Research Article

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Volume Optimization of Solid Waste Landfill Using Voronoi Diagram Geometry

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Abstract: The loose material, falling freely on the flat area, creates a geometric object similar to the roof. This object, which is in fact an embankment, the authors described and examined in previous works. They stated that the top view of such an object is a flat diagram of Voronoi. In view of the need to enlarge the landfill by combining the existing disjoint embankments, the authors were encouraged to apply the results obtained in practice. They also provided a detailed methodology for creating such embankments. In particular, they used the standard version of AutoCAD software and built a 3D model of the landfill based on solid operations. This parametric approach to design enables the rapid determination of the volume and surface area and fits in the BIM technology of designing engineering objects.

Keywords: geometry of roofs, geometry of embankments, Voronoi diagram, geometric optimization

1 Introduction

The roof geometry and Voronoi diagrams found applications in many fields ranging from photogrammetry with (semi) automatic reconstruction of city models (Bernd et al.; 2003, Dikaikou et al., 2003; Flamanc et al., 2003; Laycock and Day, 2003; Swift et al., 2003), reconstruction of roads from satellite images (Akel, 2005; Thom, 2005), in cartography (Haunert and Sester, 2004; Wang et al., 2011), in the analysis of the morphology of materials with grain structure (Akel, 2005), in medicine for the representation, reconstruction and visualization of human organs (Barequet et al., 2003), in the topographic analysis of the area,

computer graphics in the field of object shape recognition and analysis (Lawlor, 2004), in transport of ready-mixed concrete (Koźniewski et al., 2017), and in designing of earthwork organization (Koźniewski, 2018).. The solution presented in this paper is a new example of applying the above-mentioned concepts in the practical field of designing the shape of embankments.

A direct, practical reason for dealing with the problem solved at work was the need to enlarge the landfill without significantly increasing the area (base) occupied by the embankment.

The limited possibilities of expanding the waste disposal area, and more precisely lack of such possibilities, prompted the management of the Municipal Waste Utilization Plant to seek new solutions by reconstructing the geometry of the existing quarters in order to find additional optimal volume reserves for the repository. The authors of the present paper were commissioned to perform this reconstruction. The scope of work included the construction (forming) of the optimal 3D geometric model of the waste storage facility within the existing quarters with the determination of its capacity (absorption), horizontal areas (i.e. area with a small minimum inclination of 2%), surface of access roads and slopes (with a high slope of 1:0.58), after the adoption (in the assumption):

- heights: 30 meters,
- acceptable slope angle: 60° (i.e. inclination 1: 0.58), including six shelves (in a vertical distance of five meters), with a width of 1 m,
- access road with a width of 4 m for a central shelf with a width of 4 m (at a height of 15 m) and a top platform without a snail-shaped solution,
- location of the road, on a shelf with a width of 4 m (at a height of 15 m) with the possibility of moving the means of transport, with a turning radius of 13 m,
- with the minimum width of ten meters of the top platform of the headquarters.

The study did not cover any other aspects of the construction of the quarters, such as, for example, slope stability analysis, leachate removal system, post-process waste disposal system, etc.

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2 Geometry of roofs (straight skeletons) and Voronoi diagrams in application to create embankments

As a theoretical basis, the research results of Koźniewski (Koźniewski, 2004a, 2004b, 2016, 2018) regarding roof geometry and Koźniewski et al. (Koźniewski et al., 2013) concerning, among others, applications of Voronoi diagrams for embankment modeling were adopted. The source setting of roof geometry lies in the area of teaching classical descriptive geometry (Grochowski, 2018) and roof geometry (Koźniewski, 2016). The authors of this paper found a new look through the algorithmization and computer implementations of roof geometries (straight skeletons) in (Aichholzer et al., 1995; Aichholzer and Aurenhammer, 1996), and the Voronoi diagrams in (De Berg et al., 2000) and in countless publications not quoted here.

Figure 1: Embankment construction: a) on the base, which is a set of any shoreline, b) through a closed broken closed (closed chain of sections) (Koźniewski et al., 2013)

Figure 2: Roof: a) a straight skeleton for a polygon, b) 3D model of the roof (Koźniewski et al., 2013)

The method described in (Koźniewski et al., 2013), repeated and presented in this paper in the form of drawings and photographs, allows to design and model embankments from homogeneous material of any basis. The solid obtained by means of this method has an optimal volume assuming a predetermined slope angle. In addition, the same inclination angle ensures optimum cross-section area (Koźniewski et al., 2017). A virtual example of such an object is shown in Figures 1, 2 and 3. Figure 4 shows a photograph of a laboratory model.

Figure 3: The roof model: a) embankment over closed broken line with greater accuracy; b) the model of the embankment of a given height (Koźniewski et al., 2013)

Figure 4: The actual laboratory embankment model: a) top view; b) axonometric view (Koźniewski et al., 2013)

3 Construction of the model of solid waste landfill – case study

In the design of the model, the authors adopted the assumptions discussed in the introduction to this article.

3.1: As base contour, the line from the geodetic field map of the quarters in the study marked by “4A” (Karmolińska – Słotkowska, 2013) was assumed. This line was obtained from the connections of three separate (up to now) polygons. The volume of the original component field 4A was calculated at $260,000 \text{ m}^3$. In the case of municipal waste, it equals about 226,000 Mg of waste for storage (Karmolińska – Słotkowska, 2013). In the designed model, it was assumed that the field base would consist of three disjoint polygons that were in contact with one side and the way of exploitation assumed the formation of three blocks with different crown heights (Figures 5, 6). Since such a design significantly limited the maximum volume of this field, the proposal was made to connect the base during exploitation, liquidate three solids in favor of one (Figure 7) and calculate how much this change will increase the volume of this solid.

3.2: Based on the contour of the line from the geodetic map and accepted in accordance with the contract, at the client's request, a fairly large gradient slope (1:0.58), the authors of this study (E. Koźniewski and M. Orłowski) constructed a 3D model (Figures 8, 9) solid waste landfill and they made the necessary calculations (Table 1) and drawings of profile (vertical) sections, which horizontal lines were marked in Figures 10, 11. The volume of the designed storage site is about $655,461 \text{ m}^3$. This gives more than two and a half times the increase in storage capacity in rela-

Figure 5: Surveying ("outbound") map of the field of the quarters marked 4A (Karmolińska – Słotkowska, 2013)

Figure 6: The original field basis consisting of three disjoint polygons quarters marked by "4A" (source: the authors' own edition)

Figure 7: The basis of the field after merging of three disjoint pieces into one polygon (source: the authors' own edition)

tion to the size of $260,000 \text{ m}^3$ obtained in the study (Karmolińska – Słotkowska, 2013). The volume of the slope waste landfill, due to the assumed slope 1:0.58, and the theory of Voronoi diagrams (Koźniewski et al., 2013), taking into account the deliberations in paragraph 3.3 of this paper, is the maximum for a given base.

Figure 8: Virtual 3D model of the solid waste landfill – axonometric view (source: the authors' own edition)

3.3: In the aspect of the accuracy of the embankment volume calculations one should ask a question regarding the selection of the number of sides of the polygon inscribed in the base contour. For this purpose, it is advisable to carry out auxiliary calculations by increasing the number

Figure 9: Virtual 3D model of the solid waste landfill - a view with the visualization of shelves and road (source: the authors' own edition)

Figure 10: Horizontal projections of cross-section lines of solid waste landfill marked on an initial geodetic map (source: the authors' own edition)

Figure 11: Horizontal projection (top view) of the location of section lines of the solid waste landfill plotted on a 3D model view (source: the authors' own edition)

of sides of the polygon while maintaining the normality of the string of inscribed polygons. By the normal sequence of polygons inscribed in the curve, the authors adopted such a sequence that the limit of the length of the sides of polygons with the largest length is zero. In the case of our design, we first inscribed the 44-gon in the base contour, next the 84-gon. The authors created models of embankments E_{44} and E_{84} , with inclination angle of 45 degrees, for 44-gon and 84-gon, respectively: $\text{vol}(E_{44}) = 424332.8[\text{m}^3]$, $\text{vol}(E_{84}) = 425436.9[\text{m}^3]$. The quotient $q(1)$

Table 1: Volume and area of slopes (sloping) and shelves (horizontal) obtained in the study

The parameters of the designed solid waste landfill	Volume [m ³]
The volume of the designed solid waste landfill	655 461
The type of surface due to the slope	Area [m ²]
Horizontal surfaces (i.e. surfaces with a slope of 2%)	11 371
Road	634
Sloping surfaces with a slope of 60°	25 811
Sloping surfaces with a slope of 8°	8 086

Figure 12: Axonometric view of the location of section lines based on the solid waste landfill (source: the authors' own edition)

of the difference of the volume of the embankment for the 44-gon (vol(E₄₄)) and 84-gon (vol(E₈₄)) and the volume of the embankment for the 44-gon (vol(E₄₄)), i.e.

$$q = \frac{\text{vol}(E_{44}) - \text{vol}(E_{84})}{\text{vol}(E_{44})} \cdot 100\% = 0,260211\% \quad (1)$$

described in percentage points was less than 0.3%. So the authors decided that adopting the 44-sided polygon was enough. This polygon was used to build the final model of a solid waste landfill (Figures 8, 9, 12).

Figure 13: Horizontal projection of the detailed structure of the solid waste landfill (source: the authors' own edition)

3.4: The use of standard AutoCAD software, compatible with other CAD programs, allows further processing of geometric objects in other programs and inclusion in a larger BIM system. The obtained objects are then available in parametric form and can be dynamically updated, for example during the actual creation of a landfill, monitoring

the capacity of the temporary embankment, forecasting free space, etc.

4 Conclusions

- 1: The adopted landfill model was based on a polygon inscribed in the contour, which allowed to use the structure induced by the Voronoi diagram for the polygon. In the general case, with a sufficiently large number of sides of the polygon, the difference in volume between the solid built above the straight skeleton and the Voronoi diagram is small. Then one can solve the geometric roof instead of the geometric embankment, which greatly simplifies the construction.
- 2: Due to the polygonal nature of the base, large concave angles (close to the flat angle), the proposed landfill, it was possible to successfully use the roof structure instead of the Voronoi diagram. Then, the part of the dihedral and the part of the surface of the cone inscribed in this dihedral limited by the tangents form "well-fitting" surfaces.
- 3: The presented solution has allowed to increase the volume of the repository 2.5 times.
- 4: The solution in the AutoCAD environment allows for use it in BIM technology.

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