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On canonical screen for lightlike submanifolds of codimension two

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Abstract: In this paper we study two classes of lightlike submanifolds of codimension two of semi-Riemannian manifolds, according as their radical subspaces are 1-dimensional or 2-dimensional. For a large variety of both these classes, we prove the existence of integrable canonical screen distributions subject to some reasonable geometric conditions and support the results through examples.

Keywords: Half lightlike submanifold, coisotropic submanifold, canonical screen distribution, screen conformal fundamental forms
MSC (2000): 53B25, 53C50, 53B50.

1 Introduction

The general theory of lightlike submanifolds (see for example [7]) uses a non-degenerate screen distribution which (due to the degenerate induced metric) is not unique. Therefore, the induced objects of the submanifold depend upon the choice of a screen. Thus, it is reasonable to look for a canonical screen in lightlike geometry. Following [7] considerable work has been pursued to deal with the interdependence of induced objects and now there are large classes of lightlike hypersurfaces of semi-Riemannian manifolds with the choice of a canonical screen distribution (see [1, 2, 5, 10]), in some cases subject to a reasonable geometric condition. Recently, the present author has introduced the concept of induced scalar curvature for a class of lightlike hypersurfaces of Lorentzian manifolds [4] using the choice of a canonical screen distribution.

Continuing our study in this direction, a next step is to find canonical screens for

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lightlike submanifolds of codimension two [6]. There are two such classes of submanifolds explained as follows. Let (M, g) be a codimension two lightlike submanifold of an $(m+2)$-dimensional semi-Riemannian manifold $(\bar{M}, \bar{g})$ of constant index $q \geq 1$, where g is the induced degenerate tensor field of $\bar{g}$ on M [6] and $m > 1$. All manifolds are paracompact and smooth. Denote by $F(M)$ the algebra of smooth functions on M and by $\Gamma(E)$ the $F(M)$ module of smooth sections of a vector bundle E over M and the same notation for any other vector bundle. There exists a vector field $\xi \in \Gamma(TM)$, $\xi \neq 0$, such that $g(\xi, X) = 0$, for any $X \in \Gamma(TM)$. For each tangent space T_xM we consider

$$T_x M^{\perp} = \{X \in T_x \bar{M} \, : \, \bar{g}(X, U) = 0, \forall U \in T_x M\}$$

a degenerate 2-dimensional orthogonal (but not complementary) subspace of $T_x \bar{M}$. The radical subspace $Rad\, T_xM \subseteq T_xM^{\perp}$ is either a 1-dimensional or 2-dimensional subspace of $T_x M$. There exists a complementary non-degenerate distribution $S(TM)$ to $Rad\, TM$ in TM, called a screen distribution of M, with the following orthogonal distribution

$$TM = Rad\, TM \oplus_{orth} S(TM).$$

The submanifold $(M, g, S(TM))$ is called a *half lightlike submanifold* [6, 8] if $\dim(Rad\, TM) = 1$. We use the term half lightlike since for this class $TM^{\perp}$ is half lightlike. On the other hand, if $\dim(Rad\, TM) = 2$, then, $Rad\, TM = TM^{\perp}$ and $(M, g, S(TM))$ is called a *coisotropic submanifold* [11].

The objective of this paper is to show that there exist canonical screen distributions for a large variety of both the above stated classes. We deal with these two classes separately in sections 2 and 3 respectively, prove one main theorem for each class and support the results through examples.

2 Half lightlike submanifolds

Let $(M, g, S(TM))$ be a half lightlike submanifold of a semi-Riemannian manifold $(\bar{M}, \bar{g})$. Then, there exist vector fields ξ, $u \in T_x M^{\perp}$ such that

$$\bar{g}(\xi, v) = 0, \qquad \bar{g}(u, u) \neq 0, \qquad \forall v \in T_x M^{\perp}.$$

The above relations imply that $\xi \in Rad\, T_x M$. Consider the orthogonal complementary distribution $S(TM^{\perp})$ to $S(TM)$ in $T\bar{M}$. Certainly ξ and u belong to $\Gamma(S(TM^{\perp}))$. Choose u as a unit vector field, with $\bar{g}(u, u) = \epsilon = \pm 1$. We briefly summarize the following results (for details see [8]). Let $D = \text{span}\{u\}$ be a supplementary distribution to $Rad\, TM$ in $S(TM^{\perp})$. Hence we have the following orthogonal decomposition

$$S(TM^{\perp}) = D \perp D^{\perp},$$

where $D^{\perp}$ is the orthogonal complementary distribution to D in $S(TM^{\perp})$. Let F be a 1-dimensional non-null subbundle of $D^{\perp}$. Then, for any local null section ξ of $Rad(TM)$ on a coordinate neighborhood $\mathcal{U} \subset M$, there exists a uniquely defined vector field $N \in \Gamma(D^{\perp})$ satisfying

$$\bar{g}(N,\xi) = 1, \quad \bar{g}(N,N) = \bar{g}(N,W) = 0, \qquad \forall\, W \in \Gamma(S(TM)|_{\mathcal{U}}) \tag{2.1}$$

if and only if N is given by

$$N = \frac{1}{\bar{g}(\xi,\, V)}\{V - \frac{\bar{g}(V,V)}{2\,\bar{g}(\xi,\, V)}\,\xi\,\}, \quad V \in \Gamma(F_{|\mathcal{U}}) \tag{2.2}$$

such that $\bar{g}(\xi,\, V) \neq 0$. Let $\bar{\nabla}$ and ∇ be the Levi-Civita connection on $\bar{M}$ and a linear connection on M respectively and P the projection of TM on $S(TM)$. The local Gauss and Weingarten formulas are:

$$\bar{\nabla}_X Y = \nabla_X Y + D_1(X\,,\, Y)\, N + D_2(X\,,\, Y) u\,, \tag{2.3}$$

$$\bar{\nabla}_X N = -\, A_N\, X + \tau\,(X)\, N + \rho(X)\, u\,, \tag{2.4}$$

$$\bar{\nabla}_X u = -\, A_u\, X + \psi(X)\, N, \tag{2.5}$$

$$\nabla_X PY = \nabla^*_X PY + E(X\,, PY)\,\xi, \tag{2.6}$$

$$\nabla_X \xi = -\, A^*_\xi\, X - \tau\,(X)\,\xi, \quad \forall X\,,\, Y \in \Gamma\,(TM) \tag{2.7}$$

where D_1 and D_2 are the lightlike and the screen second fundamental forms of M respectively, τ, ρ and ψ are 1-forms on M. Both A_N and A_u are the shape operators of M. Also E is the local second fundamental form of $S(TM)$ with respect to $Rad\,TM$, A^*_ξ is the shape operator of the screen distribution and ∇^* is the metric connection on $S(TM)$ but, in general, ∇ is not a metric connection on M. Indeed, $\forall\, X, Y, Z \in \Gamma(TM)$, we have

$$(\nabla_X g)(Y,Z) = D_1(X,Y)\eta(Z) + D_1(X,Z)\eta(Y), \quad \eta(X) = g(X,N). \tag{2.8}$$

Using (2.1) and (2.3) - (2.7) we obtain

$$D_1(X,\xi) = 0, \quad D_1(X,PY) = g(A^*_\xi\, X, PY), \tag{2.9}$$

$$\tau(X) = \bar{g}(\nabla_X N,\xi), \quad \rho(X) = \epsilon\bar{g}(\nabla_X N,u), \tag{2.10}$$

$$\psi(X) = -\epsilon D_2(X,\xi), \qquad E(X,PY) = \bar{g}(\nabla_X PY,N). \tag{2.11}$$

Suppose a screen $S(TM)$ changes to another screen $S(TM)'$, where $\{\xi,\, N,\, W_a,\, u\,\}$ and $\{\xi,\, N',\, W'_a,\, u'\,\}$ respectively are two quasi-orthonormal frame fields for the same null section ξ. The following are the transformation equations due to this change (for details see [7, pages 164-165]).

$$W'_a = \sum_{b=1}^{m-1} A^b_a\,(W_b - \epsilon_b\,\mathbf{f}_b\,\xi)\,; \quad u' = u - \epsilon f \xi, \tag{2.12}$$

$$N' = N - \frac{1}{2}\left\{\sum_{a=1}^{m-1} \epsilon_a \mathbf{f}_a^2 + \epsilon\mathbf{f}^2\right\}\xi + \sum_{a=1}^{m+1} \mathbf{f}_a W_a + \mathbf{f}u, \tag{2.13}$$

$$\begin{aligned}\nabla'_X PY = {} & \nabla_X PY + \frac{1}{2} D_1(X,PY)\left(\sum_{a=1}^{m-1} \epsilon_a \mathbf{f}_a^2 + \epsilon\mathbf{f}^2\right)\xi \\ & + \epsilon D_2(X,PY)\mathbf{f}\xi - D_1(X,PY)\left(\sum_{a=1}^{m-1} \mathbf{f}_a\, W_a\right)\end{aligned} \tag{2.14}$$

$$D_1'(X,Y) = D_1(X,Y),\ D_2'(X,Y) = D_2(X,Y) - D_1(X,Y)\mathbf{f}, \tag{2.15}$$

$$\begin{aligned} E'(X,\ PY) &= E(X,\ PY) - \frac{1}{2}\left(||W||^2 - \epsilon\mathbf{f}^2\right) D_1(X,\ PY) \\ &\quad + g(\nabla_X PY,\ W) + \epsilon D_2(X, PY)\mathbf{f}, \end{aligned} \tag{2.16}$$

where $W = \sum_{a=1}^{m} \mathbf{f}_a W_a$. Let ω be the dual 1-form of W given by

$$\omega(X) = g(X,W), \quad \forall\, X \in \Gamma(TM). \tag{2.17}$$

Denote by $\mathcal{S}$ the first derivative of a screen distribution $S(TM)$ given by

$$\mathcal{S}(x) = \text{span}\{[X,Y]_x, \quad X_x, Y_x \in S(TM), \quad x \in M\}, \tag{2.18}$$

where $[,]$ denotes the Lie-bracket. If $S(TM)$ is integrable, then, $\mathcal{S}$ is a sub bundle of $S(TM)$. We state and prove the following theorem:

Theorem 2.1. *Let $(M, g, S(TM))$ be a half lightlike submanifold of a semi-Riemannian manifold $(\bar{M}^{m+2}, \bar{g})$ with $m > 1$. Suppose the subbundle F of $D^\perp$ admits a covariant constant non-null vector field. Then, with respect to a section ξ of $Rad\,TM$, M can admit an integrable screen $S(TM)$. Moreover, if the first derivative $\mathcal{S}$ defined by (2.18) coincides with $S(TM)$, then, $S(TM)$ is a canonical screen of M, up to an orthogonal transformation with a canonical lightlike transversal vector bundle and the screen second fundamental form E is independent of a screen distribution.*

Proof. By hypothesis, consider (without any loss of generality), along M, a unit covariant constant vector field $V \in \Gamma(F_{|\mathcal{U}})$, that is, $\bar{g}(V,V) = e = \pm 1$. To satisfy the condition given in (2.2), we choose a section ξ of $Rad\,TM$ such that $\bar{g}(V,\ \xi) \neq 0$. For convenience in calculations, we set $\bar{g}(V,\ \xi) = \theta^{-1}$. Using this and (2.2), the null transversal vector bundle of M takes the form

$$N = \theta\,(V - \frac{e\,\theta}{2}\,\xi\,). \tag{2.19}$$

Then using (2.19) in (2.4) and (2.7) we get

$$\begin{aligned} \tau(X) &= \bar{g}(\bar{\nabla}_X\, N, \xi) = X(\theta)\,\bar{g}(V,\ \xi) - \frac{e}{2}\,(\theta)^2\,\bar{g}(\bar{\nabla}_X\,\xi,\ \xi) \\ &= X(\theta)\,(\theta)^{-1} = X(\log\,\theta). \end{aligned} \tag{2.20}$$

$$\begin{aligned} \rho(X) &= \bar{g}(\bar{\nabla}_X N, u) = \bar{g}\left(\bar{\nabla}_X(\theta V) - \frac{e}{2}\bar{\nabla}_X(\theta^2\xi), u)\right) \\ &= \frac{e\,\theta^2}{2}\psi(X). \end{aligned} \tag{2.21}$$

Using above value of τ, (2.19) and (2.7) we obtain

$$\bar{\nabla}_X\, N = X(\theta)V - \frac{e}{2}\,\theta X(\theta)\xi + \frac{e}{2}\,\theta^2\ A^*_\xi X + \frac{e\theta^2}{2}\psi(X)u. \tag{2.22}$$

On the other hand, substituting the value of τ and $\rho = 0$ in (2.4), we get

$$\bar{\nabla}_X N = -A_N X + X(\theta)V - \frac{e}{2}\theta X(\theta)\,\xi + \frac{e\theta^2}{2}\psi(X)u. \tag{2.23}$$

Equating (2.21) and (2.22) we obtain

$$A_N X = -\frac{e\,\theta^2}{2} A^*_\xi X, \quad \forall X \in \Gamma(TM_{|\mathcal{U}}). \tag{2.24}$$

Since A^*_ξ is symmetric with respect to g, the Eq. (2.24) implies that A_N is also symmetric with respect to g, which further follows from [8, page 128] that the screen distribution $S(TM)$ is integrable. This means that $\mathcal{S}$ is a subbundle of $S(TM)$. Using (2.24) in the second Eq. of (2.11) we get

$$E(X, PY) = -\frac{e\,\theta^2}{2} D_1(X,Y), \quad \forall X, Y \in \Gamma(TM_{|\mathcal{U}}). \tag{2.25}$$

Using (2.24), (2.16) and $D_1' = D_1$ we obtain

$$g(\nabla_X PY, W) = \frac{1}{2}\left(||W||^2 - \epsilon \mathbf{f}^2\right) D_1(X, Y) - \epsilon D_2(X,Y)\mathbf{f} \tag{2.26}$$

$\forall\, X, Y \in \Gamma(TM_{|\mathcal{U}})$. Since the right hand side of (3.17) is symmetric in X and Y, we have $g([X,Y], W) = \omega([X,Y]) = 0$, $\forall X, Y \in \Gamma(S(TM)_{|\mathcal{U}})$, that is, ω vanishes on $\mathcal{S}$. By hypothesis, if we take $\mathcal{S} = S(TM)$, then, ω vanishes on this choice of $S(TM)$ which implies from (2.17) that $W = 0$. Therefore, the functions $\mathbf{f}_a$ vanish. Finally, substituting this data in (2.16) it is easy to see that the function $\mathbf{f}$ also vanishes. Thus, the transformation Eqs. (2.12), (2.13) and (2.14) become $W_a' = \sum_{b=1}^{m-1} A_a^b W_b$ $(1 \le a \le m-1)$, $N' = N$ and $E' = E$ where (A_a^b) is an orthogonal matrix of $S(TM)$ at any point $x \in M$. Therefore, $S(TM)$ is a canonical screen up to an orthogonal transformation with a canonical transversal vector field N and the screen fundamental form E is independent of a screen distribution. This completes the proof. □

To understand some examples of half lightlike submanifolds, satisfying theorem 2.1, we first quote the following result.

Proposition 2.2 [6]. *Let $(M, g, S(TM))$ be a half-lightlike submanifold of a semi–Riemannian manifold $\bar{M}$, with $\bar{g}$ of index $q \in \{1, \ldots, m+1\}$. Then we have the following:*

(i) *If u is spacelike then $S(TM)$ is of index $q-1$. In particular $S(TM)$ is Riemannian for $q = 1$ and Lorentzian for $q = 2$.*

(ii) *If u is timelike then $S(TM)$ is of index $q-2$. In particular $S(TM)$ is Riemannian for $q = 2$ and Lorentzian for $q = 3$.*

It follows from proposition 2.2 (i) that M can be a half lightlike submanifold of a Lorentzian manifold for which $\bar{g}(u,u) = \epsilon = 1$. Thus, it is obvious from the structure equations that we choose $\bar{g}(V,V) = e = -1$, a covariant constant timelike unit vector field. There are many examples of n-dimensional product Lorentzian spaces (such as warped product globally hyperbolic spacetime [3]) which posses at least one timelike covariant constant vector field and, therefore, can satisfy the hypothesis of the theorem 2.1. In particular, M provides a physical model of null 2-surfaces in a 4-dimensional space time of general relativity. For the remaining cases when $q = 2$, we refer to two examples given in [8, 9], both of which can satisfy the theorem 2.1.

3 Coisotropic submanifolds

For this case, $\dim(Rad\,TM) = 2$ and $Rad\,TM = TM^{\perp}$ which implies that $TM^{\perp}$ is totally lightlike. There exist fields of frames

$$\{\xi_1, \xi_2, W_1, \ldots, W_{m-2}\}$$

and

$$\{\xi_1, \xi_2, W_1, \ldots, W_{m-2}, N_1, N_2\}$$

on M and $\bar{M}$ respectively such that $Rad\,TM = \mathrm{span}\{\xi_1, \xi_2\}$ and the canonical normal null bundle $NM = \mathrm{span}\{N_1, N_2\}$ satisfying

$$\bar{g}(\xi_i, \xi_j) = \bar{g}(N_i, N_j) = 0, \quad \bar{g}(N_i, \xi_j) = \delta_{ij}, \quad \forall i, j = 1, 2.$$

Following are the Gauss and Weingarten equations [11]:

$$\bar{\nabla}_X Y = \nabla_X Y + \sum_i D_i\,(X\,,\,Y)\,N_i, \tag{3.1}$$

$$\bar{\nabla}_X N_i = -\,A_{N_i}\,X + \tau_{ij}\,(X)\,N_j, \tag{3.2}$$

$$\nabla_X PY = \nabla^*_X PY + \sum_i E_i(X\,,PY)\,\xi_i, \tag{3.3}$$

$$\nabla_X \xi_i = -\,A^*_{\xi_i}\,X - \tau_{ij}(X)\,\xi_j, \quad \forall X\,,\,Y \in \Gamma\,(TM), \tag{3.4}$$

where $i, j = 1, 2$, D_i are the local second fundamental forms of M with respect to the normals N_i, A_{N_i} are the respective shape operators of M and τ_{ij} are 1-forms on M. Also E_i are the local second fundamental forms of $S(TM)$ with respect to $Rad\,TM$, $A^*_{\xi_i}$ are the respective shape operators of the screen distribution and ∇^* is the metric connection on $S(TM)$.

$$D_1(X, \xi_1) = D_2(X, \xi_2) = 0, \quad D_i(X, PY) = g(A^*_{\xi_i}\,X, PY), \tag{3.5}$$

$$E_i(X, PY) = \bar{g}(\nabla_X PY, N_i) = g(A_{N_i} X, PY). \tag{3.6}$$

Suppose a screen $S(TM)$ changes to another screen $S(TM)'$, where $\{\xi_1, \xi_2, W'_1, \ldots, W'_{m-2}, N'_1, N'_2\}$ is another quasi-orthonormal frame fields for the same pair of null sections $\{\xi_1, \xi_2\}$. The following are the transformation equations due to this change.

$$W'_a = \sum_{b=1}^{m-2} A_a^b \left(W_b - \epsilon_b \sum_{i=1}^{2} \mathbf{f}_{ib}\,\xi_i \right) \tag{3.7}$$

$$N'_i = N_i + \sum_{j=1}^{2} N_{ij}\xi_j + \sum_{a=1}^{m-2} \mathbf{f}_{ia} W_a, \tag{3.8}$$

with the conditions

$$2N_{ii} = -\sum_{a=1}^{m-2} \epsilon_a (\mathbf{f}_{ia})^2, \quad N_{12} + N_{21} + \sum_{a=1}^{m-2} \epsilon_a \mathbf{f}_{1a}\mathbf{f}_{2a} = 0. \tag{3.9}$$

$$D'_i(X,Y) = D_i(X,Y), \quad \forall X\,,\, Y \in \Gamma\,(TM), \tag{3.10}$$

$$\nabla'_X PY = \nabla_X PY - \sum_{j=1}^{2}\left(\sum_{i=1}^{2} D_i(X,PY)N_{ij}\right)\xi_j - \sum_{a=1}^{m-2}\left(\sum_{i=1}^{2} D_i(X,PY)\mathbf{f}_{ia}\right)W_a, \tag{3.11}$$

$$E'_1(X,\, PY) = E_1(X,\, PY) - \frac{1}{2}||\mathbf{Z}_1||^2 D_1(X,PY) + N_{21}D_2(X,PY) + g(\nabla_X PY,\, \mathbf{Z}_1) - g(\mathbf{Z}_2,\mathbf{Z}_2))D_2(X,PY) \tag{3.12}$$

$$E'_2(X,\, PY) = E_2(X,\, PY) - \frac{1}{2}||\mathbf{Z}_2||^2 D_2(X,PY) + N_{12}D_1(X,PY) + g(\nabla_X PY,\, \mathbf{Z}_2) - g(\mathbf{Z}_1,\mathbf{Z}_1)D_1(X,PY) \tag{3.13}$$

where $\mathbf{Z}_i = \sum_{a=1}^{m} \mathbf{f}_{ia}\, W_a$ are two characteristic vector fields of the screen change. Let ω_i be the respective dual 1-forms of $\mathbf{Z}_i$ given by

$$\omega_i(X) = g(X,\mathbf{Z}_i), \quad \forall\, X \in \Gamma(TM). \tag{3.14}$$

It is known that the second fundamental forms and their respective shape operators of a non-degenerate submanifold are related by means of the metric tensor. Contrary to this we see from Eqs. (3.5) and (3.6) that there are interrelations between the lightlike and the screen second fundamental forms and their respective shape operators. This interrelation indicates that the lightlike geometry depends on a choice of screen distribution. While we know from Eq. (3.10) that the lightlike second fundamental forms are independent of a screen, the same is not true for the screen fundamental forms (see Eqs. (3.12) and (3.13)), which is the root of non-uniqueness anomaly in the lightlike geometry. Since, in general, it is impossible to remove this anomaly, we consider a class of coisotropic submanifolds M whose lightlike and screen fundamental forms are related by two conformal non-vanishing smooth functions in $\mathcal{F}(M)$. The motivation for this geometric restriction comes from the classical geometry of non-degenerate submanifolds for which there are only one type of fundamental forms with their one type of respective shape operators. Thus, we make the following definition.

Definition 3.1. A coisotropic submanifold $(M, g, S(TM))$ of a semi-Riemannian manifold $(\bar{M}, \bar{g})$ is called a *screen locally conformal submanifold* if its screen fundamental forms E_i are conformally related to the corresponding lightlike fundamental forms D_i by

$$E_i(X,PY) = \varphi_i D_i(X,Y), \quad \forall X,Y,\Gamma(TM|_{\mathcal{U}}), \quad i \in \{1,2\}, \tag{3.15}$$

where $\varphi'_i s$ are non-vanishing smooth functions on a neighborhood $\mathcal{U}$ in M.

In order to avoid trivial ambiguities, we will consider $\mathcal{U}$ to be connected and maximal in the sense that there is no larger domain $\mathcal{U}' \supset \mathcal{U}$ on which Eq. (3.15) holds. In case $\mathcal{U} = M$ the screen conformality is said to be global. Moreover, the above definition will also hold for a coisotropic submanifold of codimension higher than two.

Theorem 3.2. *Let $(M, g, S(TM))$ be a codimension two coisotropic screen conformal submanifold of a semi-Riemannian manifold $(\bar{M}^{m+2}, \bar{g})$ with $m > 1$. Then,*

(a) *any choice of a screen distribution is integrable and*

(b) *the two 1-forms ω_i in (3.14) vanish identically on the first derivative $\mathcal{S}$ given by (2.18).*

(c) *If $\mathcal{S}$ coincides with $S(TM)$, then, there exists a pair of null sections $\{\xi_1, \xi_2\}$ of $\Gamma(Rad\,TM)$ with respect to which $S(TM)$ is a canonical screen distribution of M, up to an orthogonal transformation with a canonical pair $\{N_1, N_2\}$ of lightlike transversal vector bundle and the screen fundamental forms E_i are independent of a screen distribution.*

Proof. Using (3.15) in (3.6) and the second equation of (3.5) we get

$$A_{N_i} X = \varphi_i \, A^*_{\xi_i} X, \quad \forall X \in \Gamma(TM_{|\mathcal{U}}). \tag{3.16}$$

Since each $A^*_{\xi_i}$ is symmetric with respect to g, Eq. (3.16) implies that each A_{N_i} is also symmetric with respect to g, which further follows from [11, page 38] that any choice of a screen distribution is integrable proving (a). Choose an integrable screen $S(TM)$. This means that $\mathcal{S}$ is a subbundle of $S(TM)$. Using (3.15) in (3.12) and $D_i' = D_i$ we obtain

$$g(\nabla_X PY, \mathbf{Z}_1) = \frac{1}{2}||\mathbf{Z}_1||^2 D_1(X, Y) + (g(\mathbf{Z}_2, \mathbf{Z}_2) - N_{21})\, D_2(X, Y) \tag{3.17}$$

$\forall\, X, Y \in \Gamma(TM_{|\mathcal{U}})$. Since the right hand side of (3.17) is symmetric in X and Y, we have $g([X, Y], \mathbf{Z}_1) = \omega_1([X, Y]) = 0$, $\forall X, Y \in \Gamma(S(TM)_{|\mathcal{U}})$, that is, ω_1 vanishes on $\mathcal{S}$. Similarly, repeating above steps for Eq. (3.13) we claim that ω_2 vanishes on $\mathcal{S}$. If we take $\mathcal{S} = S(TM)$, then, both ω_i vanish on this choice of $S(TM)$ which implies from (3.14) that both the characteristic vector fields $\mathbf{Z}_i$ vanish. Therefore, all the functions $\mathbf{f}_{ia}$ vanish. Finally, substituting this data in (3.9), (3.12) and (3.13) it is easy to see that all the functions N_{ij} also vanish. Thus, the transformation Eqs. (3.7), (3.8), (3.12) (3.13) become $W_a' = \sum_{b=1}^{m-1} A_a^b W_b$ $(1 \le a \le m-1)$, $N_i' = N_i$ and $E_i' = E_i$ where (A_a^b) is an orthogonal matrix of $S(TM)$ at any point $x \in M$. Therefore, $S(TM)$ is a canonical screen up to an orthogonal transformation with canonical transversal vector fields N_i and the screen fundamental forms E_i are independent of a screen distribution. This completes the proof. □

To present some examples of coisotropic submanifolds, satisfying theorem 3.2, we first quote the following result:

Theorem 3.3. [11, page 43]. *Let* $(M, g, S(TM))$ *be a proper totally umbilical coisotropic submanifold of a semi-Riemannian manifold of constant curvature* $(\bar{M}(c), \bar{g})$*. Then the screen distribution* $S(TM)$ *is integrable, if and only if, each* 1*-form* τ_{ij} *induced by* $S(TM)$ *satisfies* $d(Tr(\tau_{ij})) = 0$*, where* $Tr(\tau_{ij})$ *is the trace of the matrix* (τ_{ij})*.*

Since the primary result of theorem 3.2 is the existence of an integrable screen, it follows from the above theorem that a large class of totally umbilical coisotropic lightlike submanifolds of $(\bar{M}(c), \bar{g})$ are candidates for the existence of a canonical screen distribution.

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