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Torsional prestrain gradient and size dependence of initial yield for $\langle 100 \rangle$ Cu-Mn single crystals in tension

Abstract: The effect of torsional prestrain gradient on the size dependence of initial yielding during subsequent tension of Cu-Mn single crystals is investigated experimentally.

Keywords: size effect; torsional prestrain gradient; torsion-tension test.

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1 Introduction

In this short communication, we consider the effect of a strain gradient produced by torsional deformation on the strength of $\langle 100 \rangle$ single crystals for two series of Cu-1.35 and Cu-1.51 wt% Mn alloys. The prestrain gradient effect is evaluated by torsion of cylindrical specimens, followed by a reduction of specimen diameter and subsequent measurement of initial yield in tension. It is observed that specimens with the same surface strain but smaller diameter, i.e., higher torsional prestrain gradient, exhibit a reduced strength in subsequent tension. This size effect can be interpreted by an analysis of the resolved shear stresses on the two characteristic families of slip systems that are governing plastic flow in this loading configuration. For the specimens with the larger diameter, one of the slip systems (type A, i.e., the family of slip systems with direction normal to the specimen axis, as shown in Figure 1), which carries most of the plastic deformation during initial torsion, creates an extended dislocation forest (being more pronounced

near the surface of the specimen, where the strain is larger). This forest inhibits the activity on the second slip system (type B, i.e., the family of 8 slip systems with slip directions inclined to the specimen axis, as shown in Figure 1), which carries most of the plastic deformation during subsequent tension.

There has been a renewed interest in torsional deformation of small volume specimens, where nanoscale phenomena may be traced through molecular and discrete dislocation dynamics simulations [1, 2]. These studies complement earlier studies of such type for macroscopic specimens [3, 4], where the inhomogeneity of slip and dislocation clustering was recorded. Torsion experiments were also conducted in the past to determine related size effects, i.e., the dependence of initial yield [5] and flow stress [6] on the specimen diameter. A more recent study [7] has revealed inhomogeneous hardening during torsion by measuring higher hardness values at the center of the specimen. Some of these results on size-dependent flow stress and hardness for macroscopic specimens were recently interpreted [8, 9] using Aifantis' gradient plasticity theory [10]. However, none of the previous studies considered the effect of a torsional prestrain gradient on the yielding behavior on subsequent yielding. This problem is examined here using predeformed specimens in torsion, which are subsequently subjected to tension. Single Cu-Mn alloy crystals of $\langle 100 \rangle$ orientation are used. This special orientation was chosen because it closely resembles the deformation of polycrystals (multiple slip), without losing its simplicity. The solid solution hardened Cu-Mn alloy is expected to suppress or at least reduce the annihilation/recovery effects. Moreover, the deformation mode of cyclic torsion and subsequent tension was applied to reach high strains (and high strain gradients) by essentially maintaining the same crystal orientation.

Single crystals of Cu-Mn alloys were grown from induction-melted alloys (5 N Cu, 4 N Mn) under a vacuum of $<5 \times 10^{-3}$ Pa in graphite crucibles containing a seed with the desired $\langle 100 \rangle$ orientation. A Bridgman technique was applied, where the crystal grows with a constant speed of 2 cm/h and with a melting zone temperature of 1250°C. After growing, the Cu-Mn specimens were

Figure 1 (A) Stereographic projection of the axis of the <100> single crystals used. The numbers 0, 1, 2, and 3 between slashes represent the slip planes (111), ($\bar{1}\bar{1}\bar{1}$), ($\bar{1}\bar{1}1$), and ($1\bar{1}\bar{1}$), respectively. Slip directions $\alpha\beta$ (with $\alpha \neq \beta$ and $\alpha\beta = \beta\alpha$) are also shown between slashes. (B) Schematic representation of the slip systems in a cylindrical specimen of a perfect <100> single crystal.

	Mn (wt%)	Zn (ppm)	Pb (ppm)
Series 1	1.51	42	<3.3
Series 2	1.35	46	<3.3

Table 1 Weight concentrations for the two series of Cu-Mn single crystals determined by ICP mass spectrometry.

annealed in vacuum for 10 h at 900°C to achieve solute homogenization. The diameter and length of the tested specimens were about 4 and 80 mm, respectively. The crystal axis orientation was checked by both the light-scattering method of the etched surface [11] and the Laue X-ray diffraction method and is shown in the <100> stereographic projection of Figure 1A. The chemical composition of the alloys was analyzed with induction coupled plasma (ICP) mass spectrometry. Two series of specimens were used with weight concentration of the main elements as shown in Table 1. Torsion experiments were performed in a free-end torsion machine. Torsion was always forward and backward to allow the single crystals to deform by a large amount of strain without bending and without changing the orientation. Subsequent tension was performed in an 1185 Instron machine at room temperature, with the rate of tensile deformation chosen as $3 \times 10^{-5} \text{ s}^{-1}$. Slip band formation after torsion was checked with a Zeiss metallographic microscope. To investigate the slip line structure, the surface was electropolished in a 1:2 solution of orthophosphoric acid and distilled water. Figure 1B indicates the possible slip systems for a perfect <100> single crystal, which govern plastic flow in this configuration.

2 Stress-strain behavior in tension and torsion

In Figure 2A, two typical stress-strain curves (resolved shear stress vs. resolved shear strain) in tension of a specimen without predeformation and another predeformed by torsion are shown for comparison. As in [12], the initial yield stress in tension of <100> single crystals previously deformed by torsion is observed to increase with increasing amount of predeformation as a result of the development of a work hardening dislocation structure. The transition from the elastic to the plastic region occurs in the predeformed specimens in an extended microyield region, i.e., plastic yielding in this case is less definite than in simple tension (Figure 2A). This indicates that a considerable strain is needed to reconstruct the dislocation substructure after torsion into the corresponding one for tension with a similar yield stress. This is consistent with slip line numerical and experimental studies, which reveal that, in torsion, slip is activated preferentially in four regions around the <100>-oriented crystal and gradually spreads sidewise [3, 13, 14]. The resulting nonuniform circumferential work hardening is also consistent with the observation that new slip lines in subsequent tension originate in a manner that resembles single slip rather than multiple slip (which would be expected for pure tension of the <100>-oriented crystals) [15–17].

For the same reason, the torsional predeformation leads to a considerable reduction of the work hardening rate, as compared with the corresponding one for simple

Figure 2 (A) Typical τ_{res} -vs.- γ_{res} curves in tension and torsion-tension of <100> single crystals for series 1 (Cu-1.51 wt% Mn) specimens. The vertical line at a strain of $\gamma=0.16$ corresponds to unloading and reloading of the specimen. (B) Applied shear stress τ_N vs. applied shear strain γ in simple torsion of <100> single crystals for series 2 (Cu-1.35 wt% Mn) specimens.

tension of <100> crystals. This high work hardening rate arises because, as mentioned earlier, 8 slip systems are activated simultaneously and interact strongly following the onset of plastic flow. Initially, the developed dislocation structure on those eight separate slip systems is relatively “soft” for the dislocations activated during the subsequent tensile deformation with changed strain path. In any case, however, no tensile load drops are observed in tension after torsion of the <100> Cu-Mn single crystals, contrary to the findings of [4] for pure Cu single crystals oriented for single glide. This may be due to the considerable “friction” stress on dislocations in the Cu-Mn alloys and to the interaction of dislocations on four equally stressed slip planes. This prevents the activation of dislocation sources to produce a sufficient number of strain/dislocation avalanches, whereas in the pure single glide of the Cu crystals, the dislocations on their new strain path cut and readily destroy the previously built microstructure (strain softening).

In Figure 2B, a typical pure torsion curve for the applied shear stress vs. applied shear strain is given. It suggests that in the beginning of plastic deformation, a region resembling single slip sets in and is followed by a sudden increase in work hardening rate. Qualitatively, the initial plateau-like behavior may be viewed as being consistent with molecular and discrete dislocation dynamics simulations of <100> nanowires under torsion [1, 2], which exhibit such a plateau (with “serrations” due to the small resolutions used) of the effective stress-strain curve for small values of the plastic strain. The aforementioned simulations confirm the occurrence of heterogeneous plastic deformation as a result of the nucleation of dislocations and their evolution into twist boundaries at various locations along the wire diameter.

3 Effect of specimen diameter on the yield stress

The effect of the prestrain gradient built during torsion in the radial direction is examined by reducing the diameter of the specimens by electrolytic polishing before subsequent tension. Three specimens of series 1 are deformed with different amounts of pretorsion γ ($=r\Sigma\phi$, i.e., proportional to the total angle of twist), so that the total surface strains γ_s after the reduction of diameter are similar in all specimens, but the associated strain gradient $\nabla\gamma$ ($=\Sigma\phi$) is different. The subsequent tension data reveal that the calculated yield stress (critical resolved shear stress in tension for the slip system with the maximum Schmid factor) τ_{res} increases with an increase of the diameter of the specimen. Table 2 summarizes these results, showing in particular that with increasing strain gradient, the tensile flow stress decreases.

Specimen	D_{tor} (mm)	D_{ten} (mm)	$\Sigma\phi$ (°/cm)	γ_s	τ_{res} (MPa)
M6	3.8	3.8	81.0	0.27	31.1
M8	3.8	2.7	97.2	0.23	25.9
M9	3.8	2.1	124.8	0.23	20.3

Table 2 Size effect observed in tension after torsion for <100> single crystals of series 1.

D_{tor} and D_{ten} denote the diameters of the specimens in torsion and subsequent tension (after reduction of diameter), respectively, $\Sigma\phi$ is the total angle of twist per unit length, γ_s is the total surface prestrain (including forward and backward torsion) of the specimens after reduction of diameter, and τ_{res} is the critical resolved shear stress measured in tension after torsion and diameter reduction.

Motivated by the above observations, the study of the size effect is further elaborated upon by testing the specimens of series 2. Again, the specimens are deformed with an amount of pretorsion forward and backward. Then, they are subjected to a small amount of tension, and the critical resolved shear stress in tension after torsion is calculated. In the sequel, two successive reductions of the diameter of the specimens are performed, each followed by a small amount of tension, so that the distribution of the resolved shear stress along the diameter can be examined in tension. Finally, a correction of the “shift” in tensile stress for the two tests is made by subtracting the respective small increment of tensile stress produced by work hardening during the previous steps in tension. In Figure 3A, the critical resolved shear stress in tension and its dependence on the reduced diameter after torsion for the three specimens of series 2, with a different amount of twist per unit length (i.e., with different gradients of prestrain), are presented. In Figure 3B, the respective critical resolved shear stress dependence on the prior torsional surface strain is shown. In both cases, the straight lines fitting the aforementioned dependences reasonably extrapolate to the critical resolved shear stress of $\langle 100 \rangle$ single crystals in simple tension, which is indicated as the black square on the ordinate. This value is considered to exist in the center of the specimen where the strain γ may be assumed zero (the gradient hardening effect [7] in the center of the specimen can be neglected for our maximum

surface strain of $\gamma_s = 0.4$). In Figure 3A and B, the fitting lines for each specimen do not take account of the point with zero strain in the center of the specimen because such linear extrapolation would be an oversimplification. Again, a clear effect of the specimen size on the resolved shear stress is observed: while τ_{res} increases monotonically with increasing pretorsional strain γ_s (as expected), a “size effect” (as illustrated in Table 2) is observed since the τ_{res} at a fixed strain γ_s value (in Figure 3B) is lower for the larger prestrain gradient (which increases from specimen m27 to m30).

In conclusion, we emphasize that the present results are the first in the literature to report the effect of torsional prestrain gradient on the initial yield of subsequent tension. Size effects are observed in experiments carried out in two series of specimens, which are deformed to different amounts of torsional strain, thus involving different gradients of strain. With appropriate reduction of their diameters through electropolishing, specimens with similar surface shear strain are obtained. In subsequent tension, it is observed that the tensile yield stress increases with increasing specimen diameter. A detailed standard analysis of the resolved shear stresses on the 12 slip systems of $\langle 100 \rangle$ single crystals will be included in a forthcoming publication, where the effect of torsional prestrain gradient on the subsequent flow stress in tension will be additionally interpreted using the gradient plasticity theory [8, 18]. From the aforementioned standard

Figure 3 Single $\langle 100 \rangle$ crystals of series 2 (Cu-1.35 wt% Mn).

The diameter of the specimens in torsion is approximately 3.8 mm. The specimens are subjected to forward and backward torsion with an amount of twist of four times $7.5^\circ/\text{cm}$ for specimen m27, six times $7.5^\circ/\text{cm}$ for specimen m28, and eight times $7.5^\circ/\text{cm}$ for specimen m30. (A) Variation of the critical resolved shear stress measured in tension for different diameters D_{ten} (produced by electrolytic polishing). (B) Respective critical resolved shear stress dependence on the prior total surface strain.

stereographic analysis, it was determined that two types of slip systems are active during torsion: first, the most active slip systems of type A in torsion (with zero resolved shear stress in tension) and later the second most active slip systems of type B in tension with smaller resolved stress in torsion. Each slip system starts to be activated on four positions on the surface of a cross section of the specimen, which form an angle of 90° with each other and the dislocations spread out in both radial and angular directions. This has been confirmed by slip line observations. From the observed size effect, it can be concluded that most dislocation structures of type B are concentrated near the surface of the crystal as a consequence of the

strain gradient during torsion. No tensile load drops after torsion were observed in $\langle 100 \rangle$ single crystals probably due to multiple slip and strong solid solution hardening caused by the Mn solutes.

Acknowledgements: The authors wish to thank the European Community TMR Network under contract (FMRX-CT96-0062) for financial support. Also, the kind help of Dr. A. Ziegenbein and Dr. A. Nortmann during the experiments is gratefully acknowledged. Thanks are also due to Prof. D. Zachmann (Institute of GeoSciences of TUBS) for performing the chemical analyses of our specimens.

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