Research Article

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Exact solution for the thermo-elastic deformation and stress states of FG rotating spherical body

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Abstract: In this paper, a generalized solution for 1-D steady-state mechanical and thermal deformation and stresses in rotating hollow functionally graded spherical body is presented. Spherical shells are treated under mechanical and thermal loads in the form of rotational body force with heat generation. Temperature distribution is assumed to vary along the radial direction due to variable heat generation. General uniform mechanical boundary condition at inner and outer surfaces along with prescribed temperatures at both the ends are assumed as boundary conditions. In the present study, material properties are taken as power function of radius with grading parameter ranging between -2 to 3 . Governing differential equation with variable coefficient is developed and solved to find deformation and stresses. The obtained results are verified with benchmark results and are found to be in good agreement. Results show that deformation and stresses decrease with an increase in the value of grading parameter and are less as compared to the homogeneous body.

Keywords: functionally graded material, thermo-elastic analysis, power law FGM, axisymmetric body, rotating hollow sphere

Nomenclature

ϵ_r	Radial strain
ϵ_t	Tangential strain
ω	Rotation
ρ	Density of material

σ_r	Radial stress
σ_t	Tangential stress
G	Gravity

1 Introduction

FGMs are advanced class of composite materials wherein mechanical properties vary continuously at macroscopic level from surface to surface. Thermo-mechanical stresses in FG thick sphere is reported by M.R. Eslami, M.H. Babaei, R. Poultangari [1] wherein they considered a thick hollow spherical body of FG material under one dimensional steady state distributed temperature with general type of boundary conditions (mechanical and thermal). Deformation and stresses in rotating FG material pressurized thick hollow cylindrical body under thermal load is given by, M. Zamani Nejad and G.H. Rahimi [2]. Effect of material gradient on stresses of thick FG spherical pressure vessels using exponentially varying grading properties are given by M. Zamani Nejad, M. Gharibi [3]. A novel approach to stress analysis of pressurized FGM cylinder, disc and spheres is given by Naki Tutuncu, Beytullah Temel [4]. FG hollow cylindrical under thermal and pressure loading effects due to material parameters on stresses and temperature distributions are reported by Celal Evci, Mufit Gulgec [5]. Rotating disk, cylinder and sphere with variable thickness are analysed and reported in [6]. Thermo-elastic, thermo-mechanical stress analysis has been conducted in few literatures [7–10]. Semi exact solution of non uniform disk was analysed and presented in [11] and [12]. Some exact solution of cylinders are analysed [13] and [14]. The dynamic analysis of rotating doubly-curved shell structures made of FGMs on critical speed is analyzed in [15]. Thermo-elastic analysis of rotating multilayer FG-GPLRC truncated conic based on a coupled TDQM-NURBS scheme is reported by [16]. A multi objective optimization of a FG sandwich panel with mechanical loading was carried out in [17]. Indentation of materials with a linear yield strength gradient by spherical indenters are analyzed in [18]. Temperature dependent vibration analysis of functionally graded vis-

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coelastic cylindrical micro-shell is analyzed and reported in [19].

In the present work report, to determine the stress and deformation state of hollow functionally graded spherical body, the problem is moulded using Navier equation including rotational body force and variable heat generation. The validation of the present exact solution is carried out with existing literatures. Corresponding to rotational speed, body force and variable heat generation in spherical body, the stress and deformation in the FG spherical body is estimated. The existing results are reported in dimensionless form.

Figure 1: Hollow spherical body

2 Mathematical formulation

A rotating hollow spherical body of ‘ a ’ inner radius and ‘ b ’ outer radius and made of FGM material is considered. The variation of material properties of hollow sphere are function of radius ‘ r ’. Let ‘ u ’ be the displacement in the radial direction. The relation between strain and displacement are given by [1]

$$\varepsilon_r = \frac{du}{dr} = \frac{1}{E(r)} [\sigma_r - 2\vartheta\sigma_t] + \alpha T(r) \quad (1)$$

$$\varepsilon_t = \frac{u}{r} = \frac{1}{E(r)} [\sigma_t(1 - \vartheta) - \vartheta\sigma_r] + \alpha T(r) \quad (2)$$

Stress-strain relations are given by [1]

$$\sigma_r = \frac{E(r)}{(1 + \vartheta)(1 - 2\vartheta)} \left[\varepsilon_r(1 - \vartheta) + 2\vartheta\varepsilon_t - (1 + \vartheta)\alpha T(r) \right] \quad (3)$$

$$\sigma_t = \frac{E(r)}{(1 + \vartheta)(1 - 2\vartheta)} [\vartheta\varepsilon_r + \varepsilon_t - (1 + \vartheta)\alpha T(r)] \quad (4)$$

Here, ε_i and σ_i ($i = r, t$) are the strain and stress tensor. Heat conduction equation is used to determine distribution of the temperature $T(r)$, α is the thermal expansion coefficient. The equilibrium equation in radial direction, including the inertia term and body force, is given by,

$$r \frac{d}{dr} \sigma_r + 2(\sigma_r - \sigma_t) + \rho \left(\omega^2 - \frac{g}{a} \right) r^2 = 0 \quad (5)$$

The material properties of sphere are described by power law function which are given by [20]

$$E(r) = E_a(r)^{n_1} \quad (6)$$

$$\alpha(r) = \alpha_a(r)^{n_2} \quad (7)$$

$$k(r) = k_a(r)^{n_3} \quad (8)$$

$$\rho(r) = \rho_a(r)^{n_4} \quad (9)$$

$$q(r) = q_a(r)^{n_5} \quad (10)$$

Where, $E(r)$, $\alpha(r)$, $k(r)$, $\rho(r)$, $q(r)$ are elastic modulus, thermal expansion coefficient, thermal conduction coefficient density and heat generation at any radius respectively. E_a , α_a , k_a , ρ_a , q_a are material properties as described above at inner radius and n_1 , n_2 , n_3 , n_4 , n_5 are material index respectively.

Using above equations (1) to (10), the Navier equation, in terms of displacement, is given by

$$\begin{aligned} r \frac{d}{dr} \left[E\lambda \left\{ (1 - \vartheta) \frac{du}{dr} + 2\vartheta \frac{u}{r} - (1 + \vartheta) \alpha T \right\} \right] \\ + E\lambda \left[(1 - \vartheta) \frac{du}{dr} + 2\vartheta \frac{u}{r} - (1 + \vartheta) \alpha T \right] \\ - E\lambda \left[\vartheta \frac{du}{dr} + \frac{u}{r} - (1 + \vartheta) \alpha T \right] + \rho \left(\omega^2 - \frac{g}{a} \right) r^2 = 0 \end{aligned} \quad (11)$$

In above eq. 11,

$$E = E(r), \quad T = T(r), \quad \alpha = \alpha(r) \quad \text{and} \quad \rho = \rho(r) \quad (12)$$

$$\text{and} \quad \lambda = \frac{1}{(1 + \vartheta)(1 - 2\vartheta)}$$

3 Temperature formulation

The ‘1-D spherical heat conduction equation with heat generation’ under ‘steady-state’ condition and thermal boundary conditions for FG hollow spherical bodies, are given by [20]

$$\frac{1}{r^2} \frac{d}{dr} \left[r^2 k(r) \frac{dT}{dr} \right] + q = 0 \quad (13)$$

Subject to boundary conditions,

$$T(r) = T(a) \quad \text{at} \quad r = a \quad \text{and} \quad (14)$$

$$T(r) = T(b) \quad \text{at} \quad r = b \quad (15)$$

Where, $T(r)$ is the temperature at any radius, $T(a)$ and $T(b)$ are the temperature at inner and outer radius respectively.

Differentiating eq. (13) gives the Navier equation for temperature as follows.

$$A_1 r^2 T'' + B_1 r T' + C_1 T = \gamma_1 r^{n_5 - n_3 + 2} \quad (16)$$

Where,

$$A_1 = k_a \quad (17)$$

$$B_1 = k_a (n_3 + 2) \quad (18)$$

$$C_1 = 0 \quad (19)$$

$$\gamma_1 = -q_a \quad (20)$$

$$P_3 = 0 \quad (21)$$

$$P_4 = \frac{A_1 - B_1}{A_1} = -n_3 - 1 \quad (22)$$

P_3 and P_4 are roots of general solution of eq. 16. Solving eq. 16 analytically yields

$$T(r) = Q_3 + Q_4 r^{P_4} + \beta_1 r^{n_5 - n_3 + 2} \quad (23)$$

$$\frac{dT}{dr} = Q_4 P_4 r^{P_4 - 1} + \beta_1 (n_5 - n_3 + 2) r^{n_5 - n_3 + 1} \quad (24)$$

Where,

$$\beta_1 = \frac{\gamma_1}{A_1 [(n_5 - n_3 + 2)(n_5 - n_3 + 1)] + B_1 [n_5 - n_3 + 2] + C_1} \quad (25)$$

Using the boundary conditions, the value of Q_3 and Q_4 yields [1]

$$Q_4 = \frac{T_a - T_b}{a^{P_4} - b^{P_4}} - \frac{\beta_1 (a^{n_5 - n_3 + 2} - b^{n_5 - n_3 + 2})}{a^{P_4} - b^{P_4}} \quad (26)$$

$$Q_3 = T_a - \beta_1 a^{n_5 - n_3 + 2} - Q_4 a^{P_4} \quad (27)$$

4 Solutions of displacement equation

Navier equation given in eq. (11) needs a separate calculation for function $T(r)$. Once the function $T(r)$ is known, the equation is solved analytically. Substituting $T(r)$ in eq. 11 gives

$$Ar^2 u'' + Bru' + Cu = Ur^{n_2 + P_4 + 1} + Vr^{n_2 - n_3 + n_5 + 3} + Wr^{n_2 + 1} + Sr^{n_4 - n_1 + 3} \quad (28)$$

Where,

$$A = E_a \lambda (1 - \vartheta) \quad (29)$$

$$B = n_1 E_a \lambda (1 - \vartheta) + 2 E_a \lambda (1 - \vartheta) \quad (30)$$

$$C = 2 E_a \lambda \vartheta n_1 + 2 E_a \lambda \vartheta - 2 E_a \lambda \quad (31)$$

$$U = \frac{1}{(1 - 2\vartheta)} \left[\frac{E_a \alpha_a Q_4 P_4 + E_a \alpha_a Q_4 n_1}{+ E_a \alpha_a Q_4 n_2} \right] \quad (32)$$

$$V = \frac{\beta_1 E_a \alpha_a}{(1 - 2\vartheta)} [n_5 - n_3 + n_1 + n_2 + 2] \quad (33)$$

$$W = \frac{Q_3 E_a \alpha_a}{(1 - 2\vartheta)} [n_1 + n_2] \quad (34)$$

$$S = -\rho \left[\omega^2 - \left(\frac{g}{a} \right) \right] \quad (35)$$

The Navier equation in terms of radial displacement u , in eq. (28) is Euler differential equation of non-homogeneous form which possesses general as well as particular solutions.

The general solution, u_g is assuming,

$$u_g(r) = Qr^P \quad (36)$$

Substituting eq. (36) in homogeneous form of eq. (28), one gets,

$$AP^2 + (B - A)P + C = 0 \quad (37)$$

Eq. (37) has two real roots P_1 and P_2 as,

$$P_{1,2} = \frac{(A - B) \pm \sqrt{(B - A)^2 - 4AC}}{2A} \quad (38)$$

Thus, the general solution is given by,

$$u_g(r) = Q_1 r^{P_1} + Q_2 r^{P_2} \quad (39)$$

The particular solution $u_p(r)$ is assuming in the form,

$$u_p(r) = Ir^{n_2+P_4+1} + Jr^{n_2-n_3+n_5+3} + Lr^{n_2+1} + Mr^{n_4-n_1+3} \quad (40)$$

Substituting eq. (40) in eq. (28), one obtains,

$$\begin{aligned} & \left[A(n_2 + P_4 + 1)(n_2 + P_4) + B(n_2 + P_4 + 1) \right. \\ & \left. + C \right] Ir^{n_2+P_4+1} \\ & + \left[A(n_2 - n_3 + n_5 + 3)(n_2 - n_3 + n_5 + 2) \right. \\ & \left. + B(n_2 - n_3 + n_5 + 3) + C \right] Jr^{n_2-n_3+n_5+3} \\ & + \left[A(n_2 + 1)n_2 + B(n_2 + 1) + C \right] Lr^{n_2+1} \\ & + \left[A(n_4 - n_1 + 3)(n_4 - n_1 + 2) + B(n_4 - n_1 + 3) \right. \\ & \left. + C \right] Mr^{n_4-n_1+3} = Ur^{n_2+P_4+1} + Vr^{n_2-n_3+n_5+3} + Wr^{n_2+1} \\ & + Sr^{n_4-n_1+3} \end{aligned} \quad (41)$$

Equating the coefficient of identical power,

$$I = \frac{U}{A[(n_2 + P_4 + 1)(n_2 + P_4)] + B[n_2 + P_4 + 1] + C} \quad (42)$$

$$J = \frac{V}{\text{Denominator}} \quad (43)$$

Where

$$\begin{aligned} \text{Denominator} = & A[(n_2 - n_3 + n_5 + 3)(n_2 - n_3 + n_5 + 2)] \\ & + B[(n_2 - n_3 + n_5 + 3)] + C \end{aligned}$$

$$L = \frac{W}{A[(n_2 + 1)(n_2)] + B[(n_2 + 1)] + C} \quad (44)$$

$$M = \quad (45)$$

$$\frac{S}{A[(n_4 - n_1 + 3)(n_4 - n_1 + 2)] + B[(n_4 - n_1 + 3)] + C}$$

Overall solution for $u(r)$ is given by,

$$u(r) = u_g(r) + u_p(r) \quad (46)$$

Thus

$$u(r) = Q_1 r^{P_1} + Q_2 r^{P_2} + Ir^{n_2+P_4+1} + Jr^{n_2-n_3+n_5+3} + Lr^{n_2+1} + Mr^{n_4-n_1+3} \quad (47)$$

Substituting eq. (47) in eq. (1)-(4), the strains and stresses are obtained as,

$$\begin{aligned} \varepsilon_r = & Q_1 p_1 r^{P_1-1} + Q_2 p_2 r^{P_2-1} + I(n_2 + P_4 + 1) r^{n_2+P_4} \\ & + J(n_2 - n_3 + n_5 + 3) r^{n_2-n_3+n_5+2} + L(n_2 + 1) r^{n_2} \\ & + M(n_4 - n_1 + 3) r^{n_4-n_1+2} \end{aligned} \quad (48)$$

$$\begin{aligned} \varepsilon_t = & Q_1 r^{P_1-1} + Q_2 r^{P_2-1} + Ir^{n_2+P_4} + Jr^{n_2-n_3+n_5+2} \\ & + Lr^{n_2} + Mr^{n_4-n_1+2} \end{aligned} \quad (49)$$

$$\sigma_r = \quad (50)$$

$$E\lambda \left[\begin{aligned} & Q_1 \{ (1 - \vartheta) P_1 + 2\vartheta \} r^{n_1+P_1-1} \\ & + Q_2 \{ (1 - \vartheta) P_2 + 2\vartheta \} r^{n_1+P_2-1} \\ & + Ir^{n_1+n_2+P_4} [(1 - \vartheta)(n_2 + P_4 + 1) + 2\vartheta] \\ & + Jr^{n_1+n_2-n_3+n_5+2} [(1 - \vartheta)(n_2 - n_3 + n_5 + 3) + 2\vartheta] \\ & + Lr^{n_1+n_2} [(1 - \vartheta)(n_2 + 1) + 2\vartheta] \\ & + Mr^{n_4+2} [(1 - \vartheta)(n_4 - n_1 + 3) + 2\vartheta] - (1 - \vartheta)\alpha \\ & [Q_3 r^{n_1+n_2} + Q_4 r^{n_1+n_2+P_4} + \beta_1 r^{n_1+n_2+n_5-n_3+2}] \end{aligned} \right]$$

To determine the constant Q_1 and Q_2 , the boundary conditions arising out of mechanical loading may be used. The mechanical boundary conditions in the inner surface and outer surface are as follows [1]:

$$\sigma_r(a) = -p_a \quad \text{and} \quad \sigma_r(b) = -p_b \quad (51)$$

Upon substituting eq. (51) in eq. (50), the integration constants becomes

$$Q_1 = \frac{\phi_{22}X - \phi_{12}Y}{\phi_{11}\phi_{22} - \phi_{12}\phi_{21}} \quad (52)$$

$$Q_2 = \frac{\phi_{11}Y - \phi_{21}X}{\phi_{11}\phi_{22} - \phi_{12}\phi_{21}} \quad (53)$$

Where,

$$\phi_{11} = E_a \lambda [P_1(1 - \nu) + 2\nu] a^{n_1+P_1-1} \quad (54)$$

$$\phi_{12} = E_a \lambda [P_2(1 - \nu) + 2\nu] a^{n_1+P_2-1} \quad (55)$$

$$\phi_{21} = E_a \lambda [P_1(1 - \nu) + 2\nu] b^{n_1+P_1-1} \quad (56)$$

$$\phi_{22} = E_a \lambda [P_2(1 - \nu) + 2\nu] b^{n_1+P_2-1} \quad (57)$$

$$X = -p_a - Z(a), \quad Y = -p_b - Z(b) \quad (58)$$

$$\begin{aligned} Z(a) = & E_a \lambda I a^{n_1+n_2+P_4} [(1 - \vartheta)(n_2 + P_4 + 1) + 2\vartheta] \\ & + J a^{n_1+n_2-n_3+n_5+2} [(1 - \vartheta)(n_2 - n_3 + n_5 + 3) + 2\vartheta] \\ & + L a^{n_1+n_2} [(1 - \vartheta)(n_2 + 1) + 2\vartheta] \\ & + M a^{n_4+2} [(1 - \vartheta)(n_4 - n_1 + 3) + 2\vartheta] - (1 - \vartheta)\alpha \\ & [Q_3 a^{n_1+n_2} + Q_4 a^{n_1+n_2+P_4} + \beta_1 a^{n_1+n_2+n_5-n_3+2}] \end{aligned} \quad (59)$$

$$\begin{aligned} Z(b) = & E_a \lambda I b^{n_1+n_2+P_4} [(1 - \vartheta)(n_2 + P_4 + 1) + 2\vartheta] \\ & + J b^{n_1+n_2-n_3+n_5+2} [(1 - \vartheta)(n_2 - n_3 + n_5 + 3) + 2\vartheta] \\ & + L b^{n_1+n_2} [(1 - \vartheta)(n_2 + 1) + 2\vartheta] \\ & + M b^{n_4+2} [(1 - \vartheta)(n_4 - n_1 + 3) + 2\vartheta] - (1 - \vartheta)\alpha \\ & [Q_3 b^{n_1+n_2} + Q_4 b^{n_1+n_2+P_4} + \beta_1 b^{n_1+n_2+n_5-n_3+2}] \end{aligned} \quad (60)$$

5 Results and discussion

5.1 Internal pressure and temperature

The numerical values of different system parameters considered in the work are as follows: Inner and outer radii of hollow sphere are assumed to be $a = 1$ m, $b = 1.2$ m, Poisson's ratio, $\nu = 0.3$, since material properties are taken in accordance with equation 6 to 10. The internal properties are assumed as, modulus of elasticity $E_a = 200$ GPa, thermal coefficient of expansion $\alpha_a = 1.2 \times 10^{-6}$ per $^{\circ}\text{C}$, thermal conduction coefficient $k_a = 15$ W/mk, density $\rho_a = 7800$ kg/m³, heat generation $q = 50 \times 10^3$ kJ/m³ and rotation $\omega = 50$ rad/s and Gravity $g = 9.81$ m/s². The boundary condition for temperature are taken as, $T(a) = 10^{\circ}\text{C}$ and $T(b) = 0^{\circ}\text{C}$. Boundary conditions for stress calculations under mechanical loading are taken as, internal pressure = 50 MPa and external pressure = 0, Grading parameter n is chosen as -2 to 3 and identical for all ($n_1 = n_2 = n_3 = n_4 = n_5 = n$) [1].

Figure 2: Radially distributed elastic modulus

Figure 3: Radially distributed thermal expansion coefficient

Figure 4: Radially distributed density

Figure 5: Radially distributed thermal conduction coefficient

Figure 2 to Figure 5 shows the variation of material properties such as modulus of elasticity, thermal expansion coefficient, density, thermal conduction coefficient respectively for different values of n . It is clear from the figure, that for grading parameter $n = 0$, all mechanical properties are constant whereas for positive grading parameter, the material properties increase from inner to outer radius while for negative grading parameter, the material properties decrease from inner to outer radius.

Comparison of current results with benchmark reports [1]

Validation of present work is carried out with M.R. Eslami *et al.* [1] and comparisons are presented in Figure 6 to Figure 10. The results obtained are found to be good agreement with [1]. This establishes validity of the mathematical formulation and the MATLAB source code and is further used for the investigation of FG hollow spherical having different non-linear material behaviour under the effect of rotation, gravitational force and internal heat generation.

Figure 6: Radially distributed temperature

Figure 9: Radially distributed tangential stress

Figure 7: Radially distributed displacement

Figure 10: Radially distributed von-Mises stress for $b/a = 1.2$

Figure 8: Radially distributed radial stress

Figure 6 shows distribution of temperature for different grading parameter. It is clear from figure that the temperature reduces from inner to outer radius and it is clear from figure that as the grading parameter increases, the temperature further reduces. Figure 7 shows that as the grading parameter increases, radial displacement also decreases from inner to outer radius. Figure 8 shows the as

the grading parameter increases, radial stress at any point in the sphere decreases but the distribution of radial stress for any given value of grading parameter increases along radius from inner to outer of hollow sphere body. Figure 9 shows the plot of tangential stress. In this figure, for $n < 1$ the tangential stress decreases along radial direction from inner to outer but when $n > 1$, the tangential stress is increases along radial direction from inner to outer and for $n = 1$, tangential stress is uniform along the radial direction. Stress distribution along the radial direction is investigated in terms of von-Mises stress distribution which is reported in Figure 10 for aspect ratio $b/a = 1.2$. It is clear from figure that for $r/a < 1.09$ (approx.), as the grading parameter increases, the von-Mises stress decreases but for $r/a > 1.09$ (approx.), the situation is reversed and the von-Mises stress increases as the grading parameter increases. The von-Mises stress is almost uniform for grading parameter $n = 3$ along the radial direction.

Case 1: Rotating spherical body

In this case study a hollow spherical body with a rotational motion is investigated. The effect in hollow spherical body due to the rotation motion shows in graphs. The outcomes values of all are higher as compared to without rotation case as in benchmark.

Figure 11 shows the distribution of temperature for different grading parameter. It is clear from figure that the temperature reduces along radial direction from inner to outer and it is also observed that as the grading parameter increases, the temperature reduces. Figure 12 shows that as the grading parameter increases, radial displacement also decreases from inner to outer radii.

Figure 13 shows that as the grading parameter increases, radial stress decreases but radial stress increases from inner to outer radii of hollow sphere body. Figure 14 shows the plot of tangential stress. In this figure for $n < 1$ the tangential stress decreases along radial direction from inner to outer but when $n > 1$, the tangential stress is increases along radial direction from inner to outer and for n

Figure 11: Radially distributed temperature

Figure 12: Radially distributed displacement

Figure 13: Radially distributed radial stress

Figure 14: Radially distributed tangential stress

Figure 15: Radially distributed von-Mises stress

= 1 shows that tangential stress is uniform along the radial direction. Stress distribution along the radial direction is investigate in von-Mises stress distribution which shows in Figure 15 for aspect ratio $b/a = 1.2$. It is clear from figure that for $r/a < 1.084$ (approx.), as the grading parameter increases, the von-Mises stress decreases but for $r/a >$

Figure 16: Radially distributed temperature

1.084 (approx.), the situation is reversed and the von-Mises stress increases as the grading parameter increases.

Case 2: Spherical body with gravity

In this case, a hollow spherical body with gravitational force is investigated. The effect in hollow spherical body due to the gravitational force is shown in the following graphs. Figure 16 shows distribution of temperature for different grading parameter. Similar to previous results, Figure 17 shows that as the grading parameter increases, radial displacement decreases from inner to outer radii. Figure 18 shows that as the grading parameter increases, radial stress decreases but radial stress increases along radius from inner to outer radii of hollow sphere body. In Figure 19, tangential stress is plotted. In this figure, for $n < 1$ the tangential stress decreases along radial direction from inner to outer radii but for $n > 1$, the tangential stress increases along radial direction from inner to outer radii and at $n = 1$, tangential stress is uniform along the radial di-

Figure 17: Radially distributed displacement

rection. Stress distribution along the radial direction is investigated in terms of von-Mises stress distribution which is shown in Figure 20 for aspect ratio $b/a = 1.2$. It is clear from figure that for $r/a < 1.095$ (approx.), as the grading parameter increases, the von-Mises stress decreases but for $r/a > 1.095$ (approx.), the situation is reversed and the von-Mises stress increases as the grading parameter increases.

Figure 18: Radially distributed radial stress

Figure 19: Radially distributed tangential stress

Figure 20: Radially distributed von-Mises stress

The von-Mises stress is almost uniform for grading parameter $n = 3$ along the radial direction.

Case 3: Spherical body with variable heat generation

In this case, a hollow spherical body with variable heat generation is investigated and reported in Figure 21 to 24 wherein displacement and stresses are plotted. It is observed from figure that Figure 24 and 25, that in case of variable heat generation in the hollow sphere, the tangential stress and von-Mises stress are larger than as reported in [1].

Figure 21: Radially distributed temperature

Figure 22: Radially distributed displacement

Figure 23: Radially distributed radial stress

Figure 24: Radially distributed tangential stress

Figure 25: Radially distributed von-Mises stress

Case 4: Spherical body with rotation and gravitational force

In this case, a hollow spherical body subjected to rotation and gravitational force is investigated. The results obtained for displacement and stresses shown in figure Fig-

Figure 26: Radially distributed temperature

Figure 27: Radially distributed displacement

Figure 28: Radially distributed radial stress

ure 27 to 29. Similar observations are obtained for displacement and stress as in case 1.

Figure 29: Radially distributed tangential stress

Figure 30: Radially distributed von-Mises stress

Case 5: Spherical body with rotation and internal heat generation

Under this case study, a hollow spherical body with a rotational and variable heat generation is investigated. The results obtained for displacement and stresses is shown in figure Figure 31 to 35. It is clear from Figure 34 the tangential stress increases when the body subjected to internal heat generation.

Figure 31: Radially distributed temperature

Figure 34: Radially distributed tangential stress

Figure 32: Radially distributed displacement

Figure 35: Radially distributed von-Mises stress

Figure 33: Radially distributed radial stress

Figure 36: Radially distributed temperature

Case 6: Spherical body with variable heat generation and gravity force

In this case study a hollow spherical body subjected to gravitational force and variable heat generation is investigated. Similar observations are obtained for displacement and stresses under this case as in case 3.

Case 7: Spherical body with rotation, gravitational force and variable heat generation

In this case, a hollow spherical body with a rotational, gravitation force and variable heat generation is investigated. The radial distribution of temperature is shown in

Figure 37: Radially distributed displacement

Figure 40: Radially distributed von-Mises stress

Figure 38: Radially distributed radial stress

Figure 41: Radially distributed temperature

Figure 39: Radially distributed tangential stress

Figure 42: Radially distributed displacement

Figure 41 and displacement and stresses are shown in figures Figure 42 to 45.

In the graphs presented in all the cases reported above, it has been observed that nature of von-Mises stresses for r/a in the range of 1.08 to 1.1 (approx.) is reversed i.e. for $r/a < 1.08$ (approx.) the von-Mises stress is inversely proportional to the grading parameters and for $r/a > 1.1$ (ap-

prox.) the von-Mises stress is directly proportional to the grading parameter. The values are given in Table 1 for $b/a = 1.08$ for corresponding to grading parameters, and different loading condition.

Figure 43: Radially distributed radial stress

Figure 44: Radially distributed tangential stress

Figure 45: Radially distributed von-Mises stress

6 Conclusion

The present paper reports the exact solution for thermo-elastic deformation and stress state of functionally graded rotating spherical body. Power law function is used for material grading along the radial direction. Stresses are ob-

Table 1: For different cases 1-7 the von-Mises stresses (σ^*) at $b/a = 1.08$

CASES	Grading parameter					
	-2	-1	0	1	2	3
Case 1	0.21	0.21	0.21	0.21	0.21	0.21
Case 2	0.20	0.20	0.19	0.19	0.19	0.19
Case 3	0.19	0.19	0.19	0.19	0.19	0.18
Case 4	0.21	0.21	0.21	0.21	0.21	0.21
Case 5	0.21	0.21	0.21	0.21	0.21	0.21
Case 6	0.19	0.19	0.19	0.19	0.19	0.18
Case 7	0.21	0.21	0.21	0.21	0.21	0.21

tained from the solution of Navier equation through direct method and the effect due to grading parameter, rotational speed, gravitational force and variable heat generation are studied on the stresses and displacement field. This paper shows the general mathematical formulation and solution technique based on material grading law for the FG spherical body.

- The effect of grading parameter was studied on internally pressurised hollow functionally graded spherical body and found that with the increments of grading parameter, the strength of hollow spherical body improves.
- Effect of grading parameter on radial displacement of hollow functionally graded spherical body was investigated and noticed that as grading parameter is increased, the displacement of hollow spherical body reduces continually.
- Displacement reduces continuously along radial direction from inner to outer side of hollow spherical body under all cases reported for the present study.
- Effect of grading parameter on radial stress of hollow functionally graded spherical body was investigated and noticed that as grading parameter increases, the radial stress recedes.
- Effect of grading parameter on tangential stress of hollow functionally graded spherical body was investigated and noticed that for $n < 1$ the tangential stress decreases along radial direction from inner to outer radii but for $n > 1$, the tangential stress increases along radial direction from inner to outer radii and at $n = 1$, tangential stress is almost uniform along the radial direction.
- To study the overall stress distribution along the radial direction, the von-Mises stress is plotted along the radial direction for $b/a = 1.2$. The von-Mises stress is almost uniform along radial direction for $n = 3$. It is reported that von-Mises stress is higher at

inner radius and continuously reduces along till the outer radius is reached.

- It is clear from all graphs of von-Mises stress that the von-Mises stress decreases as grading parameter increases till a critical value of b/a is reached. Beyond this, von-Mises stress increases as the grading parameter increases.

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