Research Article

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Proton internal pressure distribution suggests a simple proton structure

<https://doi.org/10.1515/jmbm-2019-0001>

Received Jan 25, 2019; accepted Jun 10, 2019

Abstract: Understanding the origin of quark confinement in hadrons remains one of the most challenging problems in modern physics. Recently, the pressure distribution inside the proton was measured via deeply virtual Compton scattering. Surprisingly, strong repulsive pressure up to 10^{35} pascals, the highest so far measured in our universe, was obtained near the center of the proton up to 0.6 fm, combined with strong binding energy at larger distances. We show here that this profile can be derived semi-quantitatively without any adjustable parameters using the rotating lepton model of composite particles (RLM), i.e. a proton structure comprising a ring of three gravitationally attracting rotating ultrarelativistic quarks. The RLM synthesizes Newton's gravitational law, Einstein's special relativity, and the de Broglie's wavelength expression, thereby conforming with quantum mechanics, and also yields a simple analytical formula for the proton radius and for the maximum measured pressure which are in excellent agreement with the experimental values.

Keywords: Hadron structure; Proton structure; Internal pressure distribution; Rotating Lepton Model; Relativistic neutrinos; Quarks; Gravitational mass

1 Introduction

The proton consists of fundamental particles called quarks and gluons. Gluons carry the force that binds quarks together. Quarks are always confined in the composite particles in which they are located. The origin of quark confinement is still a subject of intensive study in modern physics. Recently, for the first time, the pressure distribution inside the proton was measured via deeply virtual Compton scat-

tering [1–4]. Strong repulsive pressure up to 10^{35} pascals, the highest so far measured in our universe, was obtained near the center of the proton up to 0.6 fm, combined with strong binding energy at larger distances. This recent pioneering experimental [1, 4] and theoretical [1–3] work on the deeply virtual Compton scattering has opened a new area of research on the fundamental gravitational properties of protons, neutrons and nuclei and has underlined that gravity plays an important, perhaps dominant, role inside protons and other hadrons [1–3].

The dominant role of gravity inside hadrons has also emerged in recent years from the rotating lepton model of composite particles (RLM) [5, 6], in which gravity causes confinement of highly energetic neutrinos in bound rotational states and thus leads to formation of quarks, hadrons and bosons [5–8]. This model follows exactly the steps of the Bohr treatment of the H atom and contains no adjustable parameters (Figure 1). It synthesizes Newton's gravitational law, Einstein's special relativity [9], and de Broglie's wavelength expression, thereby conforming with quantum mechanics. The model does not require any new theory. Furthermore, it fits extremely well with current experimental evidence for the masses and other properties of hadrons. The simple structure of the RLM for the proton is also suggested by the earlier proposed [10] bagel-shaped proton geometry (Figure 1). Without any adjustable parameters, the RLM predicts the masses of hadrons [5, 6], but also of bosons [7, 8] with an accuracy of one percent [5–8, 11, 12].

Two important suggestions have emerged from the RLM analysis [5, 6]: First, quarks and hadrons consist primarily of rotating neutrinos, and their mass is due to the kinetic energy of the rotating neutrinos. Second, the Strong Force can be viewed as the relativistic gravitational force. Here the RLM is used to describe quantitatively the measured pressure distribution in protons and to derive simple analytical formulae for the proton radius and for the maximum measured pressure of 10^{35} pascals. We also show that a proton structure comprising a ring of three rotating ultrarelativistic quarks with radius 0.63 fm describes the proton's measured pressure profile semi-quantitatively without any adjustable parameters.

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Figure 1: Rotating lepton model (RLM) of the proton. Top: Schematic comparison and synthesis of the bagel shape model of protons computed via model wave functions, constructed with Poincaré invariance [10], and of the three-rotating neutrino RLM model of protons [5–7]. Particle size is dictated by the quark Compton wavelength $\lambda_q = \hbar/m_q c$. The central particle is a positron of negligible speed, thus negligible gravitational mass [7]. Bottom: Confining and repulsive forces in a proton according to [1] and to the RLM. Point O is the center of rotation and point D is the midpoint of A and B.

Similar to the Bohr model of the H atom, but utilizing gravitational instead of electrostatic forces, the RLM for the proton comprises only two equations, i.e.

$$F = \gamma m_o v^2 / r_e = \frac{G m_g^2}{\sqrt{3} r_e^2} \quad (1)$$

where r_e is the radius of the circle defined by the centers of the three rotating particles, which form an equilateral triangle, m_o is the neutrino rest mass, and m_g is the neutrino gravitational mass, together with the corresponding de Broglie wavelength equation

$$\gamma m_o v r_e = n \hbar \quad (2)$$

which is the historical basis of quantum mechanics and introduces quantization via the integer number n . Account-

ing for the equivalence principle, m_g equals the inertial mass, m_i , which as Einstein showed in his pioneering special relativity paper in 1905 [9], is equal, for linear motion, to the longitudinal mass, $\gamma^3 m_o$, [9, 13]. Originally derived for linear motion [9], this useful result has been shown [5, 6], via the use of instantaneous reference frames [13], to remain valid for arbitrary motion including circular motion. Thus it follows

$$m_g = m_i = \gamma^3 m_o. \quad (3)$$

Solution of equations (1), (2) and (3) for $n = 1$, the ground state, gives

$$\gamma_q = 3^{1/12} (m_{Pl}/m_o)^{1/3}; \quad (4)$$

$$m_p = 3\gamma_q m_o = 3m_q = 3^{13/12} (m_{Pl} m_o^2)^{1/3}$$

where $m_{Pl} (= (\hbar c/G)^{1/2})$ is the Planck mass and the subscript “q” denotes quark.

Setting $m_p = 938.272 \text{ MeV}/c^2$, the proton mass, in equation (4), one computes $m_o = 0.0437 \text{ eV}/c^2$, which remarkably is within the experimental limits of the mass of the heaviest electron neutrinos ($0.048 \pm 0.01 \text{ eV}/c^2$) [14, 15]. Consequently, the rest mass of quarks appears to be that of electron neutrinos. Also the radius, r_e , of the proton computed as the quark de Broglie wavelength from equation (2) for $n = 1$ and $v \approx c$, is given by

$$r_e = \lambda_q = \hbar/\gamma_q m_o c = \hbar/m_q c = 3\hbar/m_p c = 0.63 \text{ fm}, \quad (5)$$

in very good agreement with the experimental value [1, 16]. It is worth noting that the first equation (4), together with (3), dictate that the gravitational mass, $\gamma_q^3 m_o$, of the relativistic rotational quarks is very close to the Planck mass, i.e.

$$m_g = \gamma_q^3 m_o = 3^{1/4} m_{Pl}. \quad (6)$$

Consequently the relativistic gravitational force and potential energy computed from eq (1) is

$$F = \frac{G m_g^2}{\sqrt{3} r_e^2} = \frac{G(\hbar c/G)}{r_e^2} = \frac{\hbar c}{r_e^2}; \quad U = -\frac{\hbar c}{r_e} \quad (7)$$

which are the values anticipated for the strong force between two quarks [16]. These values are a factor of $\alpha^{-1} = 137.035$ stronger than the Coulombic attraction and potential energy of an e^+e^- pair at the same distance.

2 Results

2.1 Force and pressure in the proton

The rotating quark ring structure of the proton according to the RLM allows for the quantitative description of the

potential energy and pressure distribution inside a proton and their comparison with those measured in the pioneering reference [1]. We first show in Figure 1, bottom, that the rotating quark ring of the RLM creates two types of attractive strong relativistic gravitational forces. First, those pointing inward and confining the three neutrinos to their rotational orbit at $r = \lambda_q = 0.63$ fm (Figure 1, bottom) and second, those pointing outwards and giving, due to their outward direction, the impression of repulsive forces (and pressure [1]) but in reality being equal and symmetric with the confining ones on the two sides of the rotating quark ring.

Thus, considering the distance of the quarks A and B fixed, and quark C at a variable location r on the perpendicular bisector of AB (Figure 2, bottom) one obtains, as shown in the methods section, that the total gravitational force, F , exerted by quarks A and B to quark C in the x direction is

$$F(r) = \frac{-2\hbar c [r + (\lambda_q/2)]}{[r^2 + r\lambda_q + \lambda_q^2]^{3/2}} \quad (8)$$

which, in dimensionless form, is given by

$$F(z) = \frac{-\hbar c}{(\lambda_q/2)^2} \left[\frac{2(z+1)}{[(z+1)^2 + 3]^{3/2}} \right] \quad (9)$$

where

$$z = r/(\lambda_q/2) \quad (10)$$

Equation (8) is plotted in Figure 3, as $F(r)/4\pi$ vs r and z , since this quantity is equal to $r^2 p(r)$ (expressed in 10^{-2} GeV/fm) which has been used in the y axis of ref. [1] as shown in Figures 2 and 3. In this way, direct comparison of equation (8) with reference [1] is possible. The shaded part of Figure 3 is the one compared in Figure 2 with the experimental data of [1]. There is remarkable semiquantitative agreement between reference [1] and equations (8) and (9).

2.2 Asymptotic freedom and confinement

It is also worth noting that the force expressions (8) and (9) describe in a simple way the two key characteristics of the Strong Force, *i.e.* asymptotic freedom ($F = 0$ at $z+1 = 0$, *i.e.* at $r = -\lambda_q/2 = -0.315$ fm) and confinement, *i.e.* $dF/dr > 0$ up to the force minimum at $z+1 = \sqrt{3}/2$, thus $r=0.071$ fm, in excellent agreement with the experimental results [1], as shown in Figures 2 and 3.

Figure 2: Comparison of measurements [1] and of the present RLM results: Measured [1] radial force and pressure distributions $r^2 p(r)$ and $p(r)$, respectively, in the proton vs the radial distance r from the center of the proton (thick black line) and comparison with those computed here via the rotating lepton model, (RLM), (thick yellow line). Uncertainties in [1] shown in the Figure correspond to one standard deviation. Also shown is the geometry of the rotating lepton model used here to compute the force $F(r) = 4\pi r^2 p(r)$, where $F(r)/4\pi = r^2 p(r)$, which is the quantity plotted in [1]. Thus the figure allows for direct comparison of the data of [1] (thick black line) with the present results (computed from equation (8), thick yellow line). The two distance scales (r of the present work and $r_{[1]}$ of reference [1]) have been superposed so that they coincide at the pressure crossover point ($p(r) = 0$) which has been calculated from first principles via the RLM. The agreement between the two curves (measured and theoretical), *i.e.* black and yellow, is astonishing especially given the absence of any adjustable parameters.

2.3 Potential energy

Utilizing $F = -dU/dr$, where U is the potential energy of quark C due to its interaction with A and B one obtains

$$U(z) = -\frac{2}{3} m_p c^2 \left[\frac{2}{[(z+1)^2 + 3]^{1/2}} \right] \quad (11)$$

Figure 3: Force and pressure in the proton. Plot of eqs (8) and (9) showing the radial force exerted on quark C, by quarks A and B, as a function of the distance from the center of the proton. At $r = 0$ the total force exerted on the three particles in the x direction vanishes and the rotational radius equals the quark Compton wavelength λ_q . The pressure $p(r)$ at $r = \lambda_q/2$ (i.e. $z = 1$) is $5.5 \cdot 10^{34}$ Pa.

which is plotted in Figure 4. The minimum U value occurs at $z = -1$, i.e. at $r = -\lambda_q/2$, and equals

$$U_{\min} = -\frac{4}{3\sqrt{3}} m_p c^2 = -0.723 \text{ GeV} \quad (12)$$

while the corresponding value at the rotational center $z = 0$ and $r = 0$ fm is

$$U_o = -2(m_p c^2/3) = -0.626 \text{ GeV} \quad (13)$$

for quark C and

$$U_p = 3U_o = -2m_p c^2 = -2(938.3 \text{ GeV}) \quad (14)$$

for the entire proton, as shown in Figure 4. This is consistent with the virial theorem, i.e.

$$U_p = -2T_p \quad (15)$$

as shown in Figure 4, since, according to the RLM, the kinetic energy, T_p , of the relativistic neutrinos is the rest energy of the proton.

2.4 Composite mass dependence of force and pressure

Equations (8) to (14) show that, while the potential energies $U_{\min}$ and U_o are of the order of -1 GeV, the forces, F ,

Figure 4: Potential energy in the proton. Gravitational potential energy distribution of quark C (thick line) and of the three quarks together (thick dotted line) in the proton. The negative of the potential energy value at $r = 0$ (the minimum value allowed by the Compton wavelength λ_q) is twice the proton rest energy $m_p c^2$. The latter is determined by the kinetic energy, T_p , of the rotating neutrinos.

in the proton are on the order of

$$F \approx \frac{4\hbar c}{\lambda_q^2} = \frac{4(m_p c^2/3)^2}{\hbar c} = 3.16 \cdot 10^5 \text{ N} = 2 \text{ GeV/fm} \quad (16)$$

and the pressure, p , computed from $p = F/4\pi r^2$ is of the order of

$$p \approx \frac{4\hbar c}{\pi \lambda_q^4} = \frac{4}{\pi} \frac{(m_p c^2/3)^4}{(\hbar c)^3} = 2.56 \cdot 10^{35} \text{ Pa} \quad (17)$$

in very good agreement with reference [1]. Indeed, as shown in Figures 2, 3 and 4, these values provide a good measure of the interquark forces and pressure inside the proton, in very good agreement with the experimental values measured in [1].

3 Discussion

In summary, there is at least semiquantitative agreement between the RLM and the recent pioneering experimental results for force and pressure in the proton [1]. This agreement provides strong support for the simple rotating quark model and demonstrates the excellent quality of the experimental data of references [1] and [4] and of their theoretical treatment [1]. It also underlines the importance of

gravitational forces inside the proton and suggests that, as previously proposed [5], the Strong Force can be viewed as relativistic gravity. This suggests that the simple methodology used here to model the proton structure and to compute the mass and basic properties of protons can be also applied to other hadrons as well.

Indeed the use of RLM has been explored recently to compute the masses of several hadrons [5, 6], but also bosons [7, 8], with an accuracy of one percent [5–8, 11, 12], using similar simple rotating lepton structures, the components of which are chosen on the basis of the decay products of the corresponding composite particles. It thus appears that the present methodology may be of broader usefulness for the study of the structure and mass of composite particles.

4 Methods

4.1 Derivation of the force equations (8) and (9)

Figure 5 shows the model geometry. Particles A, B and C, of gravitational mass m_g each, lie on a circle of center M and radius R. The distance of points A and B is fixed and equal to $\sqrt{3}\lambda_q$. Point C can move on the perpendicular bisector, x, of AB. When point C coincides with point C*, then the isoscele triangle ABC becomes equilateral, i.e. (AC*)=(BC*)=(AB) and point M coincides with point O. The distances OA, OB are always equal to λ_q . Point C' is the antidiometric of point C on the circle of radius, R, thus (CC')=2R.

One notes in Figure 5 that

$$\cos \varphi = \frac{(\lambda_q/2) + r}{(AC)} \quad (18)$$

and

$$\cos \varphi = \frac{(AC)}{2R} \quad (19)$$

Consequently

$$[\lambda_q/2 + r] 2R = (AC)^2 = 4R^2 \cos^2 \varphi \quad (20)$$

Therefore

$$\lambda_q/2 + r = 2R \cos^2 \varphi \quad (21)$$

When C reaches C^* , then $R = \lambda_q$ and $\varphi = 30^\circ$, thus $\cos^2 \varphi = \frac{3}{4}$ and equation (21) gives $3\lambda_q/2 = 3R/2$ confirming its validity.

Figure 5: Model geometry: Particles A, B and C lie on the dotted circle with fixed distance $(AB)=\sqrt{3}\lambda_q$. Particle C can move on the x axis. When it is at point C*, the triangle ABC is equilateral.

We denote F_A and F_R the gravitational forces exerted by particles A and B on particle C. It is

$$F_A = F_B = \frac{-Gm_g^2}{(AC)^2} \quad (22)$$

and we consider the centripetal force, F , acting on particle C. It is

$$F = 2F_A \cos \varphi = \frac{-2Gm_g^2}{(AC)^2} \cos \varphi \quad (23)$$

Noting from (18) that

$$\cos \varphi = \frac{r + (\lambda_q/2)}{(AC)}$$

we can write equation (23) as

$$F = \frac{-2Gm_g^2 [r + (\lambda_q/2)]}{(AC)^3} \quad (24)$$

From Pythagoras's theorem it follows

$$(AC)^2 = (AD)^2 + [r + (\lambda_q/2)]^2 = 3\lambda_q^2/4 + r^2 + \lambda_q r + \lambda_q^2/4 \quad (25)$$

thus

$$(AC)^2 = \lambda_q^2 + \lambda_q r + r^2 \quad (26)$$

From (24) and (26) it follows

$$F = \frac{-(2Gm_g^2) [r + (\lambda_q/2)]}{[r^2 + r\lambda_q + \lambda_q^2]^{3/2}} \quad (27)$$

For $m_g = m_{pl}$ equation (27) gives equation (25), i.e. accounting for $m_g = m_{pl} = (\hbar c/G)^{1/2}$, it is

$$F = \frac{-2G(\hbar c/G) [r + (\lambda_q/2)]}{[r^2 + r\lambda_q + \lambda_q^2]^{3/2}} = \frac{-2\hbar c [r + (\lambda_q/2)]}{[r^2 + r\lambda_q + \lambda_q^2]^{3/2}} \quad (28)$$

which is equation (8). Introducing $z = r/(\lambda_q/2)$ one obtains equation (9).

4.2 Force extrema

The force component in $\bar{x}$ of Figure 5 is given by

$$F(z) = \frac{-4\hbar c}{\lambda_q^2} \frac{2(z+1)}{[(z+1)^2 + 3]^{3/2}} \quad (29)$$

and its minimum occurs when

$$2 \left[(z+1)^2 + 3 \right]^{3/2} - 2(z+1) \frac{3}{2} \left[(z+1)^2 + 3 \right]^{1/2} 2(z+1) = 0 \quad (30)$$

which leads to

$$2 \left[(z+1)^2 + 3 \right]^{3/2} = 6(z+1)^2 \left[(z+1)^2 + 3 \right]^{1/2} \quad (31)$$

and to

$$2 \left[(z+1)^2 + 3 \right] = 6(z+1)^2 \quad (32)$$

Defining

$$(z+1)^2 = y \quad (33)$$

it follows from (32) that

$$2(y+3) = 6y \Rightarrow y = \frac{3}{2} \quad (34)$$

Consequently it follows

$$(z+1)^2 = \frac{3}{2} \quad z+1 = \frac{\sqrt{3}}{\sqrt{2}} \quad (35)$$

which implies that the minimum occurs at

$$z = \sqrt{3/2} - 1 = 0.225 \quad (36)$$

consequently

$$r = (\lambda_q/2)z = 0.071 \text{ fm} \quad (37)$$

and the maximum at

$$z = -1 - \sqrt{3/2} = -2.22 \quad (38)$$

therefore at

$$r = (\lambda_q/2)z = -0.70 \text{ fm} \quad (39)$$

in good agreement with the plots of Figures 2 and 3.

Acknowledgement: We are very thankful to Professor Donna Greenberg from Harvard University for numerous helpful discussions and style corrections.

5 Conclusions

The recently measured for the first time internal pressure profile in a proton [1] is described semiquantitatively by the Rotating Lepton Model (RLM) of composite particles. There is excellent agreement between model and experiment regarding the radius (0.63 fm) and the maximum pressure ($\sim 2.5 \cdot 10^{35}$ Pa).

This agreement provides very strong support for the validity of the rotating lepton model (RLM) and its great advantage over the standard model (SM) which neglects gravitational forces and does not allow for any such pressure computations except perhaps via inclusion of more adjustable parameters.

The RLM also explains that the strong repulsive pressure reported in [1] near the center of the proton is in reality due to the strong gravitational attraction by the rotating neutrino ring, which attracts matter equally strongly inside and outside its radius.

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