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Fractional Order on the Impact of Climate Change With Dominant Earth's Fluctuations

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Abstract: In this investigation, fractional order model on the impact of climate change with dominant Earth's fluctuations is given. The solution of the model was obtained using modified Laplace Adomian decomposition method. The result is compared with the result obtained from integer solution. We observed that what is seen in the fractional part takes longer to be seen in the integer part. We also observed that regardless of any choice we make to mitigate climate change, the impact will still persist due to the effect of Earth's fluctuations.

Keywords: Earth's fluctuations, climate change, thermocline, damped constant, Laplace Adomian decomposition

MSC: 34A08; 26A33

1 Introduction

Climate change represents one of the greatest environmental threats facing planet Earth today and it has become one of the most pressing scientific challenges facing society [1, 2]. The consequences of climate change could be devastating with increased human activities such as burning of fossil fuels, which has been identified as significant causes of recent climate change, often referred to as global warming [3, 4]. This global warming has resulted in large-scale, high-impact, nonlinear, and potentially abrupt and / or irreversible changes in physical and biological systems [5, 6].

As global temperatures rise, oceans get warmer and when water heats up, it expands and sea levels rise. The United Nations' Intergovernmental Panel on Climate Change (IPCC) reported that between 1993 to 2003, global sea level rose about 3 millimeters each year, and approximately half of that increase is attributed to the ocean expanding as it warms (see IPCC [1]). The global temperatures rise is one of the results of the current energy imbalance of the Earth. The energy imbalance is due to the Earth not emitting as much energy in thermal radiation as it is absorbing from the Sun's solar radiation. Greenhouse gases (GHGs) contribute to this effect by not allowing some thermal radiation to leave Earth's atmosphere, forces it to come back to the Earth to be absorbed, causing global warming [7]. This global warming is not restricted to climate change; the entire Earth is experiencing the consequences of energy imbalance. This energy imbalance could be in the form of a volcanic activity or an earthquake (seismic activities) [8]. So the dangers of global warming are far greater for humanity than implied climate change.

Therefore, to better understand and predict the climate behaviour in Pacific Ocean requires an in-depth understanding of Earth's fluctuations, which are caused by seismic activities in the Earth's crust.

In [9, 10], it was observed that in the absence of seismic activities in the Earth's crust, the Earth undergoes continuous oscillations often known as Earth hum, and the main source of energy responsible for the excitation of the fluctuations is provided by the presence of coda waves, originated by the continuous occur-

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rence of seismic activities. It was shown in [11] that the power spectrum of coda waves follows a power law that behaves as $\frac{1}{f^\alpha}$ noise. It is considered to be noise because it masks the arrival of coherent seismic phase.

In the work of White and Liu [12], Bershaskii [13] (and among others), only integer model of climate change was studied. Therefore, in this work, the primary concern is to study the impact of climate change using fractional differentials approach, since fractional calculus is an important tool for the study of dynamical systems where classical methods reveal strong limitations. Also, many phenomena in nature are modelled accurately using fractional calculus [14].

Several authors have used fractional calculus to model physical and biological phenomena in their work, and we shall therefore mention only briefly a few papers in which more comprehensive references can be found. In the work Atabong and Oyesanya [15], the fractional reaction-diffusion model as a plausible model for the invasion of tumor and its consequences in the human organ in which it resides was developed. Oyesanya and Atabong [16] also used a fractional model to study reaction diffusion for tumor invasion and therapeutic action. Long-Jye et al [17] examined the dynamics of the fractional damped Duffing equation. In this study, the fractional damped Duffing equation was transformed into a set of fractional integral equations and solved by a predictor-corrector method. Nan and Ming-Yan [18] proposed a new detection method for improving fractional order Duffing oscillator. Arafa et al. [19] used Mittag-Leffler function method to solve some nonlinear fractional differential equations. They also constructed a new solution in power series. In the work of Zakia and Toufik [20], a fractional Newell-Whitehead equation with initial condition was solved using Adomian decomposition method for two cases time-fractional order and time-space fractional order respectively. Mohammad [21] used multi-step Laplace Adomian decomposition method to solve nonlinear fractional differential equations which were presented as fractional initial value problems. Ahmed and Kir-tiwant [22] applied the Adomian decomposition and modified Adomian decomposition methods to find the approximate solution of a nonlinear fractional Volterra-Fredholm integro-differential equation.

In this work, we shall adopt modified Laplace Adomian decomposition method since it can provide a direct scheme to solve our model without the need for linearization, perturbation or any massive computation.

2 Formulation of the problem

The conceptual model of the tropical pacific oscillations for positive upper ocean temperature anomalies T driving shallow thermocline depth anomalies u , is described by the following equations [12, 13]:

$$\frac{du}{dt} = -T + \lambda \quad (1)$$

and

$$\frac{dT}{dt} = -aT + (u + bu^3) + k \cos \omega t \quad (2)$$

where, λ is ambient noise sources, $k \cos \theta$ is a solar forcing, a and b are coefficients of damping and nonlinearity respectively.

By differentiating (1) with respect to t and substituting in (2), we have

$$\frac{d^2u}{dt^2} = aT - (u + bu^3) - k \cos \omega t. \quad (3)$$

Now substituting (1) in (3), we have

$$\frac{d^2u}{dt^2} = a \left(\lambda - \frac{du}{dt} \right) - (u + bu^3) - k \cos \omega t. \quad (4)$$

Now we shall let $\lambda = \frac{1}{f^\alpha}$ as in [10] to have

$$\frac{d^2u}{dt^2} = a \left(\frac{1}{f^\alpha} - \frac{du}{dt} \right) - (u + bu^3) - k \cos \omega t. \quad (5)$$

Since anthropogenic carbon dioxide CO_2 emissions are the largest greenhouse gas in the atmosphere and responsible for most of the global warming [23], then we shall incorporate its radiative forcing in the model.

According to Greiner and Semmler [23], the radiative forcing due to increase in CO_2 can be expressed as

$$R = \frac{6.3\beta_1(1-\varnothing)}{C_h} \ln\left(\frac{C}{\hat{C}}\right), \quad (6)$$

where C_h is the specific heat capacity of the Earth, which is taken to be equal to $16.7 \text{ Wm}^{-2} \text{ K}^{-1}$, $\hat{C}$ is the pre-industrial concentration, β_1 stands for all feedback, which assumed to take the values between 1.1 and 3.4 (see [6, 24]), and $\varnothing$ captures the part of the warmth generated by greenhouse effect that is absorbed by the oceans and transported from their upper layers to deep sea, and its value is taken as 0.23 [24].

Therefore, taking the radiative forcing due to increase in CO_2 into consideration, then equation (5) can now be written as

$$\frac{d^2u}{dt^2} + a\frac{du}{dt} + u + bu^3 = af^{-\alpha_*} + R - k \cos \omega t \quad (7)$$

$$\Rightarrow \ddot{u} + a\dot{u} + u + bu^3 = af^{(-\alpha_*)} + R - k \cos \omega t. \quad (8)$$

where k , ω and f are amplitude, natural frequency of solar forcing and frequency of seismic activities respectively.

The motion of the system will be established by initial conditions:

$$u(0) = \dot{u}(0) = 0. \quad (9)$$

3 Preliminary

We now start with a brief discussion of modified Laplace Adomian decomposition method (MLADM) since it can provide a direct scheme to solve our model without the need for linearization, perturbation or any massive computation.

Now we consider the general form of second order nonlinear differential equation with initial conditions in the form [25–29]:

$$Au(t) + Bu(t) + Nu(t) = f(t), \quad (10)$$

$$u(0) = a, u'(0) = b, \quad (11)$$

where A is the second order differential operator, B is the remaining linear operator, N represents a general nonlinear differential operator, $f(t)$ is known function, a and b are real constants.

Applying Laplace transform on both sides of differential equation (10), we have

$$L[Au(t)] + L[Bu(t)] + L[Nu(t)] = L[f(t)]. \quad (12)$$

Now using the differential property of Laplace transform and initial conditions (11), we obtain:

$$s^2L(u) - sa - b + L[B(u)] + L[N(u)] = L[f(t)] \Rightarrow L(u) = \frac{a}{s} + \frac{b}{s^2} - \frac{1}{s^2}L[B(u)] - \frac{1}{s^2}L[N(u)] + \frac{1}{s^2}L[f(t)]. \quad (13)$$

The MLADM represents the solution as an infinite series

$$u(t) = \sum_{n=0}^{\infty} u_n(t). \quad (14)$$

The nonlinear operator is decomposed as

$$Nu(t) = \sum_{n=0}^{\infty} A_n, \quad (15)$$

where A_n are so the Adomian polynomials defined as

$$A_n = \left[\frac{1}{n!} \frac{d}{d\varphi^n} N \left(\sum_{n=0}^{\infty} u_n(\varphi)^n \right) \right]_{\varphi=0} \quad (16)$$

Now, using (13), (15) and (16), we get

$$\sum_{n=0}^{\infty} u_n(t) = \frac{a}{s} + \frac{b}{s^2} - \frac{1}{s^2} L[B(u)] - \frac{1}{s^2} L \left[\sum_{n=0}^{\infty} A_n \right] + \frac{1}{s^2} L[f(t)]. \quad (17)$$

Comparing both sides of (17), we have

$$L[u_0(t)] = k_1(s), \quad (18)$$

$$L[u_1(t)] = k_2(s) - \frac{1}{s^2} L[B_0(u)] - \frac{1}{s^2} L[A_0(t)], \quad (19)$$

$$L[u_{n+1}(t)] = -\frac{1}{s^2} L[B_n(u)] - \frac{1}{s^2} L[A_n(x)], \quad n \geq 1, \quad (20)$$

where $k_0(s)$ and $k_1(s)$ are Laplace transform of $k_0(t)$ and $k_1(t)$ respectively.

Applying the inverse Laplace transform to equations (18)–(20) gives the following recursive relation:

$$u_0(t) = k_1(t), \quad (21)$$

$$u_1(t) = k_2(t) - L^{-1} \left\{ \frac{1}{s^2} L[B_0(u)] - \frac{1}{s^2} L[A_0(t)] \right\}, \quad (22)$$

$$u_{n+1}(t) = -L^{-1} \left\{ \frac{1}{s^2} L[B_n(u)] - \frac{1}{s^2} L[A_n(x)] \right\}, \quad n \geq 1. \quad (23)$$

4 Solution of the integer problem

Applying Laplace transform on both sides of equation (8) gives

$$s^2 L(u) - su(0) - \dot{u}(0) + a[sL(u) - u(0)] + L(u) + bL(u^3) = \frac{af^{-\alpha_*} + R}{s} - \frac{ks}{s^2 + \omega^2} \quad (24)$$

Now using the initial conditions (7) in equation (24), we have

$$L(u) = -\frac{bL(u^3)}{(s^2 + as + 1)} + \frac{af^{-\alpha_*} + R}{s(s^2 + as + 1)} - \frac{ks}{(s^2 + \omega^2)(s^2 + as + 1)}. \quad (25)$$

We now let $u(t) = \sum_{n=0}^{\infty} u_n(t)$ and $u^3 = \sum_{n=0}^{\infty} A_n$ to have

$$L(u_0 + u_1 + u_2 + u_3 + \dots) = \frac{af^{-\alpha_*} + R}{s(s^2 + as + 1)} - \frac{ks}{(s^2 + \omega^2)(s^2 + as + 1)} - \frac{b}{(s^2 + as + 1)} (A_0 + A_1 + A_2 + \dots). \quad (26)$$

Comparing both sides of (26), we have

$$L(u_0) = \frac{af^{-\alpha_*}}{s(s^2 + as + 1)} - \frac{ks}{(s^2 + \omega^2)(s^2 + as + 1)}.$$

$$\Rightarrow u_0 = L^{-1} \left\{ \frac{af^{-\alpha} + R}{s(s^2 + as + 1)} - \frac{ks}{(s^2 + \omega^2)(s^2 + as + 1)} \right\}. \quad (27)$$

$$u_0 = (af^{-\alpha} + R) \left(1 - e^{-\frac{a}{2}t} \cos \frac{\sqrt{4-a^2}}{2}t + \frac{2}{\sqrt{4-a^2}} e^{-\frac{a}{2}t} \sin \frac{\sqrt{4-a^2}}{2}t \right) - \frac{k}{(\omega^2 - 1)^2 + \omega^2 a^2} \left[a\omega \sin \omega t - (\omega^2 - 1) \cos \omega t + (\omega^2 - 1) e^{-\frac{a}{2}t} \cos \frac{\sqrt{4-a^2}}{2}t - \frac{2a}{\sqrt{4-a^2}} e^{-\frac{a}{2}t} \sin \frac{\sqrt{4-a^2}}{2}t \right]. \quad (28)$$

The Adomian polynomials for the nonlinear term u^3 are computed as follows:

$$A_0 = u_0^3, A_1 = 3u_0^2 u_1, A_2 = 3u_0 u_1^2 + 3u_0^2 u_2, \text{ etc.} \quad (29)$$

Hence,

$$L(u_1) = -\frac{bA_0}{s^2 + as + 1} = -\frac{bu_0^3}{s^2 + as + 1} \Rightarrow u_1 = L^{-1} \left[-\frac{bu_0^3}{s^2 + as + 1} \right]. \quad (30)$$

$$u_1 = -\frac{abu_0^3}{\sqrt{4-a^2}} e^{-\frac{a}{2}t} \sin \frac{\sqrt{4-a^2}}{2}t. \quad (31)$$

For u_2 , we have

$$L(u_2) = -\frac{bA_1}{s^2 + as + 1} = -\frac{3bu_0^2 u_1}{s^2 + as + 1} \Rightarrow u_2 = L^{-1} \left[-\frac{3bu_0^2 u_1}{s^2 + as + 1} \right]. \quad (32)$$

$$\therefore u_2 = -\frac{3bu_0^2 u_1}{\sqrt{4-a^2}} e^{-\frac{a}{2}t} \sin \frac{\sqrt{4-a^2}}{2}t. \quad (33)$$

For u_3 , we have

$$L(u_3) = -\frac{bA_2}{s^2 + as + 1} = -\frac{3bu_0^2 u_2 + 3bu_0 u_1^2}{s^2 + as + 1} \Rightarrow u_3 = L^{-1} \left[-\frac{3bu_0^2 u_2 + 3bu_0 u_1^2}{s^2 + as + 1} \right]. \quad (34)$$

$$\therefore u_3 = -\frac{3ab}{\sqrt{4-a^2}} \{u_0^2 u_2 + u_0 u_1^2\} e^{-\frac{a}{2}t} \sin \frac{\sqrt{4-a^2}}{2}t. \quad (35)$$

Thus,

$$u = u_0 + u_1 + u_2 + u_3 + \dots. \quad (36)$$

5 Fractional formulation of the climate change model

We can now move to the fractional derivatives of the equation (8) by using the proposed fractional derivatives in [30], which give

$$\frac{d}{dt} \rightarrow \frac{1}{\sigma^{1-\beta}} \frac{d^\beta}{dt^\beta}, \quad 0 < \beta \leq 1, \quad (37)$$

$$\frac{d^2}{dt^2} \rightarrow \frac{1}{\sigma^{2-\alpha}} \frac{d^\alpha}{dt^\alpha}, \quad 1 < \alpha \leq 2, \quad (38)$$

where σ represent the fractional time components in the system, which must have dimension of seconds and consistent with the dimension of ordinary derivative operator.

Therefore, using (37) and (38), the model (8) can be written in terms of fractional derivatives as

$$\frac{d^\alpha u}{dt^\alpha} + a\sigma^{1+\beta-\alpha} \frac{d^\beta u}{dt^\beta} + \sigma^{2-\alpha} u + b\sigma^{2-\alpha} u^3 = a\sigma^{2-\alpha} f^{-\alpha} + \sigma^{2-\alpha} R - \sigma^{2-\alpha} k \cos \omega t. \quad (39)$$

We let $\sigma^{1+\beta-\alpha} = m$ seconds and $\sigma^{2-\alpha} = n$ seconds to have

$$D_t^\alpha u + amD_t^\beta u + nu + nbu^3 = anf^{-\alpha^*} + nR - nk \cos \omega t, \quad (40)$$

with initial conditions

$$u(0) = D_t^\beta u(0) = 0, \quad (41)$$

where D_t^α and D_t^β are describing the orders of the fractional time derivatives.

In the analysis of fractional calculus, it is necessary to use the appropriate definition of fractional derivative. There are several definitions of fractional derivative (see [31–35]), but in this work, we shall adopt Caputo definition since we can define initial conditions for fractional differential equations which can be handled by using the idea of ordinary derivative.

6 Definition special functions in fractional calculus

Definition 6.1 Caputo fractional derivative with order α for a function $u(t)$ is defined as

$$D_{t_0}^\alpha u(t) = \frac{1}{\Gamma(q-\alpha)} \int_{t_0}^t (t-\tau)^{q-\alpha-1} u^{(q)}(\tau) d\tau, \quad (42)$$

where $0 < q-1 \leq \alpha < q$, $q \in \mathbb{Z}_+$, and $t = t_0$ is the initial time and $\Gamma(\cdot)$ is the Gamma function.

Definition 6.2 In a Caputo sense, the Laplace transform of fractional derivative with order α for a function $u(t)$ is defined as

$$\int_0^\infty e^{-vt} D_{t_0}^\alpha u(t) dt = v^\alpha U(v) - \sum_{m=0}^n v^{\alpha-m-1} u^{(m)}(0), \quad (n-1 < \alpha \leq n). \quad (43)$$

Definition 6.3 A two parameter function of the Mittag-Leffler (ML) type is defined as

$$E_{\alpha,\beta}(z) = \sum_{j=0}^\infty \frac{z^j}{(\alpha j + \beta)}, \quad \alpha > 0, \beta > 0. \quad (44)$$

Definition 6.4 The Laplace transform of ML is given as

$$L\left\{t^{a+m+\alpha-1} E_{\alpha,\beta}(z)\right\} = \frac{m! s^{\alpha-\beta}}{(s^\alpha - a)^{m+1}}. \quad (45)$$

7 Solution of the fractional problem

To solve equation (40), we use MLADM.

Now applying Laplace transform on both sides of (40) gives

$$s^\alpha L(u) - su(0) - D_t^\beta u(0) + am \left[s^\beta L(u) - u(0) \right] + nL(u) + nbL(u^3) = \frac{anf^{-\alpha^*} + nR}{s} - \frac{nks}{s^2 + \omega^2}. \quad (46)$$

Now using the initial conditions (41), we have

$$L(u) = \frac{anf^{-\alpha^*} + nR}{s(s^\alpha + ams^\beta + n)} - \frac{nks}{(s^2 + \omega^2)s^\alpha + ams^\beta + n} - \frac{nbL(u^3)}{s^\alpha + ams^\beta + n}. \quad (47)$$

We now let $u(t) = \sum_{n=0}^\infty u_n(t)$ and $u^3 = \sum_{n=0}^\infty A_n$ to have

$$\begin{aligned} L(u_0 + u_1 + u_2 + u_3 + \dots) &= \frac{anf^{-\alpha^*} + nR}{s(s^\alpha + ams^\beta + n)} - \frac{nks}{(s^2 + \omega^2)(s^\alpha + ams^\beta + n)} \\ &\quad - \frac{nb}{(s^\alpha + ams^\beta + n)} (A_0 + A_1 + A_2 + \dots). \end{aligned} \quad (48)$$

Comparing both sides of (47), we have

$$\begin{aligned} L(u_0) &= \frac{anf^{-\alpha^*} + nR}{s(s^\alpha + ams^\beta + n)} - \frac{nks}{(s^2 + \omega^2)(s^\alpha + ams^\beta + n)} \\ \Rightarrow u_0 &= L^{-1} \left\{ \frac{anf^{-\alpha^*} + nR}{s(s^\alpha + ams^\beta + n)} - \frac{nks}{(s^2 + \omega^2)(s^\alpha + ams^\beta + n)} \right\} = \\ &= \frac{af^{-\alpha^*} + R}{n} L^{-1} \left\{ \frac{1}{s} - \frac{s^{\alpha-1}}{s^\alpha + ams^\beta + n} - \frac{ams^{\beta-1}}{s^\alpha + ams^\beta + n} \right\} + nk \left\{ \frac{s^{-1} - s^{-3}\omega^2 + \frac{1}{2!}s^{-5}\omega^4 + \dots}{s^\alpha + ams^\beta + n} \right\}. \end{aligned} \quad (49)$$

$$\begin{aligned} \therefore u_0 &= \frac{af^{-\alpha^*} + R}{n} t - \frac{af^{-\alpha^*} + R}{n} \left\{ \sum_{n=0}^{\infty} \frac{(-1)^j}{j!} n^j t^{\alpha j} E_{\alpha-\beta, \beta j+1}^j (-amt^{\alpha-\beta}) \right\} \\ &- \frac{af^{-\alpha^*} + R}{n} \left\{ am \sum_{n=0}^{\infty} \frac{(-1)^j}{j!} n^j t^{\alpha(j+1)+(\beta-1)j-1} E_{\alpha-1, \alpha+\beta j}^j (-amt^{\alpha-\beta}) \right\} \\ &+ nk \left\{ \sum_{n=0}^{\infty} \frac{(-1)^j}{j!} n^j t^{\alpha(j+1)} E_{\alpha-\beta, \beta j+1}^j (-amt^{\alpha-\beta}) \right\} - nk\omega^2 \left\{ \sum_{n=0}^{\infty} \frac{(-1)^j}{j!} n^j t^{\alpha(j+1)+2} E_{\alpha-\beta, \beta j+3}^j (-amt^{\alpha-\beta}) \right\} \\ &+ \frac{1}{2!} nk\omega^4 \left\{ \sum_{n=0}^{\infty} \frac{(-1)^j}{j!} n^j t^{\alpha(j+1)+4} E_{\alpha-\beta, \beta j+5}^j (-amt^{\alpha-\beta}) \right\} - \dots \end{aligned} \quad (50)$$

The Adomian polynomials for the nonlinear term u^3 are computed using equation (29):
Therefore,

$$L(u_1) = -\frac{nbA_0}{s^\alpha + ams^\beta + n} = -\frac{nbu_0^3}{s^\alpha + ams^\beta + n} \Rightarrow u_1 = L^{-1} \left[-\frac{nbu_0^3}{s^\alpha + ams^\beta + n} \right]. \quad (51)$$

$$\therefore u_1 = -nbu_0^3 \left\{ \sum_{n=0}^{\infty} \frac{(-1)^j}{j!} n^j t^{\alpha(j+1)-1} E_{\alpha-\beta, \alpha+\beta j}^j (-amt^{\alpha-\beta}) \right\}. \quad (52)$$

For u_2 , we have

$$L(u_2) = -\frac{nbA_1}{s^\alpha + ams^\beta + n} = -\frac{3nbu_0^3 u_1}{s^\alpha + ams^\beta + n} \Rightarrow u_2 = L^{-1} \left[-\frac{3nbu_0^3 u_1}{s^\alpha + ams^\beta + n} \right] \quad (53)$$

$$\therefore u_2 = -3nbu_0^3 u_1 \left\{ \sum_{n=0}^{\infty} \frac{(-1)^j}{j!} n^j t^{\alpha(j+1)-1} E_{\alpha-\beta, \alpha+\beta j}^j (-amt^{\alpha-\beta}) \right\}. \quad (54)$$

For u_3 , we have

$$L(u_3) = -\frac{bnA_2}{s^\alpha + ams^\beta + n} = -\frac{3nbu_0^2 u_2 + 3nbu_0 u_1^2}{s^\alpha + ams^\beta + n} \Rightarrow u_3 = L^{-1} \left[-\frac{3nbu_0^2 u_2 + 3nbu_0 u_1^2}{s^\alpha + ams^\beta + n} \right]. \quad (55)$$

$$\therefore u_3 = -(3nbu_0^2 u_2 + 3nbu_0 u_1^2) \left\{ \sum_{n=0}^{\infty} \frac{(-1)^j}{j!} n^j t^{\alpha(j+1)-1} E_{\alpha-\beta, \alpha+\beta j}^j (-amt^{\alpha-\beta}) \right\} \quad (56)$$

Hence,

$$\therefore u = u_0 + u_1 + u_2 + u_3 + \dots \quad (57)$$

8 Discussion of results

To understand the impacts of climate change with dominant Earth's fluctuations and to predict the future of climate, we generate the graphs of the approximate solutions (36) and (57) using the result of IPCC [1] assessments of R ($R = 3.7 \text{ W/m}^2$) with frequency interval $0.04\text{Hz} - 0.3\text{Hz}$ (the frequency interval of microseismic activities that constitutes the best approximation to the background equilibrium fluctuations that is recorded worldwide) [12], the exponent α_* in the interval $0.1 - 2.0$, and damped constant a in the interval $0.0 - 2.0$.

The first part of our discussion deals with integer order solution. Figure 1 showed that we have sinusoidal curve whose amplitude is maintained due to the absence of damping. In Figure 2, the graph of depth against time for values of damped constant $a = 0.2$ and $a = 0.5$ was plotted and the result showed that the solution does not lose the oscillatory characteristics completely. What it means is that regardless of any choice we make to mitigate climate change, the impact will still persist due to the effect of Earth's fluctuations. Figure 3 showed that when the values of f increases from $f = 0.05\text{Hz}$ to $f = 0.2\text{Hz}$, the amplitude decay as time passes, although not completely. This can also be attributed to the effect of Earth's fluctuations. In Figures 4, we observed that as α_* increases from 0.5 to 1.25 , the oscillation increases. This means the result could lead to flooding.

In the second part of our discussion, the solution of fractional model was investigated. Figure 5 displayed the effect of zero damping. In this figure, it can be seen that increase in time lead to increase in depth. This means the result could lead to flooding if there is no effective mitigation to control the average sea level rise. In Figure 6, the values of f and α_* are varied.

When $\alpha_* = 1.5$ and f decreases from 0.2Hz to 0.05Hz , the depth of thermocline increases, but as α_* decreases from 1.5 to 0.5 , the depth of thermocline decreases. This means that the effect caused by the changes in f depends on the changes in α_* . In Figure 7, the value of damped constant a is varied, and as a increases, the depth of thermocline decreases, showing the effect of damped constant.

Therefore, we can see that a better result was obtained when we considered a fractional order model. It was observed from the graph of the fractional order solution that for $a = 0$ (see Figure 5), there was a clear increase in the depth of thermocline, unlike the integer order solution for $a = 0$ (see Figure 1), where we have just a sinusoidal curve.

Thus, in this work, what is seen in the fractional part takes longer to be seen in the integer part; therefore, the fractional part presents a clear and better result for the analysis of the impact of climate change with dominant Earth's

Figure 1: The graph of solution (36) when $b = 1$, $f = 0.1\text{Hz}$, $\omega = 5\text{Hz}$, $\alpha_* = 0.6$ and $k = 6m$.

Figure 2: The graph of solution (36) when $b = 1$, $f = 0.1\text{Hz}$, $\omega = 5\text{Hz}$, $\alpha_* = 0.6$ and $k = 6m$.

Figure 3: The graph of solution (36) when $a = 0.2$, $b = 1$, $\omega = 5\text{Hz}$, $\alpha_* = 0.6$ and $k = 6m$.

Figure 4: The graph of solution (36) when $a = 0.2$, $b = 1$, $\omega = 5\text{Hz}$, $f = 0.1\text{Hz}$ and $k = 3m$.

Figure 5: The graph of solution (57) when $b = 1$, $\omega = 5\text{Hz}$, $n = 1s$, $m = 2s$, $\beta = 0.5$, $\alpha = 1.5$ and $k = 3m$.

Figure 6: The graph of solution (57) when $b = 1$, $\omega = 5\text{Hz}$, $n = 1\text{s}$, $m = 2\text{s}$, $\beta = 0.5$, $\alpha = 1.5$ and $k = 3m$.

Figure 7: The graph of solution (57) when $b = 1$, $\omega = 5\text{Hz}$, $n = 1\text{s}$, $m = 2\text{s}$, $f = 0.05\text{Hz}$, $\beta = 0.5$, $\alpha = 0.6$, $\alpha = 1.5$ and $k = 3m$.

9 Conclusion

In this work, we obtained a model and its fractional formulation that will help to predict the impact of climate change with dominant Earth's fluctuations in Pacific Ocean. The approximation solution of the model and its fractional formulation were obtained using modified Laplace Adomian decomposition method. Our results indicate that regardless of the choices we make to mitigate climate change, the effect will still persist, which means no mitigation practice can eradicate the effect of climate change completely. We observed that what is seen in the fractional model takes longer to be seen in the integer model. Therefore, we conclude that the fractional model presents a clear and better result for the analysis of the impact of climate change with dominant Earth's fluctuations.

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