Research Article

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Jia-Bao Liu, Imran Khalid, Mohammad Tariq Rahim, Masood Ur Rehman*, Faisal Ali and Muhammad Salman

Eccentric topological properties of a graph associated to a finite dimensional vector space

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Abstract: A topological index is actually designed by transforming a chemical structure into a number. Topological index is a graph invariant which characterizes the topology of the graph and remains invariant under graph automorphism. Eccentricity based topological indices are of great importance and play a vital role in chemical graph theory. In this article, we consider a graph (non-zero component graph) associated to a finite dimensional vector space over a finite field in the context of the following eleven eccentricity based topological indices: total eccentricity index; average eccentricity index; eccentric connectivity index; eccentric distance sum index; adjacent distance sum index; connective eccentricity index; geometric arithmetic index; atom bond connectivity index; and three versions of Zagreb indices. Relationship of the investigated indices and their dependency with respect to the involved parameters are also visualized by evaluating them numerically and by plotting their results.

Keywords: distance; degree; eccentricity; topological index

AMS Subject Classification Numbers: 15A27; 05C07; 05C12; 05C25; 05C69; 20E45

1 Introduction

Let a connected graph G having the sets $V(G)$ and $E(G)$ as the vertex set and the edge set, respectively. Starting from a vertex v and ending at a vertex u in G , a shortest alternating sequence of vertices and edges without repetition of any vertex is known as a $v-u$ geodesic. The number of edges in a $v-u$ geodesic is denoted by $d(v,u)$, and is called the distance between v and u in G . The maximum distance of a vertex v among its distances with all the vertices of G is called the eccentricity of v , denoted by $ecc(v)$. The number $diam(G) = \max_{v \in V(G)} ecc(v)$ is the diameter of G . For any vertex v of G , the number $D(v|G) = \sum_{u \in V(G)} d(v,u)$ is the distance

number of v , and the number $d(v)$ is the degree of v .

An involvement of different graph theoretic tools/parameters in the field of chemistry to deal with chemical structures is due to chemical graph theory. In chemical graph theory, an interesting sub field, namely cheminformatics, deals with chemical phenomenal “quantitative structure-activity/structure-property relationships” of chemical compounds. An emerging tool used, in the study of these phenomenal, is a topological index, which is invariant for chemical structures up to their symmetry (automorphism). Correlation of many physico-chemical properties like boiling point, stability, strain energy, etc. of chemical compounds in a chemical structure is due to its topological index (Bruckler et al., 2011; Deng et al., 2011; Klavžar and Gutman, 1996; Liu et al., 2019; Rucker and Rucker, 1999; Tang et al., 2019; Zheng et al., 2019; Zhang and Zhang, 1996). In this regard, the very first topological index was introduced by Wiener in 1947, and gave it name the path

*Corresponding author: Masood Ur Rehman, Department of Mathematics, Abbottabad University of Science and Technology, Abbottabad, Pakistan; e-mail: masoodqau27@gmail.com, masood@mail.ustc.edu.cn

Jia-Bao Liu, School of Mathematics and Physics, Anhui Jianzhu University, Hefei 230601, China; e-mail: liujiabao@163.com

Imran Khalid, Center for Advanced Studies in Pure and Applied Mathematics, Bahauddin Zakariya University Multan, 60800, Pakistan; e-mail: imran.khalid@bzu.edu.pk

Mohammad Tariq Rahim, Department of Mathematics, Abbottabad University of Science and Technology, Abbottabad, Pakistan; e-mail: tariq.rahim@nu.edu.pk

Faisal Ali, Center for Advanced Studies in Pure and Applied Mathematics, Bahauddin Zakariya University Multan, 60800, Pakistan; e-mail: faisalali@bzu.edu.pk

Muhammad Salman, Department of Mathematics, The Islamia University of Bahawalpur, 63100, Pakistan; e-mail: muhammad.salman@iub.edu.pk

number (Wiener, 1947). This index is based on the concept of distance, mathematically, defined as:

$$W(G) = \sum_{u,v \in V(G)} d(u,v),$$

and called the Wiener index of a chemical structure/graph G . Extending the study of topological index, based on the distance, many graph theorists introduced and studied various topological indices. Among these topological indices, a lot of researchers considered topological indices, defined using the eccentricity of each vertex, and produced remarkable investigations, such as: the average eccentricity of a graph and its subgraph was investigated in Dankelmann et al. (2004); the extremal properties of the average eccentricity and conjectures about the average eccentricity were obtained in Ilic (2012); lower and upper bounds of average eccentricity for trees were provided in Tang and Zhou (2012); the average eccentricity and standard deviation of Sierpinski graphs were established in Hinz and Parris (2012); the eccentricity based ABC and geometric arithmetic indices for copper oxide networks are obtained in Imran et al. (2017); in Gao et al. (2016), results about the eccentric ABC index of linear polyene parallelogram benzenoid are investigated; the total eccentricity, average eccentricity, eccentricity-based Zagreb indices, eccentricity based atom bond connectivity (ABC) and geometric arithmetic indices for oxide networks were found in Imran et al. (2018); eccentricity based topological indices of a cyclic octahedron structure were explored in Zahid et al. (2018). Furthermore, results regarding the average eccentricity index and eccentricity based geometric arithmetic index can be found in Zhang et al. (2017).

In this paper, we extend the study of eccentricity topological indices, based on the eccentricity, in chemical graph theory by involving an algebraic structures called a vector space. We consider a graph associated to a finite dimensional vector space over a finite field, and compute all the indices listed in the Table 1.

2 Graph associated to a vector space

Let $\mathbb{V}$ be a finite dimensional vector space over a field $\mathbb{F}$ with $\{\alpha_1, \alpha_2, \dots, \alpha_n\}$ as a basis and θ as the null vector. Then any vector $v \in \mathbb{V}$ can be expressed uniquely as a linear combination of the form $v = a_1 \alpha_1 + a_2 \alpha_2 + \dots + a_n \alpha_n$.

This representation of v is its basic representation with respect to $\{\alpha_1, \alpha_2, \dots, \alpha_n\}$. Now, the graph associated with $\mathbb{V}$, denoted by $\Gamma(\mathbb{V})$, is called a non-zero component graph, which is defined with respect to a basis $\{\alpha_1, \alpha_2, \dots, \alpha_n\}$ as follows: the vertex set is $V(\Gamma(\mathbb{V})) = \mathbb{V} - \theta$; and two vertices $v_1, v_2 \in V(\Gamma(\mathbb{V}))$ form an edge in $\Gamma(\mathbb{V})$ if v_1 and v_2 share at least one α_i with non-zero coefficient in their basic representation, i.e. there exists at least one α_i along which both v_1 and v_2 have non-zero components. Unless otherwise mention, we take the basis $\{\alpha_1, \alpha_2, \dots, \alpha_n\}$ on which the graph is constructed (Das, 2016). Now, we state some basic results about $\Gamma(\mathbb{V})$, which will be useful in the sequel.

Theorem 1 (Das, 2016)

If $\mathbb{V}$ be an n -dimensional vector space over a field $\mathbb{F}$ with $q \geq 2$ elements, then the order of $\Gamma(\mathbb{V})$ is $q^n - 1$ and the size of $\Gamma(\mathbb{V})$ is $\frac{q^{2n} - q^n + 1 - (2q - 1)^n}{2}$.

Theorem 2 (Das, 2016)

Let $\mathbb{V}$ be a vector space over a finite field $\mathbb{F}$ with $q \geq 2$ elements and $\Gamma(\mathbb{V})$ be its associated graph with respect to basis $\{\alpha_1, \alpha_2, \dots, \alpha_n\}$. Then, the degree of the vertex $c_1 \alpha_{i_1} + c_2 \alpha_{i_2} + \dots + c_n \alpha_{i_n}$, where $c_1, c_2, \dots, c_k \neq 0$, is $(q^k - 1)q^{n-k} - 1$.

Theorem 3 (Das, 2016)

$\Gamma(\mathbb{V})$ is connected and $\text{diam}(\Gamma(\mathbb{V})) = 2$.

Theorem 4 (Das, 2016)

$\Gamma(\mathbb{V})$ is a complete graph if and only if $\mathbb{V}$ is 1-dimensional.

Let $\mathbb{V}$ be an n -dimensional vector space, $n \geq 1$, over a field $\mathbb{F}$ of order $q \geq 2$. Then there are $\binom{n}{k}(q-1)^k$ unique vectors $\alpha_{i_1} v_1 + \alpha_{i_2} v_2 + \dots + \alpha_{i_k} v_k$ of length k in $\Gamma(\mathbb{V})$ for each $1 \leq k \leq n$. Let us denote the vertex corresponds to the i th vector of length k by v_i^k , where $1 \leq i \leq \binom{n}{k}(q-1)^k$ and $1 \leq k \leq n$.

So the vertex set of $\Gamma(\mathbb{V})$ is $\{v_i^k; 1 \leq k \leq n, 1 \leq i \leq \binom{n}{k}(q-1)^k\}$.

Let us denote the degree of a vertex $v_i^k, 1 \leq i \leq \binom{n}{k}(q-1)^k$, by d_k , then $d_k = (q^k - 1)q^{n-k} - 1$ for $1 \leq k \leq n$.

For $n=1$ and $q \geq 2$, $\Gamma(\mathbb{V})$ is a complete graph by Theorem 4, so one can trivially find all the topological indices in this particular case. We consider $\Gamma(\mathbb{V})$ for more than 1-dimensional vector spaces in the context of previously defined eccentricity based topological indices.

Table 1: Eccentricity based topological indices.

Notation	Formula	Index name (reference)
$\xi(G)$	$\sum_{v \in V(G)} ecc(v)$	Total eccentricity index (Sharma et al., 1997)
$avec(G)$	$\frac{\xi(G)}{ G }$	Average eccentricity index (Dankelmann et al., 2004)
$\xi^c(G)$	$\sum_{v \in V(G)} ecc(v)d(v)$	Eccentric connectivity index (Sharma et al., 1997)
$\xi^d(G)$	$\sum_{v \in V(G)} ecc(v)D(v G)$	Eccentric distance sum index (Gupta et al., 2002)
$\xi^{sv}(G)$	$\sum_{v \in V(G)} \frac{ecc(v)D(v G)}{d(v)}$	Adjacent eccentric distance sum index (Sardana and Madan, 2002)
$\xi^{ce}(G)$	$\sum_{v \in V(G)} \frac{d(v)}{ecc(v)}$	Connective eccentricity index (Yu and Feng, 2013)
$GA_4(G)$	$\sum_{uv \in E(G)} \frac{2\sqrt{ecc(u) \times ecc(v)}}{ecc(u) + ecc(v)}$	Geometric arithmetic index (Ghorbani and Khaki, 2010)
$ABC_5(G)$	$\sum_{uv \in E(G)} \frac{\sqrt{ecc(u) + ecc(v) - 2}}{ecc(u) \times ecc(v)}$	Atomic bond connectivity index (Farahani, 2013)
$M_1^*(G)$	$\sum_{uv \in E(G)} (ecc(u) + ecc(v))$	First Zagreb eccentricity index (Ghorbani and Hosseioninzadeh, 2012; Vukičević and Graovac, 2010)
$M_1^{**}(G)$	$\sum_{v \in V(G)} (ecc(v))^2$	Second Zagreb eccentricity index (Ghorbani and Hosseioninzadeh, 2012; Vukičević and Graovac, 2010)
$M_2^*(G)$	$\sum_{uv \in E(G)} ecc(u)ecc(v)$	Third Zagreb eccentricity index (Ghorbani and Hosseioninzadeh, 2012; Vukičević and Graovac, 2010)

3 Methodology

Some graph theoretical parameters such as path, distance, eccentricity, diameter, degree, etc. along with vertex partitioning method are used to construct some useful tools for investigating our main results. We also use combinatorial computing and binomial expansion theorem to find the required indices. Moreover, we use maple software (Maplesoft, McKinney, TX, USA) (see: <https://en.wikipedia.org/wiki/Maplesoftware>) for plotting our mathematical results, mathematical calculations and verifications, and to provide a numeric comparison of the investigated indices.

4 Construction

In this section, we construct/illustrate some useful results to compute the required indices. Let us denote the dimension of a vector space $\mathbb{V}$ with $\dim(\mathbb{V})$, and the order of a field $\mathbb{F}$ with $o(\mathbb{F})$.

Remark 5

If $\dim(\mathbb{V}) = n \geq 2$ and $o(\mathbb{F}) = q \geq 2$, then for all $1 \leq k \leq n$ and for each $1 \leq i \leq \binom{n}{k}(q-1)^k$, note that $d(v_i^k, v) = 1$ or 2, by Theorem 3, for any $v \in V - \{v_i^k\}$. Since the number of vertices

lying at distance 1 from v_i^k in $\Gamma(\mathbb{V})$ is the degree d_k of v_i^k , so the number of vertices lying at distance 2 from v_i^k is

$$|\Gamma(\mathbb{V})| - d_k - 1 = (q^n - 1) - (q^{n-k}(q^k - 1) - 1) - 1 = q^{n-k} - 1.$$

Remark 6

As the diameter of $\Gamma(\mathbb{V})$ is 2, by Theorem 3, therefore, $\text{ecc}(v_i^n) = 1$ for all $1 \leq i \leq (q-1)^n$, and $\text{ecc}(v_i^k) = 2$ for all $1 \leq i \leq \binom{n}{k}(q-1)^k$ and for each $1 \leq k \leq n-1$.

The following proposition provides the distance number of each vertex of $\Gamma(\mathbb{V})$.

Proposition 7

Let $\dim(\mathbb{V}) = n \geq 2$ and $o(\mathbb{F}) = q \geq 2$, then for any $1 \leq k \leq n$ and for each $1 \leq i \leq \binom{n}{k}(q-1)^k$, we have $D(v_i^k | \Gamma(\mathbb{V})) = q^{n-k}(q^k + 1) - 3$.

Proof

Using Remark 5, the distance number of any vertex of $\Gamma(\mathbb{V})$ is:

$$D(v_i^k | \Gamma(\mathbb{V})) = [q^{n-k}(q^k - 1) - 1](1) + [q^{n-k} - 1](2) = q^{n-k}(q^k + 1) - 3.$$

□

The notation $a \sim b$ is a type of an edge whose end vertices have degrees a and b . The following result gives edge partition of $\Gamma(\mathbb{V})$ according to the eccentricity of each vertex.

Proposition 8

Let $\dim(\mathbb{V}) = n \geq 2$ and $o(\mathbb{F}) = q \geq 2$, then the edge partition of $\Gamma(\mathbb{V})$ according to the eccentricity of each vertex is:

- (1) $\binom{(q-1)^n}{2}$ edges of type 1 ~ 1,
- (2) $(q-1)^n(q^n - (q-1)^n + 1)$ edges of type 1 ~ 2,
- (3) $\frac{1}{2}[q^{2n} - q^n - (2q-1)^n - (q-1)^n(2q^n - (q-1)^n + 1) + 1]$ edges of type 2 ~ 2.

where $a \sim b$ denotes an edge whose one end vertex has the eccentricity a and the other end vertex has the eccentricity b .

Proof

By Remark 6, there are three types of edges of $\Gamma(\mathbb{V})$: 1 ~ 1; 1 ~ 2 and 2 ~ 2. The number of edges in each type can be found as follows:

- (1) Edges of type 1 ~ 1: As $\text{ecc}(v_i^n) = 1$ for all $1 \leq i \leq (q-1)^n$ and the subgraph of $\Gamma(\mathbb{V})$ induced by the vertices v_i^n for all $1 \leq i \leq (q-1)^n$ is a complete graph $K_{(q-1)^n}$, so the number of edges of type 1 ~ 1 is the size of $K_{(q-1)^n}$, which is $\binom{(q-1)^n}{2}$.
- (2) Edges of type 1 ~ 2: In $\Gamma(\mathbb{V})$, two types of edges, 1 ~ 1 and 1 ~ 2, are counted to form a degree d_n of a vertex v_i^n , $1 \leq i \leq (q-1)^n$. Thus, the number of edges of type 1 ~ 2 is obtained from the degree d_n of v_i^n by subtracting the number of edges of type 1 ~ 1. Note that, $(q-1)^n - 1$ edges of type 1 ~ 1 are counted to form a degree d_n , because every two vertices having degree d_n are adjacent in $\Gamma(\mathbb{V})$. It follows that $d_n - (q-1)^n + 1$ edges of type 1 ~ 2 are counted to form each degree d_n . Since there are $(q-1)^n$ vertices having degree d_n , so the number of edges of type 1 ~ 2 is

$$(q-1)^n(d_n - (q-1)^n + 1) = (q-1)^n(q^n - (q-1)^n + 1).$$

- (3) Edges of type 2 ~ 2: The number of edges of type 2 ~ 2 can be obtained by the formula Size of $\Gamma(\mathbb{V})$ – the number of edges of type 1 ~ 1 – the number of edges of type 1 ~ 2, which yields that

$$\begin{aligned} & \frac{1}{2}(q^{2n} - q^n - (2q-1)^n + 1) - \frac{1}{2}(q-1)^n((q-1)^n - 1) \\ & \quad - (q-1)^n(q^n - (q-1)^n + 1) \\ & = \frac{1}{2}(q^{2n} - q^n - (2q-1)^n - (q-1)^n(2q^n - (q-1)^n + 1)). \end{aligned}$$

5 Results

The total eccentricity index of $\Gamma(\mathbb{V})$ is computed in the following result:

Theorem 9

Let $G = \Gamma(\mathbb{V})$ be a non-zero component graph of a vector space $\mathbb{V}$ of dimension $n \geq 2$ over a finite field $\mathbb{F}$ of order $q \geq 2$. Then $\xi(G) = 2(q^n - 1) - (q-1)^n$.

Proof

Using Remark 6 in the formula of total eccentricity index, we have

$$\begin{aligned}\xi(G) &= \sum_{k=1}^n \sum_{i=1}^{\binom{n}{k}(q-1)^k} \text{ecc}(v_i^k) \\ &= \left[(q^n - 1) - (q - 1)^n \right] (2) + (q - 1)^n = 2(q^n - 1) - (q - 1)^n.\end{aligned}$$

Using the total eccentricity index, computed in Theorem 9, in the formula of the average eccentricity index, we get the following result:

Theorem 10

Let $G = \Gamma(\mathbb{V})$ be a non-zero component graph of a vector space $\mathbb{V}$ of dimension $n \geq 2$ over a finite field $\mathbb{F}$ of order $q \geq 2$.

Then $\text{avec}(G) = 2 - \frac{(q-1)^n}{q^n - 1}$.

In the following result, we compute the connective eccentricity index of $\Gamma(\mathbb{V})$

Theorem 11

Let $G = \Gamma(\mathbb{V})$ be a non-zero component graph of a vector space $\mathbb{V}$ of dimension $n \geq 2$ over a finite field $\mathbb{F}$ of order $q \geq 2$. Then

$$\xi^{ce}(G) = \frac{1}{2} \left[q^n(q^n - 1) - (2q - 1)^n + (q - 1)^n(q^n - 2) + 1 \right].$$

Proof

Using the degree and eccentricity of each vertex, given in Remark 6, in the formula of connective eccentricity index, we have

$$\begin{aligned}\xi^{ce}(G) &= \sum_{k=1}^n \sum_{i=1}^{\binom{n}{k}(q-1)^k} \frac{d(v_i^k)}{\text{ecc}(v_i^k)} \\ &= \frac{\binom{n}{1}(q-1)^1((q^1-1)q^{n-1}-1)}{2} \\ &\quad + \frac{\binom{n}{2}(q-1)^2((q^2-1)q^{n-2}-1)}{2} + \dots\end{aligned}$$

$$\begin{aligned}&+ \frac{\binom{n}{n-1}(q-1)^{n-1}((q^{n-1}-1)q^{n-(n-1)}-1)}{2} \\ &+ \frac{\binom{n}{n}(q-1)^n((q^n-1)q^{n-n}-1)}{1} \\ &= \frac{1}{2} \binom{n}{1} \left[(q-1)^1 q^n - (q-1)^1 q^{n-1} - (q-1)^1 \right] \\ &+ \frac{1}{2} \binom{n}{2} \left[(q-1)^2 q^n - (q-1)^2 q^{n-2} - (q-1)^2 \right] + \dots \\ &+ \frac{1}{2} \binom{n}{n} \left[(q-1)^n q^n - (q-1)^n q^{n-n} - (q-1)^n \right] \\ &+ \frac{(q-1)^n(q^n-2)}{2}.\end{aligned}$$

Few more simplifications, using binomial expansions, provide the required results.

The eccentric connectivity index of $\Gamma(\mathbb{V})$ is computed in the next result.

Theorem 12

Let $G = \Gamma(\mathbb{V})$ be a non-zero component graph of a vector space $\mathbb{V}$ of dimension $n \geq 2$ over a finite field $\mathbb{F}$ of order $q \geq 2$. Then

$$\xi^c(G) = 2(q^{2n} - q^n + 1 - (2q - 1)^n) + (q - 1)^n.$$

Proof

Using the degree and eccentricity of each vertex, given in Remark 6, in the formula of eccentric connectivity index, we have

$$\begin{aligned}\xi^c(G) &= \sum_{k=1}^n \sum_{i=1}^{\binom{n}{k}(q-1)^k} \text{ecc}(v_i^k) d(v_i^k) \\ &= \binom{n}{1} (q-1)^1 (q^n - q^{n-1} - 1) (2) \\ &+ \binom{n}{2} (q-1)^2 (q^n - q^{n-2} - 1) (2) + \dots\end{aligned}$$

$$\begin{aligned}
& + \binom{n}{n-1} (q-1)^{n-1} (q^n - q^1 - 1) (2) \\
& + \binom{n}{n} (q-1)^n (q^n - q^{n-n} - 1) (1) \\
& = 2 \binom{n}{1} \left[(q-1)^1 q^n + (q-1)^1 q^{n-1} - (q-1)^1 \right] \\
& + 2 \binom{n}{2} \left[(q-1)^2 q^n - (q-1)^2 q^{n-2} - (q-1)^2 \right] + \dots \\
& + 2 \binom{n}{n} \left[(q-1)^n q^n - (q-1)^n q^{n-n} - (q-1)^n \right] \\
& - \binom{n}{n} \left[(q-1)^n q^n - (q-1)^n q^{n-n} - (q-1)^n \right] \\
& = q^n \left[2 \left(-1 + 1 + \binom{n}{1} (q-1) + \binom{n}{2} (q-1)^2 + \dots + \binom{n}{n} (q-1)^n \right) \right. \\
& \quad \left. - \binom{n}{n} (q-1)^n \right] \\
& - \left[2 \left(-q^n + q^n + \binom{n}{1} (q-1)^1 q^{n-1} + \dots + \binom{n}{n} (q-1)^n q^{n-n} \right) \right. \\
& \quad \left. - \binom{n}{n} (q-1)^n q^{n-n} \right] \\
& - \left[2 \left(-1 + 1 + \binom{n}{1} (q-1) + \dots + \binom{n}{n} (q-1)^n \right) - \binom{n}{n} (q-1)^n \right].
\end{aligned}$$

After some simple calculations, using binomial expansions, we get the required index.

The investigation of the eccentric distance sum index of $\Gamma(\mathbb{V})$ is given in the following result:

Theorem 13

Let $G = \Gamma(\mathbb{V})$ be a non-zero component graph of a vector space $\mathbb{V}$ of dimension $n \geq 2$ over a finite field $\mathbb{F}$ of order $q \geq 2$. Then

$$\xi^d(G) = 2 \left[(2q-1)^n + q^{2n} - 4q^n + 3 \right] - \left[(q-1)^n (q^n - 2) \right].$$

Proof

Using the eccentricity of each vertex, given in Remark 6, and the distance number of each vertex, given in

Proposition 7, in the formula of eccentric distance sum index, we have

$$\begin{aligned}
\xi^d(G) &= \sum_{k=1}^n \sum_{i=1}^{\binom{n}{k} (q-1)^k} ecc(v_i^k) D(v_i^k | G) \\
&= 2 \binom{n}{1} (q-1)^1 \left(q^{n-1} (q^1 + 1) - 3 \right) \\
&+ 2 \binom{n}{2} (q-1)^2 \left(q^{n-2} (q^2 + 1) - 3 \right) + \dots \\
&+ 2 \binom{n}{n-1} (q-1)^{n-1} \left(q^{n-(n-1)} (q^{n-1} + 1) - 3 \right) \\
&+ \binom{n}{n} (q-1)^n \left(q^{n-n} (q^n + 1) - 3 \right) \\
&= 2 \left[\binom{n}{1} (q-1)^1 (q^n + q^{n-1} - 3) + \dots + \binom{n}{n-1} (q-1)^{n-1} \right. \\
&\quad \left. (q^n + q^{n-(n-1)} - 3) \right. \\
&\quad \left. - \binom{n}{n} (q-1)^n (q^n + q^{n-n} - 3) \right] \\
&= 2q^n \left[-1 + 1 + \binom{n}{1} (q-1) + \binom{n}{2} (q-1)^2 + \dots + \binom{n}{n} (q-1)^n \right] \\
&+ 2 \left[-q^n + q^n + \binom{n}{1} (q-1) q^{n-1} + \dots + \binom{n}{n} (q-1)^n q^{n-n} \right] \\
&- 6 \left[-1 + 1 + \binom{n}{1} (q-1) + \binom{n}{2} (q-1)^2 + \dots + \binom{n}{n} (q-1)^n \right] \\
&- \left[(q-1)^n (q^n - 2) \right].
\end{aligned}$$

Using binomial expansions and by performing some simplifications, we get the required index.

The adjacent eccentric distance sum index of $\Gamma(\mathbb{V})$ is investigated in the following result:

Theorem 14

Let $G = \Gamma(\mathbb{V})$ be a non-zero component graph of a vector space $\mathbb{V}$ of dimension $n \geq 2$ over a finite field $\mathbb{F}$ of order $q \geq 2$. Then

$$\xi^{sv}(G) = (q-1)^n + 2 \sum_{j=1}^{n-1} \binom{n}{j} (q-1)^j \left(\frac{q^n + q^{n-j} - 3}{q^n - q^{n-j} - 1} \right).$$

Proof

Using the degree, eccentricity and distance number of each vertex, given in Remark 6 and Proposition 7, in the formula of adjacent eccentric distance sum index, we have

$$\begin{aligned}
 \xi^d(G) &= \sum_{k=1}^n \sum_{i=1}^{\binom{n}{k}(q-1)^k} \frac{\text{ecc}(v_i^k) D(v_i^k | G)}{d(v_i^k)} \\
 &= \sum_{k=1}^n \binom{n}{k} (q-1)^k \frac{\text{ecc}(v_i^k) (q^{n-k}(q^k+1)-3)}{\left((q^k-1)q^{n-k}-1\right)} \\
 &= \binom{n}{1} (q-1)^1 \frac{\text{ecc}(v_i^1) (q^{n-1}(q^1+1)-3)}{\left((q^1-1)q^{n-1}-1\right)} \\
 &\quad + \binom{n}{2} (q-1)^2 \frac{\text{ecc}(v_i^2) (q^{n-2}(q^2+1)-3)}{\left((q^2-1)q^{n-2}-1\right)} \\
 &\quad + \dots + \binom{n}{n-1} (q-1)^{n-1} \frac{\text{ecc}(v_i^{n-1}) (q^{n-(n-1)}(q^{n-1}+1)-3)}{\left((q^{n-1}-1)q^{n-(n-1)}-1\right)} \\
 &\quad + \binom{n}{n} (q-1)^n \frac{\text{ecc}(v_i^n) (q^{n-n}(q^n+1)-3)}{\left((q^n-1)q^{n-n}-1\right)} \\
 &= \binom{n}{1} (q-1)^1 \frac{2(q^{n-1}(q^1+1)-3)}{\left((q^1-1)q^{n-1}-1\right)} \\
 &\quad + \binom{n}{2} (q-1)^2 \frac{2(q^{n-2}(q^2+1)-3)}{\left((q^2-1)q^{n-2}-1\right)} + \dots \\
 &\quad + \binom{n}{n-1} (q-1)^{n-1} \frac{2(q^{n-(n-1)}(q^{n-1}+1)-3)}{\left((q^{n-1}-1)q^{n-(n-1)}-1\right)} \\
 &\quad + \binom{n}{n} (q-1)^n \frac{(q^{n-n}(q^n+1)-3)}{\left((q^n-1)q^{n-n}-1\right)}
 \end{aligned}$$

$$= 2 \sum_{j=1}^{n-1} \binom{n}{j} (q-1)^j \left(\frac{q^n + q^{n-j} - 3}{q^n - q^{n-j} - 1} \right) + (q-1)^n.$$

We get the required result after performing some calculations using binomial expansion.

The eccentricity based geometric arithmetic index of $\Gamma(\mathbb{V})$ is computed in the next result.

Theorem 15

Let $G = \Gamma(\mathbb{V})$ be a non-zero component graph of a vector space $\mathbb{V}$ of dimension $n \geq 2$ over a finite field $\mathbb{F}$ of order $q \geq 2$. Then

$$\begin{aligned}
 GA_4(G) &= \left(\frac{3-2\sqrt{2}}{3} \right) (q-1)^{2n} \\
 &\quad + \frac{1}{6} \left[\left(4\sqrt{2}-6 \right) (q^n+1) \right] (q-1)^n + 3q^{2n} - 3q^n - 3(2q-1)^n + 3.
 \end{aligned}$$

Proof

Using the edge partition, given in Proposition 8, in the formula of geometric arithmetic index, we have

$$\begin{aligned}
 GA_4(G) &= \sum_{u \sim v} \frac{2\sqrt{\text{ecc}(u) \times \text{ecc}(v)}}{\text{ecc}(u) + \text{ecc}(v)} \\
 &= \binom{(q-1)^n}{2} \frac{2\sqrt{1 \times 1}}{1+1} + \left[(q-1)^n (q^n - (q-1)^n + 1) \right] \frac{2\sqrt{1 \times 2}}{1+2} \\
 &\quad + \frac{1}{2} \left[q^{2n} - q^n - (2q-1)^n - (q-1)^n (2q^n - (q-1)^n + 1) + 1 \right] \frac{2\sqrt{2 \times 2}}{2+2} \\
 &= \frac{(q-1)^n ((q-1)^n - 1)}{2} + \left[(q-1)^n (q^n - (q-1)^n + 1) \right] \frac{2\sqrt{2}}{3} \\
 &\quad + \frac{1}{2} \left[q^{2n} - q^n - (2q-1)^n - (q-1)^n (2q^n - (q-1)^n + 1) + 1 \right] \\
 &= \frac{1}{6} \left[(q-1)^n \left[\frac{3(q-1)^n - 3 + (4q^n - 4(q-1)^n + 4)\sqrt{2} - 6q^n}{+3(q-1)^n - 3} \right] \right]
 \end{aligned}$$

$$\begin{aligned}
& + \frac{1}{6} \left[3q^{2n} - 3q^n - 3(2q-1)^n + 3 \right] \\
& = \frac{1}{6} \left[(q-1)^n \left[\left(6 - 4\sqrt{2} \right) (q-1)^n + \left(4\sqrt{2} - 6 \right) q^n + 4\sqrt{2} - 6 \right] \right. \\
& \quad \left. + \frac{1}{6} \left[3q^{2n} - 3q^n - 3(2q-1)^n + 3 \right] \right].
\end{aligned}$$

By performing some calculations, using binomial expansion, we get the required index.

The following result provides the formula for the eccentricity based ABC index of $\Gamma(\mathbb{V})$.

Theorem 16

Let $G = \Gamma(\mathbb{V})$ be a non-zero component graph of a vector space $\mathbb{V}$ of dimension $n \geq 2$ over a finite field $\mathbb{F}$ of order $q \geq 2$. Then

$$ABC_5(G) = \frac{1}{2\sqrt{2}} \left[(q-1)^n (1 - (q-1)^n) + q^n (q^n - 1) - (2q-1)^n + 1 \right].$$

Proof

Using the edge partition, given in Proposition 8, in the formula of ABC index, we have

$$\begin{aligned}
ABC_5(G) &= \sum_{u-v} \sqrt{\frac{ecc(u) + ecc(v) - 2}{ecc(u) \times ecc(v)}} \\
&= \left(\binom{(q-1)^n}{2} \right) \sqrt{\frac{1+1-2}{1 \times 1}} + \left[(q-1)^n (q^n - (q-1)^n + 1) \right] \sqrt{\frac{1+2-2}{1 \times 2}} \\
&\quad + \frac{1}{2} \left[q^{2n} - q^n - (2q-1)^n - (q-1)^n (2q^n - (q-1)^n + 1) + 1 \right] \sqrt{\frac{2+2-2}{2 \times 2}} \\
&= \frac{(q-1)^n ((q-1)^n - 1)}{2} (0) + \left[(q-1)^n (q^n - (q-1)^n + 1) \right] \frac{1}{\sqrt{2}} \\
&\quad + \frac{1}{2} \left[q^{2n} - q^n - (2q-1)^n - (q-1)^n (2q^n - (q-1)^n + 1) + 1 \right] \frac{1}{\sqrt{2}} \\
&= \frac{1}{2\sqrt{2}} \left[2(q-1)^n (q^n - (q-1)^n + 1) + q^{2n} - q^n - (2q-1)^n \right]
\end{aligned}$$

$$\begin{aligned}
& - \frac{1}{2\sqrt{2}} \left[(q-1)^n (2q^n - (q-1)^n + 1) - 1 \right] \\
&= \frac{1}{2\sqrt{2}} \left[(q-1)^n \left[2q^n - 2(q-1)^n + 2 - 2q^n + (q-1)^n - 1 \right] \right. \\
&\quad \left. + \left[q^{2n} - q^n - (2q-1)^n + 1 \right] \right] \\
&= \frac{1}{2\sqrt{2}} \left[(q-1)^n (1 - (q-1)^n) + q^n (q^n - 1) - (2q-1)^n + 1 \right].
\end{aligned}$$

Hence, the required result is obtained.

The investigation of the first Zagreb eccentricity index of $\Gamma(\mathbb{V})$ is given in the following result:

Theorem 17

Let $G = \Gamma(\mathbb{V})$ be a non-zero component graph of a vector space $\mathbb{V}$ of dimension $n \geq 2$ over a finite field $\mathbb{F}$ of order $q \geq 2$. Then

$$M_1^*(G) = 2q^n (q^n - 1) - q^n (q-1)^n - 2(2q-1)^n + 2.$$

Proof

Using edge partition, given in Proposition 8, in the formula of the first Zagreb eccentricity index, we have

$$\begin{aligned}
M_1^*(G) &= \sum_{u-v} [ecc(u) + ecc(v)] \\
&= \left(\binom{(q-1)^n}{2} \right) [1+1] + \left[(q-1)^n (q^n - (q-1)^n + 1) \right] [1+2] \\
&\quad + \frac{1}{2} \left[q^{2n} - q^n - (2q-1)^n - (q-1)^n (2q^n - (q-1)^n + 1) + 1 \right] [2+2] \\
&= \frac{(q-1)^n ((q-1)^n - 1)}{2} [2] + \left[(q-1)^n (q^n - (q-1)^n + 1) \right] [3] \\
&\quad + \frac{1}{2} \left[q^{2n} - q^n - (2q-1)^n - (q-1)^n (2q^n - (q-1)^n + 1) + 1 \right] [4] \\
&= (q-1)^n ((q-1)^n - 1) + 3(q-1)^n (q^n - (q-1)^n + 1)
\end{aligned}$$

$$-2\left[(q-1)^n(2q^n-(q-1)^n)+1\right]+2(q^{2n}-q^n-(2q-1)^n+1)$$

$$=(q-1)^n\left[(q-1)^n-1+3q^n-3(q-1)^n+3-4q^n+2(q-1)^n-2\right]$$

$$+2q^{2n}-2q^n-2(2q-1)^n+2.$$

Binomial expansion and some simplifications yield the required result.

The second Zagreb eccentricity index of $\Gamma(\mathbb{V})$ is computed in the next result.

Theorem 18

Let $G=\Gamma(\mathbb{V})$ be a non-zero component graph of a vector space $\mathbb{V}$ of dimension $n \geq 2$ over a finite field $\mathbb{F}$ of order $q \geq 2$. Then

$$M_1^{**}(G)=4q^n-3(q-1)^n-4.$$

Proof

By using the eccentricity of each vertex, given in Remark 6, in the formula of second Zagreb eccentricity index, we have

$$\begin{aligned} M_1^{**}(G) &= \sum_{v_i^k \in V(G)} [ecc(v_i^k)]^2 \\ &= \left[(q^n-1)-(q-1)^n\right](2)^2 + (q-1)^n(1)^2 \\ &= 4\left[(q^n-1)-(q-1)^n\right] + (q-1)^n(1) \\ &= 4q^n - 4 - 4(q-1)^n + (q-1)^n \\ &= 4q^n - 3(q-1)^n - 4. \end{aligned}$$

Hence, we get the required result.

The following result provides the formulation of the third Zagreb eccentricity index of $\Gamma(\mathbb{V})$.

Theorem 19

Let $G=\Gamma(\mathbb{V})$ be a non-zero component graph of a vector space $\mathbb{V}$ of dimension $n \geq 2$ over a finite field $\mathbb{F}$ of order $q \geq 2$. Then

$$M_2^*(G) = \frac{1}{2}(q-1)^n\left[(q-1)^n-4q^n-1\right] + 2q^n(q^n-1) - 2(2q-1)^n + 2.$$

Proof

Using edge partition, given in Proposition 8, in the formula of the third Zagreb eccentricity index, we have

$$\begin{aligned} M_2^*(G) &= \sum_{u-v} [ecc(u) \times ecc(v)] \\ &= (q-1)^n_2(1 \times 1) + \left[(q-1)^n(q^n-(q-1)^n+1)\right](1 \times 2) \\ &\quad + \frac{1}{2}\left[q^{2n}-q^n-(2q-1)^n-(q-1)^n(2q^n-(q-1)^n+1)+1\right](2 \times 2) \\ &= \frac{1}{2}(q-1)^n((q-1)^n-1) + (q-1)^n(2q^n-2(q-1)^n+2) \\ &\quad + 2q^{2n}-2q^n-2(2q-1)^n-(q-1)^n(4q^n-2(q-1)^n+2)+2 \\ &= \frac{1}{2}(q-1)^n((q-1)^n-1) + (q-1)^n(2q^n-2(q-1)^n+2) \\ &\quad + 2q^n(q^n-1)-(q-1)^n(4q^n-2(q-1)^n+2)+2-2(2q-1)^n \\ &= \frac{1}{2}(q-1)^n\left[(q-1)^n-1+2(2q^n-2(q-1)^n+2)-2(4q^n-2(q-1)^n+2)\right] \\ &\quad + 2-2(2q-1)^n+2q^n(q^n-1). \end{aligned}$$

Some easy simplifications with the help of binomial expansion give the required result.

6 Comparisons and plots

Using maple software (Maplesoft, McKinney, TX, USA), we provide a simple comparison of the investigated indices by plotting them and by constructing tables of their numeric values. We construct Tables 2 and 3 for different values of q, n , which depict that all the indices are in increasing order as the values of q and n are increasing, and their increasing behaviors are clearly shown in Figures 1-11 for certain values of q and n .

Table 2: Numeric comparison of the investigated indices at certain values of q and n .

(q,n)	$\xi(\Gamma(V))$	$avec(\Gamma(V))$	$\xi^{ce}(\Gamma(V))$	$\xi^c(\Gamma(V))$	$\xi^d(\Gamma(V))$	$\xi^{sv}(\Gamma(V))$
(2,2)	5	1.7	3	9	22	13
(2,3)	13	1.86	18	61	118	27.4
(2,4)	29	1.933	87	321	538	52.78
(2,5)	61	1.9677	390	1501	2254	98.31
(2,6)	125	1.98413	1683	6609	9082	181.78
(2,7)	253	1.992126	7098	28141	35998	337.84
(3,2)	12	1.5	38	100	118	18.4
(3,3)	44	1.69	389	1164	1298	59.47
(3,4)	144	1.800	3560	11728	12466	173.84
(3,5)	452	1.8678	31697	111396	114698	504.89
(3,6)	1392	1.91209	280808	1030240	1041778	1482.16
(3,7)	4244	1.941446	2491169	9405444	9425018	4394.89
(4,2)	21	1.4	159	393	362	27.55
(4,3)	99	1.57	2682	7407	6698	115.98
(4,4)	429	1.682	41727	125841	113258	463.55
(4,5)	1803	1.7625	639546	2061735	1874234	1867.45
(4,6)	7461	1.82198	9819999	33311673	30772442	7576.76
(4,7)	30579	1.866508	151711482	535193247	502559498	30783.34
(5,2)	32	1.3	444	1056	850	38.74
(5,3)	184	1.48	11322	29608	23842	202.09
(5,4)	992	1.590	271464	767136	629890	102.91
(5,5)	5224	1.6722	6450702	19407928	16426402	5296.83
(5,6)	139864	1.73784	153792684	487191216	425227330	27286.82
(5,7)	156248	1.790282	3689310882	12197325448	10936004962	140109.41
(6,2)	45	1.29	995	2305	1702	51.90
(6,3)	305	1.419	35930	90345	67502	323.91
(6,4)	1965	1.5173	1236215	3327985	2569402	2005.38
(6,5)	12425	1.59807	42295550	120597825	96898502	12504.09
(6,6)	77685	1.665095	1451966435	4349943865	3627765802	77834.31
(6,7)	481745	1.7209174	50107120370	156688872105	134895219302	4820222.19
(7,2)	60	1.25	1938	4404	3062	67.02
(7,3)	468	1.368	94383	230436	163298	487.53
(7,4)	3504	1.4600	4421472	11468976	8458418	3546.27
(7,5)	25836	1.53731	206381415	564182076	434882954	25919.9
(7,6)	188640	1.603427	9662640588	27672732144	22202348402	188800.62
(7,7)	1367148	1.6600829	454349137083	1356319281516	1126026281018	1367450.35

Table 3: Numeric comparison of the investigated indices at certain values of q and n .

(q,n)	$GA_4(\Gamma(V))$	$ABC_5(\Gamma(V))$	$M_1^*(\Gamma(V))$	$M_1^{**}(\Gamma(V))$	$M_2^*(\Gamma(V))$
(2,2)	1.77	1.41	4	9	0
(2,3)	14.54	10.66	52	25	44
(2,4)	79.08	56.57	304	57	288
(2,5)	373.17	265.16	1468	121	1436
(2,6)	1648.34	1168.14	6544	249	6480
(2,7)	7027.68	4974.40	28012	505	27884
(3,2)	22.63	12.73	60	20	30
(3,3)	279.85	184.55	940	80	752
(3,4)	2867.61	1985.56	10416	272	9240
(3,5)	27543.02	19335.83	103588	872	96308
(3,6)	245106.29	180685.58	103588	2720	938880
(3,7)	2336248.89	1656893.32	9125380	8360	8853572
(4,2)	91.88	42.43	240	33	132
(4,3)	10504.95	1056.42	5652	171	4275

(continued)

Table 3: (Continued).

(q,n)	$GA_4(\Gamma(V))$	$ABC_5(\Gamma(V))$	$M_1^*(\Gamma(V))$	$M_1^{**}(\Gamma(V))$	$M_2^*(\Gamma(V))$
(4,4)	30624.69	19940.41	105024	777	87528
(4,5)	504505.23	343632.68	1812660	3363	1593231
(4,6)	8187316.64	5700963.57	30324960	14193	27604332
(4,7)	132021927.06	92919045.25	499359252	58971	465917835
(5,2)	250.85	98.90	640	48	360
(5,3)	7159.07	3797.16	21544	304	15560
(5,4)	186302.87	112486.55	606880	1728	15560
(5,5)	4728625.44	3060322.79	16206904	9424	13530680
(5,6)	119095829.47	80193135.62	423187120	50208	13530680
(5,7)	2991473983.21	2991473983.21	10917309064	263344	9771518600
(6,2)	552.84	190.92	1380	65	780
(6,3)	21897.30	10468.72	63220	485	43970
(6,4)	807819.70	450313.88	2517360	3305	1902360
(6,5)	29317261.44	17866767.56	96294700	21725	76875950
(6,6)	1059751594.04	682654717.57	3620928240	139745	3013990740
(6,7)	38270493674.67	25541036167.4	134818793980	885365	116000512730
(7,2)	6063.18	326.68	2604	84	1470
(7,3)	55973.78	24278.51	156132	720	105264
(7,4)	2784943.85	1433842.85	8355984	5712	6083448
(7,5)	137026892.02	78357596.01	433483068	43896	333021036
(7,6)	6728737985.54	4122293610.46	22183653744	330624	17782989840
(7,7)	330376690613.97	212059773922.15	1125779668332	2454360	934422277164

Figure 1: Plot of the total eccentricity.

Figure 3: Plot of ξ^{ce} .

Figure 2: Plot of the average eccentricity.

Figure 4: Plot of ξ .

Figure 5: Plot of ζ^d .

Figure 8: Plot of ABC_5 .

Figure 6: Plot of ζ^{sv} .

Figure 9: Plot of M_1^* .

Figure 7: Plot of GA_4 .

Figure 10: Plot of M_1^{**} .

Figure 11: Plot of M_2^* .

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References

- Bruckler F.M., Došlić T., Graovac A., Gutman I., On a class of distance-based molecular structure descriptors. *Chem. Phys. Lett.*, 2011, 503, 336-338.
- Dankelmann P., Goddard W., Swart C.S., The average eccentricity of a graph and its subgraphs. *Utilitas Mathematica*, 2004, 65, 41-51.
- Das A., Non-zero component graph of finite dimensional vector spaces, *Commun. Algebra*, 2016, 44, 9, 3918-3926.
- Deng H., Yang J., Xia F., A general modeling of some vertex-degree based topological indices in benzenoid systems and phenylenes. *Comput. Math. Appl.*, 2011, 61, 3017-3023.
- Farahani M.R., Eccentricity version of atom bond connectivity index of benzenoid family $ABC_5(HK)$. *World Appl. Sci. J. Chem.*, 2013, 21, 1260-1265.
- Gao W., Farahani M.R., Jamil M.K., The eccentricity version of atom-bond connectivity index of lineae polycene parallelogram benzenoid $ABC_5P(n, n)$. *Acta Chim. Slov.*, 2016, 63, 376-379.
- Ghorbani M., Hosseini-zadeh M.A., A new version of Zagreb indices. *Filomat*, 2012, 6, 93-100.
- Ghorbani M., Khaki A., A note on the forth version of geometric-arithmetic index. *Optoelectron. Adv. Mat.*, 2010, 4, 2212-2215.
- Gupta S., Singh M., Madan A.K., Eccentric distance sum: A novel graph invariant for predicting biological and physical properties. *J. Math. Anal. Appl.*, 2002, 275, 386-401.
- Hinz A.M., Parris D., The average eccentricity of Sierpinski graphs. *Graph. Combinator.*, 2012, 5, 671-686.
- Ilic A., On the extremal properties of the average eccentricity. *Comput. Math. Appl.*, 2012, 64, 2877-2885.
- Imran M., Siddiqui M.K., Abunamous A.A.E., Ali D., Rafique S.H., Baig A.Q., Eccentricity based topological indices of an Oxide network. *Mathematics*, 2018, 6, No. 126.
- Imran M., Baig A.Q., Azhar M.R., Farahani M.R., Zhang X., Eccentricity based geometric-arithmetic and atom-bond connectivity indices of copper oxide CuO. *Int. J. Pure Appl. Math.*, 2017, 117, 481-502.
- Klavžar S., Gutman I., A comparison of the Schultz molecular topological index with the Wiener index. *J. Chem. Inf. Comput. Sci.*, 1996, 36, 1001-1003.
- Liu J.-B., Javaid M., Awais H.M., Zagreb indices of generalized operations on graphs. *IEEE Access*, 2019, 7, 105479-105488.
- Rucker G., Rucker C., On topological indices, boiling points, and cycloalkanes. *J. Chem. Inf. Comput. Sci.*, 1999, 39, 788-802.
- Sardana S., Madan A.K., Predicting anti-HIV activity of TIBO derivatives: A computational approach using a novel topological descriptor. *J. Mol. Model.*, 2002, 8, 258-265.
- Sharma V., Goswami R., Madan A.K., Eccentric connectivity index: A novel highly discriminating topological descriptor for structure property and structure activity studies. *J. Chem. Inf. Comput. Sci.*, 1997, 37, 273-282.
- Tang J.-H., Ali U., Javaid M., Shabbir K., Zagreb connection indices of subdivision and semi-total point operations on graphs. *J. Chem.*, 2019, 1-14, Article ID: 9846913.
- Tang Y., Zhou B., On average eccentricity. *MATCH-Commun. Math. Co.*, 2012, 67, 405-423.
- Vukičević D., Graovac A., A note on the comparison of the first and second normalized Zagreb eccentricity indices. *Acta Chim. Slov.*, 2010, 57, 524-528.
- Wiener H., Correlation of heats of isomerization and differences in heats of vaporization of isomers, among the paraffin hydrocarbons. *J. Am. Chem. Soc.*, 1947, 69, 17-20.
- Yu G., Feng L., On connective eccentricity index of graphs. *MATCH-Commun. Math. Co.*, 2013, 69, 611-628.
- Zahid M.A., Baig A.Q., Naeem M., Azhar M.R., Eccentricity based topological indices of a cyclic octahedron structure. *Mathematics*, 2018, 6, No. 141.
- Zheng L., Wang Y., Gao W., Topological indices of Hyaluronic Acid-Paclitaxel conjugates molecular structure in cancer treatment. *Open Chem.*, 2019, 17, 81-87.
- Zhang X., Baig A.Q., Azhar M.R., Farahani M.R., Imran M., The average eccentricity and eccentricity based geometric-arithmetic index of tetra sheets. *Int. J. Pure Appl. Math.*, 2017, 116, 467-479.
- Zhang H., Zhang F., The Clar covering polynomial of hexagonal systems. *Discrete Appl. Math.*, 1996, 69, 147-167.