Conference Announcement

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Theory and numerical methods for solving inverse and ill-posed problems

<https://doi.org/10.1515/jiip-2019-5001>

1 General information

The eleventh international annual scientific school-conference “Theory and numerical methods for solving inverse and ill-posed problems” will be held in August 26–September 4, 2019. It will be organized by Novosibirsk State University and Institute of Computational Mathematics and Mathematical Geophysics of the SB RAS.

The first ten school-conferences, held from 2009 to 2018, showed the relevance and scientific significance of the chosen subject. Over the past years, researchers, graduate students and undergraduates from Russia, Belarus, Ukraine, Kazakhstan, Uzbekistan, Kyrgyzstan, as well as from China, USA, Germany, France, Italy, Japan, UK, Brazil, Malaysia took part in the school-conferences. For ten years, representatives of 78 universities, 134 research institutes and more than twenty companies (Total, Baker–Hughes, Schlumberger, Rosneft and others) participated actively at school-conferences.

The scientific program of the school-conference usually consists of theory of inverse and ill-posed problems and regularization methods; numerical methods for solving inverse problems of acoustics, electrodynamics, geophysics, tomography, medicine, biology, finance and social processes.

Several new sections are planned to be included to the program: high performance and parallel computing in intellectual big data analysis, machine learning and artificial intelligence, visualization, nature-inspired algorithms.

The importance of the conference topics is due to the active development of the theory and numerical methods for solving inverse and ill-posed problems and their applications. According to data for 2018, more than 10,000 articles of this area have been added to Scopus over the past three years. The use of modern achievements of fundamental mathematics (algebra, geometry, functional analysis, mathematical physics, computational mathematics), interdisciplinarity and diversity of applications is a characteristic feature of the theory and numerical methods for solving inverse problems.

2 What are the inverse and ill-posed problems?

First publications on the inverse and ill-posed problems appeared in the first half of the 20th century. These were investigations of physicists (inverse problems of quantum scattering theory, electrodynamics, and acoustics), geophysicists (inverse problems of electrical survey, seismics, and potential theory), astronomy, and other fields of natural science. With the appearance of powerful computers, the application area of

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inverse and ill-posed problems has covered practically all the scientific disciplines that use mathematical methods. In direct problems of mathematical physics, researchers are trying to find (in explicit form or approximately) functions describing various physical phenomena, for instance, the propagation of sound, heat, seismic oscillations, electromagnetic waves, etc. In this case, the properties of the medium (the equation coefficients) as well as the initial state of the process (in the nonstationary case) or its properties on the boundary (in the case of a bounded region and/or in the stationary case) are assumed to be known. However, in practice the properties of the medium are often unknown [1–3]. This means that practitioners should formulate and solve inverse problems in which it is necessary to determine equation coefficients, or unknown initial or boundary conditions, the location, boundaries, and other properties of the region where the process being investigated takes place. In most cases, these problems are ill-posed. It means that in these problems at least one of the three properties of well-posedness is violated, namely, the conditions of existence, uniqueness, or stability of the solution with respect to small variations of the problem data. As a rule, the sought-for coefficients of the equations are the density, electrical conduction, heat conduction, and other important properties of the medium being investigated. In addition, in inverse problems it is often necessary to find the location, shape, and structure of inclusions, defects, and sources (of heat, oscillations, stress, and pollution), etc. It is not surprising that with such a wide range of applications, the theory of inverse and ill-posed problems since its inception has become one of the fastest developing areas of modern science. It is hardly possible to count the number of scientific publications in which the inverse and ill-posed problems are investigated in some way.

Inverse and ill-posed problems are solved by any sane person every minute and, as a rule, quickly and efficiently. For example, let us consider the visual perception. It was found that in a minute we can fix only a finite number of points of the surrounding world. Then how do we see everything? The brain (in this situation, a powerful personal computer) replenishes (interpolates and extrapolates) everything that the eye has not managed to fix by using the seen points. The real picture (in the general case, the three-dimensional and colored one) can be obtained using several points only when it is known (most objects and images have already been seen and sometimes touched by hands). That is, despite strong ill-posedness (non-uniqueness and instability of the solution) of the problem (reconstruction of the object being observed and its surroundings by several points) the brain solves it rather quickly. Why? The brain uses rich experience (a priori information). In general, if we wish to understand something that is rather complicated, solve a problem in which the probability of error is high enough, we, as a rule, come to an unstable (ill-posed) problem. It can be said that someone (especially if he is apt to search for non-standard solution methods) permanently faces ill-posed problems. Actually, everyone understands how easily one can make mistakes trying to reconstruct the past with the help of some facts of the present (find out the motives and details of a crime on the basis of available evidence, understand the reasons for the origination of a disease and stages of its development using the results of medical examination, etc.) or investigate the future (foresee the life path of a child, the development of a country or, in general, of a rather complicated process). Or get into inaccessible zones and understand what happens there (investigate human internals, detect mineral deposits, learn new information about the Universe, etc.). Any attempt to extend the range of direct (sense, visual, aural, etc.) perception of the surrounding world leads us to ill-posed problems. One can say that having learnt how to solve stable (well-posed) problems, mathematicians turned to more complicated unstable (inverse and ill-posed) problems. However, historically it is not true, the man has always been surrounded by ill-posed problems and mathematicians have tried to solve them (not using the terms “inverse problem” and “ill-posed problem”). For instance, the problem of differentiation of an approximately specified function belongs to weakly ill-posed problems. The problem of determining the Earth’s shape by its shadow on the Moon surface solved by Aristotle is an example of successfully solved inverse problem.

Without going into all details of mathematical definitions, it should be noted that inverse and ill-posed problems have an important common property, namely, instability of the solution with respect to small errors of measurement data. In most interesting cases, inverse problems are ill-posed and, vice versa, an ill-posed problem can, as a rule, be reduced to an inverse one with respect to some direct (well-posed) problem. However, since historically inverse and ill-posed problems have often been formulated and studied independently and in parallel, the both terms are used now in the scientific literature.

In conclusion, it can be said that specialists in inverse and ill-posed problems investigate the properties and methods of regularizing unstable problems. In other words, mathematicians are trying to create and study stable methods of approximating unstable mappings. From the point of view of information theory, specialists in inverse and ill-posed problems study the properties of mapping data tables with very large epsilon-entropy into tables with very small epsilon-entropy.

3 Historical remarks

As it often happens, mathematicians took new ideas and statements of inverse and ill-posed problems from natural sciences and even from philosophy. Plato's idea that only shadows on cave walls and echo (the inverse problem data) are accessible for us in the process of cognition was a precursor for Aristotle's stating and solving the problem of reconstructing the Earth's shape by its shadow on the Moon (projective geometry). The physical concept of instantaneous velocity served as a basis for Newton's discovery of the derivative. Even now many researchers must deal with instability (ill-posedness) of numerical differentiation of approximately specified functions. Rayleigh's investigations of acoustics brought him to the question of finding variable density of a string by its sound (the inverse problem of acoustics). This anticipated the development of seismic prospecting and, on the other hand, of the theory of spectral inverse problems. Study of the motions of celestial bodies by A. Legendre and K. Gauss brought them to ill-posed problems for nonlinear systems of algebraic equations, which resulted in the appearance of the least-squares method. A. Cauchy proposed the quickest descent method for finding the minimum of the function of several variables. In 1948 L. V. Kantorovich generalized, developed, and applied these ideas to operator equations in Hilbert spaces. Now the quickest descent and the conjugate gradient methods are among the most popular ones in solving of ill-posed problems. It is interesting that L. V. Kantorovich was the first one to notice that the method he proposed converges to the functional if a problem is ill-posed.

In the 20th century, mathematicians took up statements of inverse and ill-posed problems. The proposition that there are no ill-posed problems, but there are weak-posed problems discouraged some researchers, but inspired the others to look for new statements and methods for regularization of inverse and ill-posed problems. R. Courant's conviction that unstable problems do not have a physical meaning did not prevent him from brilliantly solving the strongly ill-posed problem of reconstructing a function by its spherical average. In the years 1953-1955, S.L. Sobolev was the scientific supervisor of V.K. Ivanov's Doctoral Thesis "Investigations on the Inverse Problem of Potential Theory". The classical Cauchy-Kovalevskaya theorem allows us to assert that the solution of a wide range of inverse and ill-posed problems exists and is unique, but only in the class of analytical functions. L. V. Ovsyannikov proved that the requirement of analyticity with respect to the outgoing variable can be considerably weakened. V. G. Romanov during the developing of Ovsyannikov-Nirenberg method of Banach spaces scales showed that for a wide range of inverse problems one could get rid of the analyticity condition with respect to two variables: the outgoing space variable and the time variable. These investigations opened the door to studies of multi-dimensional inverse problems of geophysics, in which a horizontally layered medium is the base model.

The field of inverse problems was first discovered and introduced in Mathematics by Soviet-Armenian physicist, Viktor Ambartsumian. While being a student, Ambartsumian thoroughly studied the theory of atomic structure, the formation of energy levels, and the Schrödinger equation and its properties, and when he mastered the theory of eigenvalues of differential equations, he pointed out the apparent analogy between discrete energy levels and the eigenvalues of differential equations. Then he asked: given a family of eigenvalues, is it possible to find the form of the equations whose eigenvalues they are? Essentially Ambartsumian was examining the inverse Sturm-Liouville problem, which dealt with determining the equations of a vibrating string. This work was published in 1929 in the German physics journal "Zeitschrift für Physik" and remained in obscurity for quite a long time. After the decades, describing this situation Ambartsumian said, "If an astronomer publishes an article with a mathematical content in a physics journal, then most likely it will be forgotten." Nevertheless, by the end of the World War II, this article, written by the 20-year-old

Ambartsumian, was found by Swedish mathematicians and became the starting point for the whole area of research on inverse problems, the foundation of the entire discipline.

4 International program committee

Well-known specialists take part in preparation of the scientific program of the school-conference including Members of the Russian Academy of Sciences Yu. E. Ershov, S. K. Godunov, S. S. Goncharov, V. G. Romanov (Sobolev Institute of Mathematics, Novosibirsk), M. A. Guzev (Institute of Applied Mathematics FEB RAS, Vladivostok), K. V. Rudakov, Yu. G. Yevtushenko and Yu. I. Zhuravlev (Federal Research Center “Computer Science and Control” RAS, Moscow), N. A. Kolchanov (Institute of Cytology and Genetics SB RAS, Novosibirsk), A. N. Konovalov, G. G. Lazareva, G. A. Mikhailov (Institute of Computational Mathematics and Mathematical Geophysics SB RAS, Novosibirsk), V. A. Sadovnichii (Lomonosov Moscow State University), R. Z. Sagdeev (International Tomography Center SB RAS, Novosibirsk), E. E. Tyrtysnikov (Institute of Numerical Mathematics RAS, Moscow), V. M. Fomin, A. N. Shiplyuk (Khristianovich Institute of Theoretical and Applied Mechanics SB RAS, Novosibirsk), B. N. Chetverushkin (Keldysh Institute of Applied Mathematics, Moscow), Yu. I. Shokin, A. M. Fedotov (Institute of Computational Technologies of SB RAS, Novosibirsk), M. I. Epov (Trofimuk Institute of Petroleum Geology and Geophysics SB RAS, Novosibirsk), V. V. Vasin (N. N. Krasovskii Institute of Mathematics and Mechanics, Yekaterinburg), I. B. Petrov, A. A. Shananin (Moscow Institute of Physics and Technology), M. P. Fedoruk (Novosibirsk State University), V. V. Shaidurov (Institute of Computational Modeling SB RAS, Krasnoyarsk). There will be several distinguished professors from abroad: V. Isakov and A. L. Bougheim (Wichita State University, USA), G. Bao (Zhejiang University, Hangzhou, China), J. Cheng (Fudan University, Shanghai, China), A. Hasanoglu (Izmit University, Izmit, Turkey), D. N. Hao (Institute of Mathematics, Hanoi, Vietnam), A. Louis (Saarland University, Saarbrücken, Germany), O. Scherzer (Vienna University, Austria), Y. Wang (Institute of Geology and Geophysics, Beijing, China), Sh. Zhang (Tianjin University of Finance and Economics, China), T. Sh. Kalmenov (Center for Physical and Mathematical Research MES RK, Almaty, Kazakhstan), M. A. Bektemesov (Abay Kazakh National Pedagogical University, Almaty, Kazakhstan).

5 Contact information

The conference will enable young researchers to get acquainted with modern achievements and trends in the field of theory and numerical methods for solving inverse and ill-posed problems. Each of the participants will be able to make a presentation (plenary, invited, sectional or poster).

The conference will contribute the formation of new scientific areas at the junction of computing technologies, parallel computing, visualization, nature-like algorithms and non-destructive testing.

Web-site: http://conf.ict.nsc.ru/tcmiip2019/en/general_info.

Acknowledgment: The Ministry of Education and Science of Russian Federation (4.1.3 The Joint Laboratories of NSU-NSC SB RAS).

References

- [1] S. I. Kabanikhin, Definitions and examples of inverse and ill-posed problems, *J. Inverse Ill-Posed Probl.* **16** (2008), no. 4, 317–357.
- [2] S. I. Kabanikhin, *Inverse and Ill-Posed Problems. Theory and Applications*, Inverse Ill-posed Probl. Ser. 55, Walter de Gruyter, Berlin, 2012.
- [3] S. I. Kabanikhin, A. D. Satybaev and M. A. Shishlenin, *Direct Methods of Solving Multidimensional Inverse Hyperbolic Problems*, Inverse Ill-posed Probl. Ser., VSP, Utrecht, 2005.