H_2)) \right\} \\ &\geq \bigwedge_{H \in L^Y} \bigwedge_{x_a \in P(L^X)} \bigwedge_{x_a \not\leq H_1' \geq f_L^{\leftarrow}(H)'} \left\{ (\mathcal{T}_{sp})_1(H_1) \mapsto (\mathcal{T}_{sp})_2(f_L^{\rightarrow}(H_2)) \right\} \\ &\geq \bigwedge_{H \in L^Y} \left\{ (\mathcal{T}_{sp})_1(H) \mapsto (\mathcal{T}_{sp})_2(f_L^{\rightarrow}(H)) \right\} = \text{SPo}(f). \end{aligned}$$

For any $G \in L^X$, we have to prove the following

$$\bigwedge_{y_b \not\leq (f_L^{\rightarrow}(G))'} spQ_{y_b}^{(\mathcal{T}_{sp})_2}(f_L^{\rightarrow}(G)) = \bigwedge_{f(x)_a \not\leq (f_L^{\rightarrow}(G))'} spQ_{f(x)_a}^{(\mathcal{T}_{sp})_2}(f_L^{\rightarrow}(G)).$$

It is obvious that

$$\bigwedge_{y_b \not\leq (f_L^{\rightarrow}(G))'} spQ_{y_b}^{(\mathcal{T}_{sp})_2}(f_L^{\rightarrow}(G)) \leq \bigwedge_{f(x)_a \not\leq (f_L^{\rightarrow}(G))'} spQ_{f(x)_a}^{(\mathcal{T}_{sp})_2}(f_L^{\rightarrow}(G)).$$

We will show that

$$\bigwedge_{y_b \not\leq (f_L^{\rightarrow}(G))'} spQ_{y_b}^{(\mathcal{T}_{sp})_2}(f_L^{\rightarrow}(G)) \geq \bigwedge_{f(x)_a \not\leq (f_L^{\rightarrow}(G))'} spQ_{f(x)_a}^{(\mathcal{T}_{sp})_2}(f_L^{\rightarrow}(G)).$$

For each $y_b \in P(L^Y)$ with $y_b \not\leq f_L^{\rightarrow}(G)'$, we obtain $\mu \not\leq (f_L^{\rightarrow}(G)(y))' = \bigwedge_{f(x)=y} G(x)'$. Then there exists $x \in X$ such that $f(x) = y$ and $b \leq G(x)'$. This implies $b \not\leq \bigwedge_{f(x)=f(z)} G(z)' = f_L^{\rightarrow}(G)(f(x))'$. Hence $f(x)_\mu \leq f_L^{\rightarrow}(G)'$. On the other hand, since

$$\bigwedge_{f(x)_a \not\leq (f_L^{\rightarrow}(G))'} spQ_{f(x)_a}^{(\mathcal{T}_{sp})_2}(f_L^{\rightarrow}(G)) \leq spQ_{f(x)_a}^{(\mathcal{T}_{sp})_2}(f_L^{\rightarrow}(G)) = spQ_{y_b}^{(\mathcal{T}_{sp})_2}(f_L^{\rightarrow}(G)),$$

then

$$\bigwedge_{f(x)_a \not\leq (f_L^{\rightarrow}(G))'} spQ_{f(x)_a}^{(\mathcal{T}_{sp})_2}(f_L^{\rightarrow}(G)) \leq \bigwedge_{y_b \not\leq (f_L^{\rightarrow}(G))'} spQ_{y_b}^{(\mathcal{T}_{sp})_2}(f_L^{\rightarrow}(G)).$$

Therefore

$$\bigwedge_{y_b \not\leq (f_L^{\rightarrow}(G))'} spQ_{y_b}^{(\mathcal{T}_{sp})_2}(f_L^{\rightarrow}(G)) \leq \bigwedge_{f(x)_a \not\leq (f_L^{\rightarrow}(G))'} spQ_{f(x)_a}^{(\mathcal{T}_{sp})_2}(f_L^{\rightarrow}(G)).$$

(2)

$$\begin{aligned} & \bigwedge_{G \in L^X} \left\{ \bigwedge_{x_a \not\leq G'} spQ_{x_a}^{(\mathcal{T}_{sp})_1}(f^{\leftarrow}(f^{\rightarrow}(G))) \mapsto \bigwedge_{f(x)_a \not\leq f^{\rightarrow}(G)'} spQ_{f(x)_a}^{(\mathcal{T}_{sp})_2}(f^{\rightarrow}(G)) \right\} \\ &\geq \bigwedge_{H \in L^Y} \bigwedge_{x_a \in P(L^X)} \left\{ spQ_{x_a}^{(\mathcal{T}_{sp})_1}(f^{\leftarrow}(H)) \mapsto spQ_{f(x)_a}^{(\mathcal{T}_{sp})_2}(H) \right\} \end{aligned}$$

for each $\mathbf{c} \in M$, such that

$$\mathbf{c} \triangleleft \bigwedge_{H \in L^Y} \bigwedge_{x_a \in P(L^X)} \left\{ spQ_{x_a}^{(\mathcal{T}_{sp})_1}(f_L^{\leftarrow}(H)) \mapsto spQ_{f(x)_a}^{(\mathcal{T}_{sp})_1}(H) \right\}.$$

Then $\mathbf{c} \leq spQ_{x_a}^{(\mathcal{T}_{sp})_1}(f_L^{\leftarrow}(H)) \mapsto spQ_{f(x)_a}^{(\mathcal{T}_{sp})_2}(H)$ for any $H \in L^Y$ and $x_a \in P(L^X)$. By Lemma 2.1(1), we have $\mathbf{c} \wedge spQ_{x_a}^{(\mathcal{T}_{sp})_1}(f_L^{\leftarrow}(H)) \leq spQ_{f(x)_a}^{(\mathcal{T}_{sp})_2}(H)$. For all $G \in L^X$ and $f(x)_a \leq f_L^{\rightarrow}(G)'$, we obtain $a \leq f_L^{\rightarrow}(G)(f(x))' = \bigwedge_{f(x)=f(z)} G(z)'$. Then there exists $z \in X$ such that $f(z) = f(x)$ and $a \leq G(z)'$. This implies $a \leq G(z)'$. On the other hand, since

$$\mathbf{c} \wedge \bigwedge_{x_a \notin G'} spQ_{x_a}^{(\mathcal{T}_{sp})_1}(f_L^{\leftarrow}(f_L^{\rightarrow}(G))) \leq \mathbf{c} \wedge spQ_{z_a}^{(\mathcal{T}_{sp})_1}(f_L^{\leftarrow}(f_L^{\rightarrow}(G))) \leq spQ_{f(z)_a}^{(\mathcal{T}_{sp})_2}(f_L^{\rightarrow}(G)) = spQ_{f(x)_a}^{(\mathcal{T}_{sp})_2}(f_L^{\rightarrow}(G)).$$

The following is valid

$$\mathbf{c} \wedge \bigwedge_{x_a \notin G'} spQ_{x_a}^{(\mathcal{T}_{sp})_1}(f_L^{\leftarrow}(f_L^{\rightarrow}(G))) \leq \bigwedge_{f(x)_a \notin f_L^{\rightarrow}(G)'} spQ_{f(x)_a}^{(\mathcal{T}_{sp})_2}(f_L^{\rightarrow}(G)).$$

By Lemma 2.1(1), we obtain

$$\mathbf{c} \leq \bigwedge_{x_a \notin G'} spQ_{x_a}^{(\mathcal{T}_{sp})_1}(f_L^{\leftarrow}(f_L^{\rightarrow}(G))) \mapsto \bigwedge_{f(x)_a \notin f_L^{\rightarrow}(G)'} spQ_{f(x)_a}^{(\mathcal{T}_{sp})_2}(f_L^{\rightarrow}(G)).$$

Thus

$$\mathbf{c} \leq \bigwedge_{G \in L^X} \left\{ \bigwedge_{x_a \notin G'} spQ_{x_a}^{(\mathcal{T}_{sp})_1}(f_L^{\leftarrow}(f_L^{\rightarrow}(G))) \leq \bigwedge_{f(x)_a \notin f_L^{\rightarrow}(G)'} spQ_{f(x)_a}^{(\mathcal{T}_{sp})_2}(f_L^{\rightarrow}(G)) \right\}.$$

By the arbitrariness of $\mathbf{c}$, we have

$$\begin{aligned} & \bigwedge_{G \in L^X} \left\{ \bigwedge_{x_a \notin G'} spQ_{x_a}^{(\mathcal{T}_{sp})_1}(f_L^{\leftarrow}(f_L^{\rightarrow}(G))) \leq \bigwedge_{f(x)_a \notin f_L^{\rightarrow}(G)'} spQ_{f(x)_a}^{(\mathcal{T}_{sp})_2}(f_L^{\rightarrow}(G)) \right\} \\ & \geq \bigwedge_{H \in L^Y} \bigwedge_{x_a \in P(L^X)} \left\{ \bigwedge_{x_a \notin G'} spQ_{x_a}^{(\mathcal{T}_{sp})_1}(f_L^{\leftarrow}(G)) \leq \bigwedge_{f(x)_a \notin f_L^{\rightarrow}(G)'} spQ_{f(x)_a}^{(\mathcal{T}_{sp})_2}(f_L^{\rightarrow}(G)) \right\}. \end{aligned}$$

For each $G \in L^X$ and $G \leq f_L^{\leftarrow}(f_L^{\rightarrow}(G))$, it can be verified that

$$\begin{aligned} \bigwedge_{G \in L^X} \left\{ (\mathcal{T}_{sp})_1(G) \mapsto (\mathcal{T}_{sp})_2(f_L^{\rightarrow}(G)) \right\} &= \bigwedge_{G \in L^X} \left\{ \bigwedge_{x_a \notin G'} \left\{ spQ_{x_a}^{(\mathcal{T}_{sp})_1}(G) \mapsto \bigwedge_{y_b \notin f^{\rightarrow}(G)'} spQ_{y_b}^{(\mathcal{T}_{sp})_2}(f_L^{\rightarrow}(G)) \right\} \right\} \\ &= \bigwedge_{G \in L^X} \left\{ \bigwedge_{x_a \notin G'} spQ_{x_a}^{(\mathcal{T}_{sp})_1}(G) \mapsto \bigwedge_{f(x)_a \notin f_L^{\rightarrow}(G)'} spQ_{f(x)_a}^{(\mathcal{T}_{sp})_2}(f_L^{\rightarrow}(G)) \right\} \\ &\geq \bigwedge_{G \in L^X} \left\{ \bigwedge_{x_a \notin G'} spQ_{x_a}^{(\mathcal{T}_{sp})_1}(f_L^{\leftarrow}(f_L^{\rightarrow}(G))) \mapsto \bigwedge_{f(x)_a \notin f_L^{\rightarrow}(G)'} spQ_{f(x)_a}^{(\mathcal{T}_{sp})_2}(f_L^{\rightarrow}(G)) \right\} \\ &\geq \bigwedge_{H \in L^Y} \left\{ \bigwedge_{x_a \in P(L^X)} spQ_{x_a}^{(\mathcal{T}_{sp})_1}(f_L^{\leftarrow}(H)) \mapsto spQ_{f(x)_a}^{(\mathcal{T}_{sp})_2}(H) \right\}. \end{aligned}$$

The desired equality is obtained. $\square$

4 Semi-precompactness, semi-preconnectedness, semipre- T_1 , and semipre- T_2 degree in (L, M) -fuzzy pretopological spaces

Theorem 4.1. Let $(X, \mathcal{T}_1)$ and $(Y, \mathcal{T}_2)$ be two (L, M) -fpts's, $f : X \rightarrow Y$ be a mapping and $L = M$. Then $\mathbf{spcom}_{(\mathcal{T}_{sp})_1}(G) \wedge \mathbf{SPi}(f) \leq \mathbf{spcom}_{(\mathcal{T}_{sp})_2}(f_L^{\rightarrow}(G))$ for all $G \in L^X$.

Proof. Take any $\mathbf{c} \in M$ such that $\mathbf{c} \triangleleft \mathbf{spcom}_{(\mathcal{T}_{sp})_1}(G) \wedge \mathbf{Spi}(f)$. Then

$$\mathbf{c} \triangleleft \mathbf{Spi}(f) = \bigwedge_{H \in L^Y} \left((\mathcal{T}_{sp})_2(H) \mapsto (\mathcal{T}_{sp})_1(f_L^{\leftarrow}(H)) \right),$$

and

$$\mathbf{c} \triangleleft \mathbf{spcom}_{(\mathcal{T}_{sp})_1}(G) = \bigwedge_{\mathcal{G} \in L^X} \left\{ \left(\bigwedge_{G_1 \in \mathcal{G}} (\mathcal{T}_{sp})_1(G_1) \wedge \bigwedge_{x \in X} \left(G' \vee \bigvee_{G_1 \in \mathcal{G}} G_1 \right)(x) \right) \mapsto \bigvee_{\mathcal{H} \in 2^{(\mathcal{G})}} \bigwedge_{x \in X} \left(G' \vee \bigvee_{G_1 \in \mathcal{H}} G_1 \right)(x) \right\}$$

□

Then for all $H \in L^Y$ and $\mathcal{G} \subseteq L^X$, by Lemma 2.2, we have $\mathbf{c} \leq (\mathcal{T}_{sp})_2(H) \mapsto (\mathcal{T}_{sp})_1(f_L^{\leftarrow}(H))$, and

$$\mathbf{c} \leq \left(\bigwedge_{G \in \mathcal{U}} (\mathcal{T}_{sp})_1(G_1) \wedge \bigwedge_{x \in X} \left(G' \vee \bigvee_{G_1 \in \mathcal{U}} G_1 \right)(x) \right) \mapsto \bigvee_{\mathcal{H} \in 2^{(\mathcal{G})}} \bigwedge_{x \in X} \left(G' \vee \bigvee_{G_1 \in \mathcal{H}} G_1 \right)(x).$$

By Lemma 2.1(1), we can obtain

$$\mathbf{c} \wedge (\mathcal{T}_{sp})_2(H) \leq (\mathcal{T}_{sp})_1(f_L^{\leftarrow}(H)), \forall H \in L^Y$$

and

$$\mathbf{c} \wedge \bigwedge_{G_1 \in \mathcal{G}} (\mathcal{T}_{sp})_1(G_1) \wedge \bigwedge_{x \in X} \left(G' \vee \bigvee_{G_1 \in \mathcal{G}} G_1 \right)(x) \leq \bigvee_{\mathcal{H} \in 2^{(\mathcal{G})}} \bigwedge_{x \in X} \left(G' \vee \bigvee_{G_1 \in \mathcal{H}} G_1 \right)(x).$$

In order to prove that for all $H \subseteq L^Y$,

$$\begin{aligned} \mathbf{c} \leq \mathbf{spcom}_{(\mathcal{T}_{sp})_2}(f_L^{\rightarrow}(G)) &= \bigwedge_{\mathcal{H} \in L^Y} \left\{ \left(\bigwedge_{H_1 \in \mathcal{H}} (\mathcal{T}_{sp})_2(H_1) \wedge \bigwedge_{y \in Y} \left(f_L^{\rightarrow}(G)' \vee \bigvee_{H_1 \in \mathcal{H}} H_1 \right)(y) \right) \right. \\ &\quad \left. \mapsto \bigvee_{\mathcal{D} \in 2^{(\mathcal{H})}} \bigwedge_{y \in Y} \left(f_L^{\rightarrow}(G)' \vee \bigvee_{H_1 \in \mathcal{D}} H_1 \right)(y) \right\}, \end{aligned}$$

let $f_L^{\leftarrow}(\mathcal{H}) = \{f_L^{\leftarrow}(H_1) \mid H_1 \in \mathcal{H}\} \subseteq L^X$. Then we have

$$\begin{aligned} \mathbf{c} \wedge \bigwedge_{H_1 \in \mathcal{H}} (\mathcal{T}_{sp})_2(H_1) \wedge \bigwedge_{y \in Y} \left(f_L^{\rightarrow}(G)' \vee \bigvee_{H_1 \in \mathcal{H}} H_1 \right)(y) &\leq \mathbf{c} \wedge \bigwedge_{H_1 \in \mathcal{H}} (\mathcal{T}_{sp})_1(f_L^{\leftarrow}(H_1)) \wedge \bigwedge_{y \in Y} \left(f_L^{\rightarrow}(G)' \vee \bigvee_{H_1 \in \mathcal{H}} H_1 \right)(y) \\ &= \mathbf{c} \wedge \bigwedge_{H_1 \in \mathcal{H}} (\mathcal{T}_{sp})_1(f_L^{\leftarrow}(H_1)) \wedge \bigwedge_{x \in X} \left(G' \vee \bigvee_{H_1 \in \mathcal{H}} f_L^{\leftarrow}(H_1) \right)(x) \\ &= \mathbf{c} \wedge \bigwedge_{G_1 \in f_L^{\leftarrow}(\mathcal{H})} (\mathcal{T}_{sp})_1(G_1) \wedge \bigwedge_{x \in X} \left(G' \vee \bigvee_{G_1 \in f_L^{\leftarrow}(\mathcal{H})} G_1 \right)(x) \\ &\leq \bigvee_{\mathcal{V} \in 2^{(f_L^{\leftarrow}(\mathcal{H}))}} \bigwedge_{x \in X} \left(G' \vee \bigvee_{G_1 \in \mathcal{V}} G_1 \right)(x) \\ &= \bigvee_{\mathcal{D} \in 2^{(\mathcal{H})}} \bigwedge_{x \in X} \left(G' \vee \bigvee_{H_1 \in \mathcal{D}} f_L^{\leftarrow}(H_1) \right)(x) \\ &= \bigvee_{\mathcal{D} \in 2^{(\mathcal{H})}} \bigwedge_{y \in Y} \left(f_L^{\rightarrow}(G)' \vee \bigvee_{H_1 \in \mathcal{D}} H_1 \right)(y). \end{aligned}$$

By Lemma 2.1(1), we have

$$\mathbf{c} \leq \left(\bigwedge_{H_1 \in \mathcal{H}} (\mathcal{T}_{sp})_2(H_1) \wedge \bigwedge_{y \in Y} \left(f_L^{\rightarrow}(G)' \vee \bigvee_{H_1 \in \mathcal{H}} H_1 \right)(y) \right) \mapsto \bigvee_{\mathcal{D} \in 2^{(\mathcal{H})}} \bigwedge_{y \in Y} \left(f_L^{\rightarrow}(G)' \vee \bigvee_{H_1 \in \mathcal{D}} H_1 \right)(y).$$

Therefore

$$\begin{aligned} \mathbf{c} &\leq \bigwedge_{\mathcal{H} \subseteq L^Y} \left\{ \left(\bigwedge_{H_1 \in \mathcal{H}} (\mathcal{T}_{sp})_2(H_1) \wedge \bigwedge_{y \in Y} \left(f_L^{\rightarrow}(G)' \vee \bigvee_{H_1 \in \mathcal{H}} H_1 \right)(y) \right) \right. \\ &\quad \left. \mapsto \bigvee_{\mathcal{D} \in 2^{\mathcal{H}}} \bigwedge_{y \in Y} \left(f_L^{\rightarrow}(G)' \vee \bigvee_{H_1 \in \mathcal{D}} H_1 \right)(y) \right\} = \mathbf{spcom}_{(\mathcal{T}_{sp})_2}(f_L^{\rightarrow}(G)). \end{aligned}$$

By the arbitrariness of $\mathbf{c}$, we obtain that $\mathbf{spcom}_{(\mathcal{T}_{sp})_1}(G) \wedge \mathbf{SPi}(f) \leq \mathbf{spcom}_{(\mathcal{T}_{sp})_2}(f_L^{\rightarrow}(G))$.

The following corollaries are direct results from Theorem 4.1.

Corollary 4.2. Let $f : (X, \mathcal{T}_1) \rightarrow (Y, \mathcal{T}_2)$ be a surjective mapping between (L, M) -fpts's X and Y and $L = M$, then $\mathbf{spcom}_{(\mathcal{T}_{sp})_1}(\underline{1}_L^X) \wedge \mathbf{SPi}(f) \leq \mathbf{spcom}_{(\mathcal{T}_{sp})_2}(\underline{1}_L^Y)$.

In general topology, if G is connected and f is continuous, then $f(G)$ is connected. Now we generalize it to the setting of (L, M) -fpt as follows.

Theorem 4.3. Let $f : (X, \mathcal{T}_1) \rightarrow (Y, \mathcal{T}_2)$ be a mapping between (L, M) -fpts's X and Y , then $\mathbf{spcom}_{(\mathcal{T}_{sp})_1}(G) \wedge \mathbf{SPi}(f) \leq \mathbf{spcom}_{(\mathcal{T}_{sp})_2}(f_L^{\rightarrow}(G))$ for all $G \in L^X$.

Theorem 4.4. Let $f : (X, \mathcal{T}_1) \rightarrow (Y, \mathcal{T}_2)$ be a mapping between (L, M) -fpts's X and Y , then $\mathbf{SPc}_{(\mathcal{T}_{sp})_2}(f_L^{\rightarrow}(G))' \wedge \mathbf{SPi}(f) \leq \mathbf{SPc}_{(\mathcal{T}_{sp})_1}(G)'$ for all $G \in L^X$.

Proof. Take any $\mathbf{c} \in M$ such that $\mathbf{c} \triangleleft \mathbf{spCon}_{(\mathcal{T}_{sp})_2}(f_L^{\rightarrow}(G))' \wedge \mathbf{SPi}(f)$. By Theorems 2.21 and 3.4 (2), we obtain

$$\mathbf{c} \triangleleft \mathbf{SPc}_{(\mathcal{T}_{sp})_2}(f_L^{\rightarrow}(G))' = \bigvee_{\substack{f_L^{\rightarrow}(G) \wedge H_1 \neq \underline{0}_L^Y, f_L^{\rightarrow}(G) \wedge H_2 \neq \underline{0}_L^Y, \\ f_L^{\rightarrow}(G) \wedge H_1 \wedge H_2 = \underline{0}_L^Y, f_L^{\rightarrow}(G) \leq H_1 \vee H_2}} \left\{ (\mathcal{T}_{sp})_2(H'_1) \vee (\mathcal{T}_{sp})_2(H'_2) \right\},$$

and

$$\mathbf{c} \triangleleft \mathbf{SPi}(f) = \bigwedge_{V_3 \in L^Y} \left\{ (\mathcal{T}_{sp})_2(V'_3) \mapsto (\mathcal{T}_{sp})_1(f_L^{\rightarrow}(V_3)') \right\}.$$

This implies that there exist $H_1, H_2 \in L^Y$ with $f_L^{\rightarrow}(G) \wedge H_1 \neq \underline{0}_L^Y, f_L^{\rightarrow}(G) \wedge H_2 \neq \underline{0}_L^Y, f_L^{\rightarrow}(G) \wedge H_1 \wedge H_2 = \underline{0}_L^Y, f_L^{\rightarrow}(G) \leq H_1 \vee H_2$ such that $\mathbf{c} \leq (\mathcal{T}_{sp})_2(H'_1) \wedge (\mathcal{T}_{sp})_2(H'_2)$, and $\mathbf{c} \leq (\mathcal{T}_{sp})_2(H'_3) \mapsto (\mathcal{T}_{sp})_1(f_L^{\rightarrow}(H_3)')$ for all $H_3 \in L^Y$. This implies that there exist $H_1, H_2 \in L^Y$ with $G \wedge f_L^{\leftarrow}(H_1) \neq \underline{0}_L^X, G \wedge f_L^{\leftarrow}(H_2) \neq \underline{0}_L^X, G \wedge f_L^{\leftarrow}(H_1) \wedge f_L^{\leftarrow}(H_2) = \underline{0}_L^X, G \leq f_L^{\leftarrow}(H_1) \vee f_L^{\leftarrow}(H_2)$ where

$$\mathbf{c} \leq (\mathcal{T}_{sp})_2(H'_1) \wedge (\mathcal{T}_{sp})_2(H'_2), \quad (4.1)$$

and

$$\mathbf{c} \wedge (\mathcal{T}_{sp})_2(H'_3) \leq (\mathcal{T}_{sp})_1(f_L^{\leftarrow}(H_3)'), \text{ for all } H_3 \in L^Y. \quad (4.2)$$

From Eqs. (4.1) and (4.2), the following is obtained,

$$\begin{aligned} \mathbf{c} &= \mathbf{c} \wedge (\mathcal{T}_{sp})_2(H'_1) \wedge (\mathcal{T}_{sp})_2(H'_2) \leq (\mathcal{T}_{sp})_1(f_L^{\leftarrow}(H_1)') \wedge (\mathcal{T}_{sp})_1(f_L^{\leftarrow}(H_2)') \\ &\leq \bigvee_{\substack{G \wedge G_1 \neq \underline{0}_L^X, G \wedge G_2 \neq \underline{0}_L^X, \\ G \wedge G_1 \wedge G_2 \neq \underline{0}_L^X, G \leq G_1 \vee G_2}} \left\{ (\mathcal{T}_{sp})_1(G'_1) \vee (\mathcal{T}_{sp})_1(H'_2) \right\} = \mathbf{SPc}_{(\mathcal{T}_{sp})_1}(G)'. \end{aligned}$$

Hence, $\mathbf{c}$ is arbitrary, we have $\mathbf{SPc}_{(\mathcal{T}_{sp})_2}(f_L^{\rightarrow}(G))' \wedge \mathbf{SPi}(f) \leq \mathbf{SPc}_{(\mathcal{T}_{sp})_1}(G)'$. $\square$

Lemma 4.5. Let $f : (X, \mathcal{T}_1) \rightarrow (Y, \mathcal{T}_2)$ be a bijective mapping between (L, M) -fuzzy pretopological spaces X and Y . Then

- (1) **Semipre- $T_1(X, \mathcal{T}_1) \wedge \mathbf{SPo}(f) \leq \mathbf{Semipre-}T_1(Y, \mathcal{T}_2)$.**
 (2) **Semipre- $T_2(X, \mathcal{T}_1) \wedge \mathbf{SPo}(f) \leq \mathbf{Semipre-}T_2(Y, \mathcal{T}_2)$.**

Proof. (1) Take any $\mathbf{c} \in M$, such that

$$\mathbf{c} \triangleleft \mathbf{Semipre-}T_1(X, \mathcal{T}_1) \wedge \mathbf{SPo}(f) = \bigwedge_{g_1 \not\leq g_2} \bigvee_{g_1 \not\leq G \geq g_2} (\mathcal{T}_{sp})_1(G') \wedge \bigwedge_{G_1 \in L^X} \left((\mathcal{T}_{sp})_1(G_1) \mapsto (\mathcal{T}_{sp})_2(f_L^{-\rightarrow}(G_1)) \right).$$

Then for any $g_1, g_2 \in P(L^X)$ with $g_1 \not\leq g_2$, there exists $G \in L^X$ such that $g_1 \not\leq G \geq g_2$ and $\mathbf{c} \leq (\mathcal{T}_{sp})_1(G')$. For any $G_1 \in L^X$, $\mathbf{c} \leq (\mathcal{T}_{sp})_1(G_1) \mapsto (\mathcal{T}_{sp})_2(f_L^{-\rightarrow}(G_1))$. By Lemma 2.1 (1), we have $\mathbf{c} \wedge (\mathcal{T}_{sp})_1(G_1) \leq (\mathcal{T}_{sp})_2(f_L^{-\rightarrow}(G_1))$. In order to prove

$$\mathbf{c} \leq \mathbf{Semipre-}T_1(Y, \mathcal{T}_2) = \bigwedge_{h_1 \not\leq h_2} \bigvee_{h_1 \not\leq H \geq h_2} (\mathcal{T}_{sp})_2(H'),$$

let $h_1, h_2 \in P(L^Y)$ with $h_1 \not\leq h_2$. Since f is a bijective mapping, there exist $g_1, g_2 \in J(L^X)$ with $g_1 \not\leq g_2$, such that $h_1 = f_L^{-\rightarrow}(g_1)$ and $h_2 = f_L^{-\rightarrow}(g_2)$. From $g_1 \not\leq g_2$, there exists $G \in L^X$, with $g_1 \not\leq G \geq g_2$ such that $\mathbf{c} \leq (\mathcal{T}_{sp})_1(G')$. Then $h_1 = f_L^{-\rightarrow}(g_1) \not\leq f_L^{-\rightarrow}(G) \geq f_L^{-\rightarrow}(g_2) = h_2$. Since f is a bijective mapping, we have

$$\mathbf{c} = \mathbf{c} \wedge (\mathcal{T}_{sp})_1(G') \leq (\mathcal{T}_{sp})_2(f_L^{-\rightarrow}(G')) = (\mathcal{T}_{sp})_2(f_L^{-\rightarrow}(G')).$$

This implies

$$\mathbf{c} \leq \bigwedge_{h_1 \not\leq h_2} \bigvee_{h_1 \not\leq H \geq h_2} (\mathcal{T}_{sp})_2(H') = \mathbf{Semipre-}T_1(Y, \mathcal{T}_2).$$

By the arbitrariness of $\mathbf{c}$, we obtain

$$\mathbf{Semipre-}T_1(X, \mathcal{T}_1) \wedge \mathbf{SPo}(f) \leq T_1(Y, \mathcal{T}_2).$$

Other case is similarly proved. □

Lemma 4.6. Let $f : (X, \mathcal{T}_1) \rightarrow (Y, \mathcal{T}_2)$ be a bijective mapping between (L, M) -fuzzy pretopological spaces X and Y . Then

- (1) **Semipre- $T_1(Y, \mathcal{T}_2) \wedge \mathbf{SPi}(f) \leq \mathbf{Semipre-}T_1(X, \mathcal{T}_1)$.**
 (2) **Semipre- $T_2(Y, \mathcal{T}_2) \wedge \mathbf{SPi}(f) \leq \mathbf{Semipre-}T_2(X, \mathcal{T}_1)$.**

Proof. It is proved in the same way as Lemma 4.5. □

By combining Lemmas 4.5, 4.6 and Definition 3.2(1), we can state the following theorem.

Theorem 4.7. Let $f : (X, \mathcal{T}_1) \rightarrow (Y, \mathcal{T}_2)$ be a bijective mapping between (L, M) -fpts's X and Y . Then

- (1) **Semipre- $T_1(X, \mathcal{T}_1) \wedge \mathbf{Semipre-Hom}(f) \leq \mathbf{Semipre-}T_1(Y, \mathcal{T}_2)$, $\mathbf{Semipre-}T_1(X, \mathcal{T}_2) \wedge \mathbf{Semipre-Hom}(f) \leq \mathbf{Semipre-}T_1(X, \mathcal{T}_1)$.**
 (2) **Semipre- $T_2(X, \mathcal{T}_1) \wedge \mathbf{Semipre-Hom}(f) \leq \mathbf{Semipre-}T_2(Y, \mathcal{T}_2)$, $\mathbf{Semipre-}T_2(Y, \mathcal{T}_2) \wedge \mathbf{Semipre-Hom}(f) \leq \mathbf{Semipre-}T_2(X, \mathcal{T}_1)$.**

The following corollary is obtained from Theorem 4.8.

Corollary 4.8. Let $f : (X, \mathcal{T}_1) \rightarrow (Y, \mathcal{T}_2)$ be a bijective mapping between (L, M) -fpts's X and Y . Then

- (1) **Semipre- $T_1(X, \mathcal{T}_1) \wedge \mathbf{Semipre-Hom}(f) = \mathbf{Semipre-}T_1(Y, \mathcal{T}_2) \wedge \mathbf{Semipre-Hom}(f)$.**
 (2) **Semipre- $T_2(X, \mathcal{T}_1) \wedge \mathbf{Semipre-Hom}(f) = \mathbf{Semipre-}T_2(Y, \mathcal{T}_2) \wedge \mathbf{Semipre-Hom}(f)$.**

5 Conclusion

In this study, we introduced and showed new degrees of weak forms of functions in (L, M) -fuzzy pretopological spaces by using the implication operations and Ghareeb's operators. We also investigated some properties of semi-preopen, semi-precontinuous, and semi-preirresolute degree of functions in (L, M) -fuzzy pretopology. We also proved that the function can be regarded as semi-preopenness, semi-precontinuity, and semi-preirresoluteness to some degree. Furthermore, various relationships with semi-precompactness, semi-preconnectedness, $\text{Semipre-}T_1$, and $\text{Semipre-}T_2$ have been constructed and analyzed.

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