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A Constitutive Equation for Predicting Elevated Temperature Flow Behavior of BFe10-1-2 Cupronickel Alloy through Double Multiple Nonlinear Regression

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Abstract: Constitutive analysis for elevated temperature flow behavior of BFe10-1-2 alloy was carried out by using experimental stress–strain data from isothermal hot compression tests on a Gleeble-3800 thermo-mechanical simulator, in a wide of temperature range of 1,023–1,273 K, and strain rate range of $0.001\text{--}10\text{ s}^{-1}$. A constitutive equation based on double multiple nonlinear regression (DMNR) was proposed considering the independent effects of strain, strain rate, temperature and their interrelation. The predicted flow stress data obtained from the developed equation based on DMNR was compared with the experimental data. Correlation coefficient (R), average absolute relative error (AARE) and relative errors were introduced to verify the validity of the developed constitutive equation. The results showed that the developed constitutive equation based on DMNR could predict flow stress of BFe10-1-2 alloy with good correlation and generalization.

Keywords: BFe10-1-2 cupronickel alloy, constitutive equation, double multivariate nonlinear regression, flow stress

Introduction

As a typical Cu–Ni alloy, BFe10-1-2 cupronickel alloy has been widely employed as cooling-condition material in shipping and seawater desalted industry due to its attractive combinations of characteristics in terms of excellent

corrosion resistance and anti-fouling properties [1]. Nowadays, the main processing method of this kind alloy is semi-solid casting ingots and followed hot extrusion, which results in many problems, such as long process time, high energy consumption, low product yield and high cost [2–5]. Hence, a thorough study on elevated temperature flow behavior of BFe10-1-2 cupronickel alloy is of vital importance to properly design the processing parameters. However, the high temperature deformation behavior of metals is a very complex process. The softening and hardening mechanisms are affected by processing parameters such as deformation temperature, strain rate and strain [6]. Nowadays, finite element method (FEM) has become a common tool to find out the optimum thermo-mechanical process parameters. Constitutive equation can represent the flow behavior of metal material, and is used as input to the FEM code for simulating the response of materials under the specified processing conditions [7]. Reliability of the outputs of these simulations depends primarily on the accuracy of prediction of the constitutive equation. Therefore, the reliability of simulation results is seriously influenced by the accuracy of the constitutive equation.

For the past few years, several empirical, semi-empirical, phenomenological and physically based constitutive models have been proposed. Among these constitutive models, the Johnson-Cook (JC) model, the Zerilli-Armstrong (ZA) model and the Arrhenius-type model are currently a part of commercial FEM software. The JC model has been widely employed owing to the fact that it requires less number of test data for evaluation of the material constants and can be used for many metal materials in a wide range of deformation temperatures and strain rates [8]. The ZA model considers coupled strain and temperature effects, and has been employed to analyze different face-centered cubic and body-centered cubic materials over various strain rates at the temperatures between room temperature and $0.6T_m$ (T_m is the melting point) [9]. The Arrhenius-type equation has been successfully used to describe the high temperature flow behavior of metal materials, such as martensitic steel [10], AZ61 Mg alloy [11] and aluminum matrix

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composites [12]. On the basis of the orthogonal experiment and variance analysis, Xiao [13] proposed a constitutive model to describe the elevated temperature flow behavior of TiNiNb alloy. However, this constitutive model ignores the combined effect of influence factors on flow stress, which may decrease the accuracy of the constitutive equation. Then, a double multiple nonlinear regression (DMNR) method with higher accuracy is proposed by Yuan [14] to predict the flow stress of Ti-6Al-4V alloy, and then was intruded by Hussain [15] to study thermo-mechanical processing of INCONEL 718 alloy. Thereafter, Li [16] proposed a modified DMNR constitutive equation to predict the flow stress of 1,060 pure aluminum with a good correlation and precision. Then this method was used by Cai [17] to analyze the flow behavior of BFe10-1-2 cupronickel alloy, and could give an accurate and precise estimate of the flow stress. However, modified DMNR constitutive model is very complex, and thereby the time required for evaluating these material constants of the constitutive equation is much longer than that of the other models. To sum up, an ideal constitutive equation should involve a reasonable number of material constants that can be calculated from limited experimental data, and can predict the flow behavior of the material with adequate accuracy and reliability over a wide processing range. The main purpose of this study is to find out the suitable constitutive equation to predict the elevated temperature flow behavior of BFe10-1-2 cupronickel alloy with considering the combined effect of strain, strain rate and deformation temperature. To achieve this objective, isothermal hot compression tests were performed in a wide range of temperatures (1,023, 1,073, 1,123, 1,173, 1,223 and 1,273 K) and strain rates of 0.001, 0.01, 0.1, 1 and 10 s^{-1} . Finally, comparative analysis between calculated and actual results was conducted to verify the developed constitutive equation.

Experimental procedure

The chemical composition (wt. %) of BFe10-1-2 cupronickel alloy investigated in the present study is: Ni = 10.80, Mn = 2, Fe = 1.38, Cu = bal. The original microstructure of as received BFe10-1-2 alloy is shown in Figure 1.

The studied alloy is processed into cylindrical specimens with a height of 15 mm and a diameter of 10 mm for hot compression tests. In order to minimize the influence of friction, the flat ends of each specimen were recessed a

Figure 1: Microstructure of as received BFe10-1-2 cupronickel alloy.

depth of 0.1 mm groove to entrap the lubricant, as shown in Figure 2. The specimens prior to isothermal compression were heated to the test temperature at a rate of 1°C/s and held for 3 min on the deformation conditions for the purpose of eliminating thermal gradients. Thereafter, isothermal compression experiments were carried out on a Gleeble-3800 thermo-simulation simulator in the strain rate range of $0.001\text{--}10\text{ s}^{-1}$ and the temperatures range of 1,023–1,273 K. According to the practical production, the compression ratio is selected as 55 %. Therefore, the true strain of specimen after deformation is about 0.8. After deformation, the specimens were cooled to room temperature in air. The true stress and strain data is acquired from load-stroke data by follows:

Figure 2: Typical appearance of the BFe10-1-2 cupronickel alloy specimens.

$$\sigma = P(1 - e)/A$$

$$\varepsilon = \ln(1 - e)$$

where σ is the true stress (MPa), ε is the true strain, P is the loading force (KN), A is the section area of the original specimens (mm^2), and e is the compression ratio.

Results and discussion

Flow stress

Figure 3 illustrates the flow stress curves at the constant strain rate of 1,173 K (Figure 3(a)) and at a constant deformation temperature of 10 s^{-1} (Figure 3(b)). It can be seen that deformation temperature and strain rate have obvious influence on flow stress curves. The flow stress increases with the increase of strain rate, and decreases with the increase of deformation temperature. At the initial strain (lower than 0.05), the stress increases rapidly due to the continuous accumulation of dislocations. Then, a near steady-state region is achieved in which the flow stress is observed to remain nearly constant with further straining, which give the dynamic recovery (DRV) characteristics of the material.

Double multivariate nonlinear regression

The high temperature flow behavior of BFe10-1-2 cupronickel alloy represented by the constitutive equation consisted with flow stress and influence parameters. Figure 4 gives the relationship between the flow stress and influence parameters. As can be seen from Figure 4, x_i ($i = 1, 2, 3, \dots, n$) are the test factors; h_j ($j = 1, 2, 3, \dots, m$) are the

analysis factors; y is the objective function representing the flow stress which is set as, and y is a pan-function of x_i and h_j , $y = F[h_1, h_2, \dots, h_m] = f(x_1, x_2, \dots, x_n)$; ω_{ij} and ω_j are the converged weights, which represent the contribution of test factors x_i to analysis factors h_j and analysis factors h_j to objective function y , respectively. The analysis factors h_j are the function of test factors x_i and can be obtained by converged weights ω_{ij} which represents the contribution of x_i on h_j . Therefore, the objective function y can be obtained from the contribution functions $f[h_j]$, and the converged weights ω_j represent the contribution of functions $f[h_j]$ on the objective function y . Nonlinear regression based on the experimental data is employed to obtain the contribution function that defines the relationship between analysis factor h_j and objective function y described as $f_j = f[h_j]$ (a contribution function). The converged weights ω_j can be calculated by using the multivariate nonlinear regression based on least square algorithm.

In order to acquire the constitutive equation for the elevated temperature deformation behavior of BFe10-1-2 cupronickel alloy, independent factors strain, strain rate and deformation temperature are defined as the test factors x_i . The analysis factors are strain, strain rate, deformation temperature, the combined effect of strain and strain rate, the combined effect of strain and deformation temperature, and the combined effect of strain rate and deformation temperature, as shown in eq. (1).

$$\begin{aligned} h_1(x_1) &= \varepsilon \\ h_2(x_2) &= \dot{\varepsilon} \\ h_3(x_3) &= 1/T \\ h_4(x_1, x_2) &= \varepsilon/\dot{\varepsilon} \\ h_5(x_2, x_3) &= T \ln \dot{\varepsilon} \\ h_6(x_3, x_1) &= 1/(T\varepsilon) \end{aligned} \quad (1)$$

Figure 3: Flow curves of BFe10-1-2 cupronickel alloy obtained at various deformation conditions (a) 1,073 K, (b) 10 s^{-1} .

Figure 4: Relationship between the flow stress and influence parameters.

After determination of all the analysis factors mentioned above, taking the mean flow stress values of each analysis factor, the contribution function f_j can be acquired by using the nonlinear regression on the basis of the experimental data scatter plot, where:

$$\begin{aligned}
 f_1 &= f_\varepsilon[h_1(x_1)] \\
 f_2 &= f_{\dot{\varepsilon}}[h_2(x_2)] \\
 f_3 &= f_T[h_3(x_3)] \\
 f_4 &= f_{\varepsilon-\dot{\varepsilon}}[h_4(x_1, x_2)] \\
 f_5 &= f_{\varepsilon-T}[h_5(x_2, x_3)] \\
 f_6 &= f_{\varepsilon-T}[h_6(x_3, x_1)]
 \end{aligned} \quad (2)$$

After the determination of all the contribution function f_j mentioned above, the constitutive equation based on the double multivariate nonlinear regression (DMNR) can be obtained. During high temperature deformation, constitutive equation of metal materials can be expressed as:

$$\sigma = f(\varepsilon, \dot{\varepsilon}, T)$$

The effect of strain, strain rate and temperature on the flow stress can be expressed in another form as [13]:

$$\sigma = \sigma_0 f_\varepsilon f_{\dot{\varepsilon}} f_T \quad (3)$$

where σ_0 is the initial stress of BFe10-1-2 cupronickel alloy under current experimental conditions; f_ε , $f_{\dot{\varepsilon}}$ and f_T are the influence coefficients of the strain, strain rate and temperature, respectively. On the basis of the analysis of the relationship between analysis factors h_j and the regression function y , eq. (3) can be modified as follows:

$$\sigma = \sigma_0 \cdot f_\varepsilon \cdot f_{\dot{\varepsilon}} \cdot f_T \cdot f_{\varepsilon-\dot{\varepsilon}} \cdot f_{\varepsilon-T} \cdot f_{\varepsilon-T} \quad (4)$$

Then, eq. (4) can be simplified as eq. (5):

$$\sigma = f_0 \cdot f_1 \cdot f_2 \cdot f_3 \cdot f_4 \cdot f_5 \cdot f_6 \quad (5)$$

where $f_0 = \sigma_0$, $f_1 = f_\varepsilon$, $f_2 = f_{\dot{\varepsilon}}$, $f_3 = f_T$, $f_4 = f_{\varepsilon-\dot{\varepsilon}}$, $f_5 = f_{\varepsilon-T}$ and $f_6 = f_{\varepsilon-T}$. Taking natural logarithms of both sides of eq. (5), eq. (6) can be obtained as follows:

$$\ln \sigma = \ln f_0 + \ln f_1 + \ln f_2 + \ln f_3 + \ln f_4 + \ln f_5 + \ln f_6 \quad (6)$$

Considering the converged weights ω_j , the following expression can be derived:

$$\begin{aligned}
 \ln \sigma &= \ln f_0 + \omega_1 \ln f_1 + \omega_2 \ln f_2 + \omega_3 \ln f_3 + \omega_4 \ln f_4 + \omega_5 \ln f_5 \\
 &\quad + \omega_6 \ln f_6
 \end{aligned} \quad (7)$$

Then eq. (7) can be rewritten as:

$$\ln \sigma = \ln f_0 + \sum_{j=1}^6 \omega_j \ln f_j \quad (8)$$

Based on DMNR, ω_j and f_0 in the constitutive equation mentioned above can be acquired, and the obtained constitutive equation based on DMNR can be expressed as:

$$\sigma = f_0 \cdot f_1^{\omega_1} \cdot f_2^{\omega_2} \cdot f_3^{\omega_3} \cdot f_4^{\omega_4} \cdot f_5^{\omega_5} \cdot f_6^{\omega_6} = f_0 \prod_{j=1}^m f_j^{\omega_j} \quad (9)$$

As for BFe10-1-2 cupronickel alloy, the constitutive equation based on the DMNR is given by following formula:

$$\sigma = f_0 \cdot f_{\varepsilon}^{\omega_1} \cdot f_{\dot{\varepsilon}}^{\omega_2} \cdot f_T^{\omega_3} \cdot f_{\varepsilon-\dot{\varepsilon}}^{\omega_4} \cdot f_{\dot{\varepsilon}-T}^{\omega_5} \cdot f_{\varepsilon-T}^{\omega_6} \quad (10)$$

The classification of the test factors and analysis factors according to independent factors and interactive factors, and levels of BFe10-1-2 cupronickel alloy are given in Table 1.

Determination of the contribution functions f_j

The mean flow stress values $\bar{\sigma}_{x_i-x_i}$ are employed to determine the constitution equation. $\bar{\sigma}_{\varepsilon}$ is the mean value of flow stress at all temperatures and strain rates, $\bar{\sigma}_{\dot{\varepsilon}}$ is the mean flow stress at all temperatures and strain, $\bar{\sigma}_T$ is the mean flow stress at all strain and strain rate, $\bar{\sigma}_{\varepsilon-\dot{\varepsilon}}$ is the mean value of flow stress at all temperatures, $\bar{\sigma}_{\dot{\varepsilon}-T}$ is the mean value of flow stress at all strain, and $\bar{\sigma}_{\varepsilon-T}$ is the mean value of all strain rates. K_{ε} , $K_{\dot{\varepsilon}}$ and K_T are the levels

Table 1: List of factors of BFe10-1-2 cupronickel alloy.

Level	Single factor			Interactive factor		
	f_{ε}	$f_{\dot{\varepsilon}}$	f_T	$f_{\varepsilon-\dot{\varepsilon}}$	$f_{\dot{\varepsilon}-T}$	$f_{\varepsilon-T}$
1	$\varepsilon_i \left(\begin{array}{l} i=1, 2, \dots, 8. \\ \varepsilon_i=0.1, 0.2, \dots, 0.8. \end{array} \right)$	0.001	1,023	$\varepsilon_i - 0.001$	1,023-0.001	$\varepsilon_i - 1, 023$
2			1,073		1,073-0.001	$\varepsilon_i - 1, 073$
3			1,123		1,123-0.001	$\varepsilon_i - 1, 123$
4			1,173		1,173-0.001	$\varepsilon_i - 1, 173$
5			1,223		1,223-0.001	$\varepsilon_i - 1, 223$
6			1,273		1,273-0.001	$\varepsilon_i - 1, 273$
7		0.01	1,023	$\varepsilon_i - 0.01$	1,023-0.01	$\varepsilon_i - 1, 023$
8			1,073		1,073-0.01	$\varepsilon_i - 1, 073$
9			1,123		1,123-0.01	$\varepsilon_i - 1, 123$
10			1,173		1,173-0.01	$\varepsilon_i - 1, 173$
11			1,223		1,223-0.01	$\varepsilon_i - 1, 223$
12			1,273		1,273-0.01	$\varepsilon_i - 1, 273$
13		0.1	1,023	$\varepsilon_i - 0.1$	1,023-0.1	$\varepsilon_i - 1, 023$
14			1,073		1,073-0.1	$\varepsilon_i - 1, 073$
15			1,123		1,123-0.1	$\varepsilon_i - 1, 123$
16			1,173		1,173-0.1	$\varepsilon_i - 1, 173$
17			1,223		1,223-0.1	$\varepsilon_i - 1, 223$
18			1,273		1,273-0.1	$\varepsilon_i - 1, 273$
19		1	1,023	$\varepsilon_i - 1$	1,023-1	$\varepsilon_i - 1, 023$
20			1,073		1,073-1	$\varepsilon_i - 1, 073$
21			1,123		1,123-1	$\varepsilon_i - 1, 123$
22			1,173		1,173-1	$\varepsilon_i - 1, 173$
23			1,223		1,223-1	$\varepsilon_i - 1, 223$
24			1,273		1,273-1	$\varepsilon_i - 1, 273$
25		10	1,023	$\varepsilon_i - 10$	1,023-10	$\varepsilon_i - 1, 023$
26			1,073		1,073-10	$\varepsilon_i - 1, 073$
27			1,123		1,123-10	$\varepsilon_i - 1, 123$
28			1,173		1,173-10	$\varepsilon_i - 1, 173$
29			1,223		1,223-10	$\varepsilon_i - 1, 223$
30			1,273		1,273-10	$\varepsilon_i - 1, 273$
...						
240	0.8	10	1273	0.8-10	1,273-10	0.8-1273

of strain, strain rate and deformation temperature, as given in eq. (11):

$$\begin{aligned}
 \bar{\sigma}_\varepsilon &= \sum_{T-\dot{\varepsilon}} \sigma / (K_T \cdot K_{\dot{\varepsilon}}) \\
 \bar{\sigma}_{\dot{\varepsilon}} &= \sum_{T-\varepsilon} \sigma / (K_T \cdot K_{\dot{\varepsilon}}) \\
 \bar{\sigma}_T &= \sum_{\varepsilon-\dot{\varepsilon}} \sigma / (K_{\varepsilon} \cdot K_{\dot{\varepsilon}}) \\
 \bar{\sigma}_{\varepsilon-\dot{\varepsilon}} &= \sum_T \sigma / K_T \\
 \bar{\sigma}_{\dot{\varepsilon}-T} &= \sum_{\varepsilon} \sigma / K_{\varepsilon} \\
 \bar{\sigma}_{\varepsilon-T} &= \sum_{\dot{\varepsilon}} \sigma / K_{\dot{\varepsilon}}
 \end{aligned}
 \quad (11)$$

In this research, the values of strain are selected in the range of 0.1–0.8 at an interval of 0.1, the strain rates are chosen as 0.001, 0.01, 0.1, 1 and 10 s⁻¹, and temperatures are selected as 1,023–1,273 K at an interval of 50 K. Therefore, the values of K_ε , $K_{\dot{\varepsilon}}$ and K_T are 8, 5 and 6, respectively.

Figure 4 illustrates the relationship between $\bar{\sigma}_\varepsilon$ and ε . It can be seen from Figure 5 that a cubic spline fit can be employed to express the relationship between $\varepsilon_i - 1,023$ and strain (eq. (12)).

$$\bar{\sigma}_\varepsilon = 47.45855 + 113.97544\varepsilon - 150.9717\varepsilon^2 + 68.65808\varepsilon^3 \quad (12)$$

The contribution function f_ε of the independent factor ε is given by the follows:

$$f_\varepsilon = 47.45855 + 113.97544\varepsilon - 150.9717\varepsilon^2 + 68.65808\varepsilon^3 \quad (13)$$

By taking the average flow stress at different strain rates ($\bar{\sigma}_{\dot{\varepsilon}}$), the graph of $\ln(\bar{\sigma}_{\dot{\varepsilon}})$ and $\ln(\dot{\varepsilon})$ can be obtained, as illustrated in Figure 6. A linear fit can be found to represent $\ln(\bar{\sigma}_{\dot{\varepsilon}})$ and $\ln(\dot{\varepsilon})$, as given in eq. (14).

Figure 6: Relationship between $\ln(\bar{\sigma}_{\dot{\varepsilon}})$ and $\ln(\dot{\varepsilon})$.

$$\ln(\bar{\sigma}_{\dot{\varepsilon}}) = 4.44243 + 0.09768 \ln(\dot{\varepsilon}) \quad (14)$$

Figure 5: Relationship between $\bar{\sigma}_\varepsilon$ and ε .

Thereafter, by taking exponentiation of both sides of eq. (14) and neglecting the constant coefficient, the contribution function $f_{\dot{\epsilon}}$ of the independent factor $\dot{\epsilon}$ can be obtained as follows:

$$f_{\dot{\epsilon}} = \dot{\epsilon}^{0.09768} \quad (15)$$

It can be found from eq. (15) that the mean value of strain rate sensitivity exponent m for BFe10-1-2 cupronickel alloy is 0.09768. Taking slopes between the adjacent two points of strain rate in Figure 6, the relationship between m and strain rate can be obtained, as illustrated in Figure 7. It should be noted that the maximum value of m is only 0.114. eq. (16) gives a simple function with a certain physical meaning [18]:

$$\sigma^5 = A \cdot \dot{\epsilon} \cdot \exp(Q_{dm}/RT) \quad (16)$$

where A is a constant term, Q_{dm} represents the activation energy for dislocation directional migration, and R is the universal gas constant ($8.314 \text{ J} \cdot \text{mol}^{-1} \cdot \text{K}^{-1}$). It can be found from eq. (16) that m value maintains at a steady value of 0.2 under steady-state deformation undergoing DRV, and this value is in good agreement with the test results AZ31 [19] alloy (about 0.19). The values of m keep at a higher level over 0.3 during hot processing with DRX softening mechanism [18]. Therefore, the flow softening of BFe10-1-2 cupronickel alloy is led mainly by DRV and not by DRX. Figure 8 gives the metallographic structure of BFe10-1-2 alloy under the deformation condition of 1,023 K in 0.01 s^{-1}

Figure 7: Relationship between m and $\ln(\dot{\epsilon})$.

Figure 8: Metallographic structure of BFe10-1-2 alloy under the deformation condition of (a) 1,023 K 0.01 s^{-1} , (b) 1,223 K 0.1 s^{-1} .

and 1,223 K in 0.1 s^{-1} . It can be seen from figures that the grains after deformation tend to be elongated perpendicular to the compression direction, but no DRX grains can be observed.

By taking the average flow stress at different temperatures ($\bar{\sigma}_T$), the graph of $\ln(\bar{\sigma}_T)$ and $1/T$ can be obtained, and a linear fit can be observed to represent $\ln(\bar{\sigma}_T)$ and $1/T$, as given in Figure 9 and eq. (17).

$$\ln(\bar{\sigma}_T) = -0.13432 + 4964.81956/T \quad (17)$$

Then taking exponentiation of both sides of eq. (17) and neglecting the constant coefficient, the contribution function f_T of the independent factor T can be obtained as follows:

$$f_T = \exp\left(\frac{4964.8196}{T}\right) = \exp\left(\frac{0.09768 \times 422.58 \times 10^3}{RT}\right) \quad (18)$$

And

$$Q = \frac{R}{m} \frac{d \ln \sigma}{d(1/T)}$$

Therefore, it can be found from eq. (18) that the average apparent activation energy Q for BFe10-1-2 cupronickel alloy is $422.58 \text{ kJ} \cdot \text{mol}^{-1}$, which is very close to the value of authors' previous study ($425.299 \text{ kJ} \cdot \text{mol}^{-1}$) [3].

The relationship between the average flow stress $\bar{\sigma}_{\varepsilon-\dot{\varepsilon}}$ and the coupled effect between strain and strain rate $\varepsilon/\dot{\varepsilon}$ is shown in Figure 10. It can be observed from figure that the lines are almost parallel indicating that the slopes of the lines vary in a very small range. The mean value of the slopes is used to determine the contribution function $f_{\varepsilon-\dot{\varepsilon}}$ of the coupled effect factor $\varepsilon/\dot{\varepsilon}$, as described in eq. (19).

$$f_{\varepsilon-\dot{\varepsilon}} = (\varepsilon/\dot{\varepsilon})^{-0.09769} \quad (19)$$

The relationship between $\bar{\sigma}_{\varepsilon-T}$ and the coupled effect between deformation temperature and strain rate $T \ln \dot{\varepsilon}$ is given in Figure 11. By taking the average fitted linear slope 0.00008908, the contribution function $f_{\varepsilon-T}$ of coupled effect between strain rate and deformation temperature $T \ln \dot{\varepsilon}$ can be obtained as follows:

$$f_{\varepsilon-T} = \dot{\varepsilon}^{0.00008908T} \quad (20)$$

The relationship between $\bar{\sigma}_{\varepsilon-T}$ and the coupled effect between strain and deformation temperature $1000/(\varepsilon T)$ is illustrated in Figure 12. It can be seen from Figure 12 that slopes of fitting linear curves at different strains exhibit obvious variation. Therefore, the influence of strain on the slopes of fitting linear curves should be taken into account in order to derive the constitutive equation more accurately. The contribution function $f_{\varepsilon-T}$ of coupled effect between strain rate and deformation temperature $1/(T\varepsilon)$ is determined as follows:

Figure 9: Relationship between $\ln(\bar{\sigma}_T)$ and $1/T$.

Figure 10: Relationship between $\ln(\bar{\sigma}_{\varepsilon-\dot{\varepsilon}})$ and $\ln(\varepsilon/\dot{\varepsilon})$.

Figure 11: Relationship between $\ln(\bar{\sigma}_{\varepsilon-T})$ and $\ln(T \ln \dot{\varepsilon})$.

$$f_{\varepsilon-T} = \exp\left(\frac{-138.92 + 5386.37\varepsilon}{T\varepsilon}\right) \quad (21)$$

$$= \exp\left(\frac{1}{RT} \frac{-1154.981 + 44782.28\varepsilon}{\varepsilon}\right)$$

Then, it can be derived from eq. (18) that the variation of apparent activation energy Q with true strain as follows:

$$Q = \frac{1}{1000m} \left(44782.28 - \frac{1154.981}{\varepsilon} \right) = 458.459 - \frac{11.824}{\varepsilon} \quad (22)$$

Table 2 gives the values of Q at different strain. It can be found that the values of Q obtained from eq. (22) and Ref [4] have little variation.

Determination of the converged weights ω_i

In order to acquire the converged weights ω_i , the logarithm on both sides of eq. (10) is taken:

Figure 12: Relationship between $\ln(\bar{\sigma}_{\varepsilon-T})$ and $1000/(\varepsilon T)$.

Table 2: The values of Q ($\text{kJ}\cdot\text{mol}^{-1}$) at various strain.

Strain	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80
eq. (20)	340.22	399.34	419.05	428.90	434.81	438.75	441.57	443.68
Ref [4]	367.28	413.14	412.11	418.57	441.40	446.07	453.26	450.57

$$\ln \sigma = \ln f_0 + \omega_1 \ln f_\varepsilon + \omega_2 \ln f_{\dot{\varepsilon}} + \omega_3 \ln f_T + \omega_4 \ln f_{\varepsilon-\dot{\varepsilon}} + \omega_5 \ln f_{\varepsilon-T} + \omega_6 \ln f_{\varepsilon-\dot{\varepsilon}-T} \quad (23)$$

Multivariate regression (MR) analysis is a highly flexible system for examining the relationship between the collection of independent variable and the dependent variable. Therefore, least-squares regression with the independent variables ($\ln f_0$, $\ln f_\varepsilon$, $\ln f_{\dot{\varepsilon}}$, $\ln f_T$, $\ln f_{\varepsilon-\dot{\varepsilon}}$,

correction coefficient f_0 and the converged weights ω_1 , ω_2 , ω_3 , ω_4 , ω_5 and ω_6 can be obtained, as given in Table 3.

Therefore, the developed constitutive model based on DMNR for BFe10-1-2 cupronickel alloy during elevated temperature deformation can be summarized as:

Table 3: The values of converged weights obtained by multivariate linear regression.

f_0	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6
0.3734	0.310	-0.894	0.937	-0.162	2.016	0.152

$\ln f_{\varepsilon-\dot{\varepsilon}-T}$ and $\ln f_{\varepsilon-T}$) and the dependent variable ($\ln \sigma$) is used to obtained the converged weights ω_j . Through the MR test, by using SPSS software, the values of

$$\left\{ \begin{array}{l} \sigma = f_0 \cdot f_\varepsilon^{0.310} \cdot f_{\dot{\varepsilon}}^{-0.894} \cdot f_T^{0.937} \cdot f_{\varepsilon-\dot{\varepsilon}}^{-0.162} \cdot f_{\varepsilon-T}^{2.016} \cdot f_{\varepsilon-\dot{\varepsilon}-T}^{0.152} \\ f_0 = 0.3734 \\ f_\varepsilon = 47.45855 + 113.97544\varepsilon - 150.9717\varepsilon^2 + 68.65808\varepsilon^3 \\ f_{\dot{\varepsilon}} = \dot{\varepsilon}^{0.09768} \\ f_T = \exp\left(\frac{4964.8196}{T}\right) \\ f_{\varepsilon-\dot{\varepsilon}} = (\varepsilon/\dot{\varepsilon})^{-0.09769} \\ f_{\varepsilon-T} = \dot{\varepsilon}^{0.00008908T} \\ f_{\varepsilon-\dot{\varepsilon}-T} = \exp\left(\frac{-138.92 + 5386.37\varepsilon}{T\varepsilon}\right) \end{array} \right. \quad (24)$$

Verification of constitutive equation

The comparisons between experimental flow stresses and predicted values by the developed constitutive equation are illustrated in Figure 13. It can be seen that the predicted flow

Figure 13: Comparison between the experimental and predicted flow stress at the temperature (a) 1,023 K, (b) 1,073 K, (c) 1,123 K, (d) 1,173 K, (e) 1,223 K and (f) 1,273 K.

stress data from the constitutive equation can track the experimental data of BFe10-1-2 cupronickel alloy throughout the entire temperature and strain rate range. Only under some deformation conditions (i.e. 1,123 K in 1 s^{-1} , and 1,173 K in 10 s^{-1}), an obvious variation between experimental and computed flow stress data can be observed.

Correlation coefficient R and average absolute relative error $AARE$ are introduced to quantify the predictability of the developed constitutive equation. These are expressed as [20]:

$$R = \frac{\sum_{i=1}^N (E_i - \bar{E})(P_i - \bar{P})}{\sqrt{\sum_{i=1}^N (E_i - \bar{E})^2 \sum_{i=1}^N (P_i - \bar{P})^2}} \quad (25)$$

$$AARE(\%) = \frac{1}{N} \sum_{i=1}^N \left| \frac{E_i - P_i}{E_i} \right| \times 100 \quad (26)$$

where E is the experimental flow stress and P is the predicted flow stress calculated from the developed constitutive equation considering strain compensation. $\bar{E}$ and $\bar{P}$ are the mean values of E and P respectively. N is the total number of data used in this study. R is a commonly employed statistical parameter and provides information on the strength of the linear relationship between the experimental and predicted data. And the $AARE$ is calculated through a term by term comparison of the relative error and therefore is an unbiased statistical parameter for determining the predictability of the

Figure 14: Correlation between the experimental and predicted flow stress data from the developed constitutive equation.

equation [21]. As can be seen from Figure 14, the values of R and $AARE$ are calculated to be 0.990 and 5.789%, respectively.

The variation between $AARE$ and R with strain, strain rate and deformation temperature is given in Figure 15. It

can be seen from Figure 15(a) there is a slight variation in R with strain (from 0.981 to 0.992). The maximum value of $AARE$ (7.696 %) and minimum value of R (0.981) are all at the strain of 0.1. The $AARE$ values at other strains are lower than 7 %, and the R values at other strains are all

Figure 15: Variation between $AARE$ and R with (a) strain, (b) strain rate and (c) deformation temperature.

higher than 0.989. In Figure 15(b), it can be seen that the maximum value $AARE$ is about 7.735 % (at the strain rate of 1 s^{-1}), with the minimum value of R (0.971). In Figure 15 (c), the maximum value of $AARE$ and minimum value of R are all at the temperature of 1,123 K, whose values are 8.306 % and 0.971, respectively.

Moreover, the performance of constitutive equation based on DMNR is further investigated by statistical analysis of the relative error. The prediction is compared with the corresponding experimental data, and subsequently the relative error is expressed as [22]:

$$\text{relative error} = \left(\frac{E_i - P_i}{E_i} \right) \times 100\% \quad (27)$$

The results of relative error are represented graphically as a typical number versus error plot, as shown in Figure 16. As can be seen from Figure 16, the relative errors exhibit Gaussian distribution. The relative errors vary from -37.53 % to 24.25 %, and the mean value of the relative errors is only -0.00327. Therefore, on the basis of the analysis of R , $AARE$ and relative error, the developed constitutive model based on DMNR can predict the elevated temperature flow behavior of BFe10-1-2 cupronickel alloy accurately.

Comparison with other constitutive models

The predictability of the proposed constitutive equation is further evaluated by comparing with the modified

Johnson Cook (M-JC) and the modified Zerilli-Armstrong (M-ZA) models. The M-JC model is expressed as follows [23]:

$$\sigma = (A_1 + B_1 \varepsilon + B_2 \varepsilon^2)(1 + C_1 \ln \dot{\varepsilon}^*) \exp[(\lambda_1 + \lambda_2 \ln \dot{\varepsilon}^*) T^*] \quad (28)$$

where $\dot{\varepsilon}^* = \dot{\varepsilon} / \dot{\varepsilon}_0$ is the dimensionless strain rate in which $\dot{\varepsilon}_0$ is the reference strain rate (s^{-1}), and $T^* = T - T_{ref}$ with T and T_{ref} representing the current and reference temperatures (K). A_1 , B_1 , B_2 , C_1 , λ_1 and λ_2 are the materials constants. The parameters of the M-JC model for BFe10-1-2 cupronickel alloy are given in Table 4 in accordance with the steps of Ref [23, 24]. Then, the M-JC constitutive equation for BFe10-1-2 cupronickel alloy was obtained as:

$$\sigma = (88.604 + 149.419\varepsilon - 99.759\varepsilon^2)(1 + 0.06877 \ln \dot{\varepsilon}^*) \times \exp[(-0.00364 + 0.000178 \ln \dot{\varepsilon}^*) T^*] \quad (29)$$

The M-ZA model is expressed as follows [25]:

$$\sigma = (C_1 + C_2 \varepsilon^n) \exp[-(C_3 + C_4 \varepsilon) T^* + (C_5 + C_6 T^*) \ln \dot{\varepsilon}^*] \quad (30)$$

where $\dot{\varepsilon}^* = \dot{\varepsilon} / \dot{\varepsilon}_0$ is the dimensionless strain rate, $\dot{\varepsilon}_0$ the reference strain rate in s^{-1} , and $T^* = T - T_{ref}$ with T and T_{ref} are the current and reference temperatures (K) respectively. C_1 , C_2 , C_3 , C_4 , C_5 , C_6 and n are the materials constants. The parameters of the M-ZA model for BFe10-1-2 cupronickel alloy are given in Table 5 in accordance with

Figure 16: Statistical analysis of the relative error.

Table 4: Parameters of the M-JC model for BFe10-1-2 cupronickel alloy.

Parameter	A_1	B_1	B_2	C_1	λ_1	λ_2
Value	88.604	149.419	-99.759	0.06877	-0.00364	0.000178

Table 5: Parameters of the M-ZA model for BFe10-1-2 cupronickel alloy.

Parameter	C_1	C_1	C_2	C_4	C_5	C_6	n
Value	67.5	88.154	0.00315	0.00117	0.0768 ± 0.0003	0.000189 ± 0.000002	0.4836

the steps of Ref [1, 25]. Then, the M-JC constitutive equation for BFe10-1-2 cupronickel alloy was obtained as:

$$\sigma = (67.5 + 88.154\epsilon^{0.4836}) \exp[-(0.00315 + 0.00117\epsilon)T^* + (0.0768 + 0.000189T^*) \ln \dot{\epsilon}^*] \quad (31)$$

The comparison between experimental flow stress and predicted data by the M-JC model are shown in Figure 17. The values of R for the M-JC and M-ZA models are 0.985 and 0.988 [1], respectively, which are all lower than that of the DMNR model. Meanwhile, the values of $AARE$ the M-JC and M-ZA models are 6.57 % and 6.403 % [1] respectively, which are all higher than that of the DMNR model.

Conclusions

Isothermal compression tests were employed to investigate the elevated temperature flow behavior of BFe10-1-2 cupronickel alloy. Based on this study, following are the conclusions:

- (1) A constitutive equation based on DMNR was used to predicting the elevated temperature flow behavior of cupronickel alloy by taking into account the independent factors of processing parameters and the interaction factors between them.
- (2) The calculated results showed that the average strain rate sensitivity exponent m and average deformation activation energy Q of BFe10-1-2 cupronickel alloy are 0.09768 and 422.58 kJ mol⁻¹, respectively.

Figure 17: Correlation between the experimental and predicted flow stress data from the M-JC constitutive equation.

And the variation law of Q with strain was also obtained.

- (3) The accuracy of the developed constitutive equation was quantified in terms of R , $AARE$ and relative errors. Meanwhile, the predictability of the modified parallel constitutive equation was comparable to that of the M-JC and M-ZA constitutive models. The results showed that the modified constitutive equation based on DMNR gave an accurate and precise estimate of the flow stress of BFe10-1-2 cupronickel alloy.

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