Research article

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Expanded Jones complex space model to describe arbitrary higher-order spatial states in fiber

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Abstract: As a new multiplexing dimension, spatial modes are catching increasing attentions nowadays. It is a fundamental task to establish an appropriate theoretical model to describe these spatial modes, especially higher-order spatial modes. However, existing theoretical models are only able to explain some special higher-order spatial states in fiber. The basic problem in these models is that their discussed dimensions are not enough. Indeed, to describe a higher-order spatial state, at least four dimensions are needed. In this paper, we present an expanded Jones complex space model, which is four-dimensional when a single higher-order state is discussed. The expanded Jones model is based on the discussion of an arbitrary combination of four degenerated higher-order modes. As a result, arbitrary spatial states are described. Because the number of used dimensions matches that of the problem, the descriptions of higher-order modes are more complete than other models. Also, we have verified the reliability of the expanded Jones model in our experiment. This model has the potential to simplify many analyses related to spatial modes in fiber.

Keywords: cylindrical vector mode; orbital angular momentum mode; fiber optics; theoretical model; polarization.

1 Introduction

Spatial modes are unique spatial distributions of electric fields that exhibit special properties. Three common kinds of spatial modes are the cylindrical vector (CV) mode, the linearly polarized (LP) mode and the orbital angular momentum (OAM) mode. Spatial modes are divided into different orders. Their radial distributions are determined by l th cylindrical functions (such as the Bessel function and the Laguerre function), where l ($l \geq 0$) is the order number. For $l=0$, the spatial modes are named as fundamental modes, whose behavior is similar to that of a plane light wave. Meanwhile, it is the feature for cylindrical functions that the value at origin is zero when $l > 0$. Thus, these spatial modes, named higher-order ($l > 0$) modes, all possess an intensity singularity at the center and their polarization connects with the spatial location. Due to the unique properties distinguishing that of a plane light wave, many fancy phenomena are found by using lights with higher-order spatial modes in various areas such as optical communication [1–6], optical tweezers [7, 8], high-resolution microscopy [9, 10] and data storage [11, 12].

If further subdivided, each higher-order spatial state consists of four degenerated modes. These four degenerated modes naturally coexist due to the neglected difference of propagation constants among them, which lead to a strong couple of the four degenerated modes when propagating [13, 14]. So far, a particular higher-order spatial state is the most simplified field that can be realized in fiber. Thus, it is vital to completely discuss the superposition of four degenerated modes.

It is a four-dimensional problem to describe arbitrary l th order spatial states because the system possesses four independent complex variables, that is, the complex amplitudes of the four degenerated modes. Efforts have been made to describe higher-order spatial states [15–34]. Many researchers have provided their own theoretical models to explain their recently discovered spatial states [15–34], but these models generally become invalid when the discussed spatial states are replaced by that of another paper. The most mature and widely used [14, 35–37] model is the higher-order Poincaré sphere (HOPS) model [34],

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which uses two surfaces of complementary HOPSs to totally describe a higher-order spatial state.

HOPS models simplify the analysis of spatial modes in a degree due to its visualization. However, the surface of a single sphere only depicts a two-dimensional space but not the four-dimensional space equivalent to this problem. Virtually, the HOPS model uses two two-dimensional spaces to describe a higher four-dimensional space. If several low-dimensional spaces are used to describe a higher-dimensional space, besides the complete discussion of these low-dimensional spaces, the relation among these low-dimensional spaces should be introduced at the same time. Specific to the HOPS model, to confirm an l th higher-order spatial state, besides the location of each HOPS, the relation between the two HOPSs should be declared at the same time. If so, the expression of higher-order spatial modes will not be intuitive. For example, $\hat{x}\text{OAM}_{+1} + i\hat{y}\text{OAM}_{-1} = \text{TM}_{01} + i\text{HE}_{21}^{\text{odd}}$, a hybrid state related to two HOPSs [19, 27], although it is able to tell the location on the two HOPSs, the phase difference factor i is difficult to be shown intuitively. We must also define the two HOPSs as equal amplitude and with $\frac{\pi}{2}$ phase difference.

Even for another kind of typical mode, the LP mode, the HOPS model is not able to express them conveniently for the same reason. Briefly, if an l th order state relates to two HOPSs simultaneously, using the HOPS model is difficult to describe this. Unfortunately, these states are the most common states in fiber.

Moreover, because a strong couple exists among the four degenerated modes when propagating, the intensity sum of the two modes described by a single HOPS is not invariant. That means, the radii of HOPSs are unfixed. Unlike the classical PS, the intensity sum of two components described by a single sphere is constant, because the conventional plane light wave only possess two independent components, that is, two orthogonal polarized components.

Besides, we notice some researchers expanding the HOPS model as the so-called “Hybrid-order Poincaré sphere” [38, 39]. They still use two PSs to describe the states including more than one order spatial modes, by deforming the PSs as some other forms. However, they not only use several low-dimensional spaces to describe a high-dimensional space, but also the dimension they use is less than the dimension of the spatial states they discuss. Thus, there must be some missing states.

In this paper, we present an expanded Jones complex space model to describe higher-order spatial states totally. When we just focus on a single higher order spatial state, it is four-dimensional. This model is based on the discussion of four degenerated modes with arbitrary amplitudes and

phases, which are regarded as four complex independent variables. We define a series of characteristic parameters (CPs) to make the mathematical expression closer to the real physical meanings. As can be seen, CV modes, LP modes and OAM modes are just three kinds of special cases of l th order spatial states, where some of the CPs take on specific values. Besides these special states, arbitrary spatial states like $0.49e^{2.47i}\text{EH}_{l-1,m}^{\text{even}} + 0.29e^{-1.21i}\text{EH}_{l-1,m}^{\text{odd}} + 0.29e^{1.43i}\text{HE}_{l+1,m}^{\text{even}} + 0.77e^{0.58i}\text{HE}_{l+1,m}^{\text{odd}}$ are explained, which are hard to be described by other theoretical models. Besides, we have done some experiments to verify the reliability of the expanded Jones model. Based on the measured patterns at the polarized angles of 0° , 45° , 90° , 135° , we use this model to match the real experiment results and the final simulation results correspond well with that.

All of these reported results [15–34] in previous papers can find their places in the expanded Jones model. As the most powerful competitor, the HOPS model is not able to include some of them [16, 19, 23, 27, 29, 40–43] because these spatial states are related to two HOPSs simultaneously. As a result, it is hoped that the expanded Jones model can simplify many of the problems related to spatial modes and become the dominant model.

2 Theory

2.1 Higher-order Poincaré sphere model

Before introducing the proposed expanded Jones model, it is helpful to know the limitation of the HOPS model.

The space below three dimensions is visible. If a system possesses no more than three independent components, it is able to be abstracted as a visible model. Polarization is such a system with three independent components. Intensity, polarization ellipticity and degree of polarization are the three independent components used to totally describe the polarization of light. Stokes [44] first used four components (only three of them are independent), namely Stokes parameters (SPs), to describe polarization. Before long, Poincaré objectified SPs as an isomorphic sphere [45], the so-called “Poincaré sphere (PS)”, a three-dimensional visible model to intuitively describe polarization.

Poincaré sphere is concluded in Figure 1A and a classical PS is shown in Figure 1B. Three independent SPs S_1 , S_2 and S_3 compose the Cartesian coordinates, while I , 2ϕ and 2θ form the radius, azimuthal angle and inclined angle of spherical coordinates. Each point in the PS corresponds to a particular polarized state. The radius of the PS denotes the intensity sum of both polarized parts and

Figure 1: Poincaré spheres with different orders.

(A) Diagram of Poincaré sphere depicting parameters. (B) Classical Poincaré sphere; and higher-order Poincaré sphere. (C) The +1 order Poincaré sphere and (D) the -1 order Poincaré sphere.

unpolarized parts. If a point locates inside PS, the radius of the point indicates the intensity of polarized parts. The ratio between the radius of the point inside the PS and the radius of the PS is the degree of polarization p . pI indicates the intensity of the polarized parts. Usually, the degree of polarization p is rarely studied. In most cases, completely polarized lights are defaulted, where $p = 1$. That means, all the points locate on the surface of the PS (two dimension). Owing to the visibility of PS, many questions related to polarization become simplified.

In the past few decades, spatial modes have gained increasing attention [46–49], and various theoretical models [15–34] are proposed to describe them. Among these models, the most complete one is the HOPS model [34]. The presenter expands the classical PS as two complementary HOPSs to describe the l th higher-order modes. These two HOPSs consist of a $+l$ sphere and a $-l$ sphere. We conclude the HOPS model using Jones calculus. When expressed in the Jones system, the field on HOPSs is equivalent as follows:

$$J_{\pm l}(\phi, \theta) = C \begin{pmatrix} |\cos \theta| e^{-i\theta} & -|\sin \theta| e^{i\theta} \\ |\sin \theta| e^{i\theta} & |\cos \theta| e^{-i\theta} \end{pmatrix} \begin{pmatrix} \cos(l\xi - \phi) \\ \pm \sin(l\xi - \phi) \end{pmatrix}, \quad (1)$$

where ξ is the spatial angle related to the optics axis and C is the normalized factor to keep $|J_{\pm l}(\phi, \theta)|$ unchanged. Eq. (1) is two equations, where the symbol “ $\pm$ ” is an alternative in both sides. They are the expressions in the Jones system equivalent to two HOPSs, respectively. When $l = 0$ (the symbol should be “ $-$ ”), it is the classical PS. Neglecting the normalized factor, Eq. (2) gives the physical meanings of several points on HOPSs,

$$\left\{ \begin{array}{l} J_{\pm l}(0, 0) = \begin{pmatrix} \cos l\xi \\ \pm \sin l\xi \end{pmatrix}, \quad \text{EH}_{l-1,m}^{\text{even}}/\text{TM}_{0,m} \text{ for } +l; \text{ and} \\ \quad \text{HE}_{l+1,m}^{\text{even}} \text{ for } -l \\ J_{\pm l}\left(\frac{\pi}{2}, 0\right) = \begin{pmatrix} \sin l\xi \\ \mp \cos l\xi \end{pmatrix}, \quad \text{EH}_{l-1,m}^{\text{odd}}/\text{TE}_{0,m} \text{ for } +l; \text{ and} \\ \quad \text{HE}_{l+1,m}^{\text{odd}} \text{ for } -l \\ J_{\pm l}\left(\phi, \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} e^{\pm i\left(l\xi - \phi \pm \frac{\pi}{4}\right)} \begin{pmatrix} 1 \\ i \end{pmatrix}, \quad \hat{\sigma}^+ \text{OAM}_{\pm l} \text{ for } \pm l; \\ \quad \text{North pole} \\ J_{\pm l}\left(\phi, -\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} e^{\mp i\left(l\xi - \phi \pm \frac{\pi}{4}\right)} \begin{pmatrix} 1 \\ -i \end{pmatrix}, \quad \hat{\sigma}^- \text{OAM}_{\mp l} \text{ for } \pm l; \\ \quad \text{South pole.} \end{array} \right. \quad (2)$$

Figure 1B–D shows the 0, +1 and –1 order PSs, where the +1 order and –1 order PSs describe the first order spatial modes in cooperation. The parameter ϕ makes the vector rotate counter-clockwise for $l \leq 0$ and clockwise for $l > 0$, while the parameter θ affects the polarization ellipticity. In Eq. (1), when $l=0$, the two HOPSs degenerate as classical PSs because the term $l\xi=0$. Thus, we can also obtain the information of classical PS from any negative HOPS when $\xi=0$. That means, the vectors at $\xi=0$ on negative HOPS combine the classical PS.

However, as mentioned already, it is a four-dimensional problem to completely describe the spatial states with a single order. The HOPS model uses two two-dimensional subspaces to depict that. Besides the location of the two HOPSs, the relation between the two PSs is necessary. Thus, spatial states related to two HOPSs are not able to be described by the HOPS model intuitively. To better describe higher-order spatial states, a four-dimensional theoretical model should be used.

2.2 Expanded Jones complex space model

In this paper, we propose an expanded Jones complex space model, by discussing the four degenerated modes with arbitrary amplitudes and phases.

Firstly the configuration of spatial modes is introduced. Figure 2A depicts the transformation relation among the CV modes, LP modes and OAM modes and the complete transformation relation can be found in our previous articles [50, 51]. Notice that such a four-dimensional space is not visible. Figure 2A is just the imaginary visible sketch of space beyond three dimensions for demonstration purposes. Any l th higher-order spatial state in optical fiber can be decomposed as the superposition of four degenerated

modes. If expressed in CV modes, the arbitrary spatial state is $E = AEH_{l-1,m}^{\text{even}} + BEH_{l-1,m}^{\text{odd}} + CHE_{l+1,m}^{\text{even}} + DHE_{l+1,m}^{\text{odd}}$, where the amplitudes and phases of A, B, C, D denote the independent amplitudes and phases of the four degenerated modes. If the values of A, B, C, D are defined, the field is also determined solely. These four independent complex variables can be spanned as a four-dimensional complex space. Each point in the space corresponds to a spatial state. The four degenerated modes work as a group of four bases in the complex space. In LP modes, the mode bases are $\hat{x}LP_{l,m}^{\text{even}}$, $\hat{y}LP_{l,m}^{\text{even}}$, $\hat{x}LP_{l,m}^{\text{odd}}$ and $\hat{y}LP_{l,m}^{\text{odd}}$ [51], where $\hat{x}$ represents the x polarization or $(1, 0)^T$ in Jones calculus, and $\hat{y}$ represents the y polarization or $(0, 1)^T$ in Jones calculus. While in OAM modes, the mode bases are $\hat{x}OAM_{-1}$, $\hat{y}OAM_{-1}$, $\hat{x}OAM_{+1}$ and $\hat{y}OAM_{+1}$. Similar to CV mode bases, complex vectors $(x_e, y_e, x_o, y_o)^T$ and $(x_-, y_-, x_+, y_+)^T$ denote the coordinates in respective mode bases [51], where the amplitudes and phases of x_e, y_e, x_o and y_o represent the amplitudes and phases of $\hat{x}LP_{l,m}^{\text{even}}$, $\hat{y}LP_{l,m}^{\text{even}}$, $\hat{x}LP_{l,m}^{\text{odd}}$ and $\hat{y}LP_{l,m}^{\text{odd}}$. So as for x_-, y_-, x_+ and y_+ . The spatial state expressed in different mode bases, such as $HE_{l+1,m}^{\text{even}} + iHE_{l+1,m}^{\text{odd}} = \hat{o}^+ OAM_{+1}$, is equivalent as base transformation in the four-dimensional complex space. And there are base transformation matrices connecting these three mode bases [51].

In this paper, we show the derivation in LP mode bases. Arbitrary the l th order spatial state expressed in LP mode bases is

$$\begin{aligned} \mathbf{E}_{\text{in}} &= x_e \hat{x}LP_{l,m}^{\text{even}} + y_e \hat{y}LP_{l,m}^{\text{even}} + x_o \hat{x}LP_{l,m}^{\text{odd}} + y_o \hat{y}LP_{l,m}^{\text{odd}} \\ &= F_{l,m}(r) \begin{pmatrix} \cos l\xi \begin{pmatrix} x_e \\ y_e \end{pmatrix} + \sin l\xi \begin{pmatrix} x_o \\ y_o \end{pmatrix} \end{pmatrix} \\ &= F_{l,m}(r) \begin{pmatrix} |E_1| e^{i\alpha_1} \cos l\xi + |E_3| e^{i\alpha_3} \sin l\xi \\ |E_2| e^{i\alpha_2} \cos l\xi + |E_4| e^{i\alpha_4} \sin l\xi \end{pmatrix}, \end{aligned} \quad (3)$$

Figure 2: Sketch of the expanded Jones model.

(A) The transformation among three kinds of spatial states; (B) the mathematical nature and procedure of the expanded Jones model.

where $LP_{l,m}^{\text{even}}$ is $F_{l,m}(r) \cos l\xi$ and $LP_{l,m}^{\text{odd}}$ is $F_{l,m}(r) \sin l\xi$, describing the radial and azimuthal amplitude distributions. $F_{l,m}(r)$ is the cylindrical function to describe the radial distribution of fiber, where l is the azimuthal order and m is the radial order. Because $F_{l,m}(0)=0$ for higher-order modes ($l>0$), an intensity singularity exists at the center of the intensity patterns. If without the modulation of the azimuthal function, the final pattern only determined by $F_{l,m}(r)$ is a doughnut shape. The vector $(x_e, y_e, x_o, y_o)^T$ is substituted by the equivalent exponential form $(|E_1|e^{i\alpha_1}, |E_2|e^{i\alpha_2}, |E_3|e^{i\alpha_3}, |E_4|e^{i\alpha_4})^T$ in the third row of Eq. (3). $|E_{1-4}|$ denote the amplitudes and α_{1-4} denote the phases of x_e, y_e, x_o, y_o , respectively. To get the information at different polarizations, we assume inserting an imaginary polarizer with the counter-clockwise angle ψ related to the defined x -axis. After passing the polarizer, the field is

$$\begin{aligned} \mathbf{E}_{\text{out}} &= \begin{pmatrix} \cos \psi & -\sin \psi \\ \sin \psi & \cos \psi \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \cos \psi & \sin \psi \\ -\sin \psi & \cos \psi \end{pmatrix} \mathbf{E}_{\text{in}} \\ &= F_{l,m}(r) [M_1 \cos(l\xi - \theta_1) + i M_2 \cos(l\xi - \theta_2)] \begin{pmatrix} \cos \psi \\ \sin \psi \end{pmatrix} \\ &= F_{l,m}(r) I(\xi)^{1/2} e^{i\gamma(\xi)} \begin{pmatrix} \cos \psi \\ \sin \psi \end{pmatrix}, \end{aligned} \quad (4)$$

where M_1, M_2, θ_1 and θ_2 are defined as the CPs. They are changed with the polarizer angle ψ , where

$$\begin{aligned} M_1 &= (|E_1| \cos \psi \cos \alpha_1 + |E_2| \sin \psi \cos \alpha_2)^2 \\ &\quad + (|E_3| \cos \psi \cos \alpha_3 + |E_4| \sin \psi \cos \alpha_4)^2)^{1/2} \\ M_2 &= (|E_1| \cos \psi \sin \alpha_1 + |E_2| \sin \psi \sin \alpha_2)^2 \\ &\quad + (|E_3| \cos \psi \sin \alpha_3 + |E_4| \sin \psi \sin \alpha_4)^2)^{1/2} \\ \theta_1 &= \arctan \left(\frac{|E_3| \cos \psi \cos \alpha_3 + |E_4| \sin \psi \cos \alpha_4}{|E_1| \cos \psi \cos \alpha_1 + |E_2| \sin \psi \cos \alpha_2} \right) \\ \theta_2 &= \arctan \left(\frac{|E_3| \cos \psi \sin \alpha_3 + |E_4| \sin \psi \sin \alpha_4}{|E_1| \cos \psi \sin \alpha_1 + |E_2| \sin \psi \sin \alpha_2} \right). \end{aligned} \quad (5)$$

Eq. (4) tells us that the field is determined by both the radial function $F_{l,m}(r)$ and the azimuthal function $I(\xi)^{1/2} e^{i\gamma(\xi)}$, which vary from the polarized angle ψ . The last 2×1 vector indicates the field with linearly polarized angle ψ .

We analyze the azimuthal function $I(\xi)^{1/2} e^{i\gamma(\xi)}$ at the beginning. When the imaginary polarizer rotates, different values of M_1, M_2, θ_1 and θ_2 are obtained. Using Eq. (4), the azimuthal intensity and phase after passing through the polarizer is

$$I(\xi) = M_1^2 \cos^2(l\xi - \theta_1) + M_2^2 \cos^2(l\xi - \theta_2), \quad (6)$$

$$\gamma(\xi) = \arctan \left(\frac{M_2 \cos(l\xi - \theta_2)}{M_1 \cos(l\xi - \theta_1)} \right). \quad (7)$$

The intensity and phase of a spatial state connects with both the azimuth ξ and the polarized angle ψ , which should be a two-variables function. The intensity expression Eq. (6) is able to be converted as

$$I(\xi) = I_D + I_A \cos^2(l\xi - \delta), \quad (8)$$

where

$$\begin{cases} I_A = [(M_1^2 \cos 2\theta_1 + M_2^2 \cos 2\theta_2)^2 \\ \quad + (M_1^2 \sin 2\theta_1 + M_2^2 \sin 2\theta_2)^2]^{1/2} \\ I_D = \frac{M_1^2 + M_2^2 - I_A}{2} \\ \delta = \frac{1}{2} \arctan \left(\frac{M_1^2 \sin 2\theta_1 + M_2^2 \sin 2\theta_2}{M_1^2 \cos 2\theta_1 + M_2^2 \cos 2\theta_2} \right). \end{cases} \quad (9)$$

Here, I_D and I_A indicate the direct current (DC) term and the alternative current (AC) term, respectively, which is the function of polarized angle ψ .

First, considering the term $\cos^2(l\xi - \delta)$ in Eq. (8), which changes with the spatial angle in the sinusoidal trend, it reaches the maximum when $l\xi - \delta = k\pi$ ($k \in \mathbb{Z}$) and the number of the maximum is $2l$ in the range $\xi \in [-\pi, \pi]$. If the radial field distribution $F_{l,m}(r)$ is also considered, that is the pattern of a $2l$ lobes pattern (or the pattern like an LP mode) with amplitude axes at $\xi = \frac{\delta + k\pi}{l}$. If the intensity is invariant with ξ [left term in Eq. (8)], the pattern should be a doughnut shape.

Eq. (8) tells us that, for l th order spatial states, the intensity consists of a DC term I_D , and an l th order AC term $I_A \cos^2(l\xi - \delta)$. The initial phase δ describes the angle of the maximum $\left(\xi = \frac{\delta + k\pi}{l} \right)$, or named AC angle. In physics, an arbitrary higher-order intensity pattern should be the superposition of a ring pattern with intensity I_D and a $2l$ lobes pattern with maximum intensity I_A and amplitude axes $\xi = \frac{\delta + k\pi}{l}$, as shown in the last three graphs in Figure 3C. Notice that, the radial intensity in Figure 3C has been normalized such that the maximum of $F_{l,m}(r)$ is 1. Thus, the variables I_D and I_A in Figure 3C indicate the maximum intensity directly.

It should be highlighted that Eq. (6) and Eq. (8) are just the different forms of the intensity distribution. Eq. (6) indicates that the final intensity pattern is the superposition of two $2l$ lobes patterns with different weights and

Figure 3: Corresponding relation between the expanded Jones model and mode patterns of a random l th order spatial state, where $|\mathbf{E}| = (2.4, 2.7, 3.1, 1.5)$ and $\alpha = (2.1, 0.2, 0.9, 3.1)$ or $1.72e^{2.47i}EH_{l-1,m}^{\text{even}} + 1.01e^{-1.21i}EH_{l-1,m}^{\text{odd}} + 1.01e^{1.43i}HE_{l+1,m}^{\text{even}} + 2.73e^{0.58i}HE_{l+1,m}^{\text{odd}}$ expressed in CV mode bases.

(A) The trend of four CPs, M_1 , M_2 , θ_1 and θ_2 ; (B) the trend of I_D , I_A and δ along with the polarized angle ψ and the patterns at several polarized angles when (D) $l=1$, $m=1$; (E) $l=2$, $m=1$. (C) Diagram to depict the decompositions of the hybrid state when $\psi=0$ and $l=1$. (F) Diagram to demonstrate the decomposition of patterns for arbitrary higher-order modes. Inten., intensity pattern; Infer., interference pattern.

amplitude axes, while Eq. (8) indicates that it consists of a ring pattern and a $2l$ lobes patterns. Figure 3C depicts the difference. If using Eq. (6) to describe the intensity pattern, even if we know the two composed $2l$ lobes patterns, it is still hard to intuitively imagine the final results. However, if using Eq. (8), the physical meaning is clear enough without further calculation. In the next section, we just use Eq. (8).

Also, Eq. (7) can be deformed as an equivalent form,

$$\gamma(\xi) = \arctan \left[\frac{M_2}{M_1} \cos(\theta_2 - \theta_1) + \frac{M_2}{M_1} \sin(\theta_2 - \theta_1) \tan(l\xi - \theta_1) \right], \quad (10)$$

The right term contributes a helical phase component and the left term can be regarded as a constant. When $\sin(\theta_2 - \theta_1) > 0$, the right term contributes an $l\xi$ helical phase. Thus, the final interference pattern should be a counter-clockwise vortex shape. A clockwise vortex shape

interference pattern happens when $\sin(\theta_2 - \theta_1) < 0$. Thus, it is able to predict the vortex direction of the interference pattern by the value of $\sin(\theta_2 - \theta_1)$. When $\sin(\theta_2 - \theta_1) = \pm 1$, if $M_2 = M_1$ is also satisfied, $\gamma = \pm l\xi \mp \theta_1$ are just the phases of the pure OAM modes. Supplement 1 elaborates the complete derivation from Eq. (3) to Eq. (10).

Figure 2B concludes the analytical procedure and mathematical nature of the proposed expanded Jones complex space model. First, the spatial state should be expressed in a specific group of mode bases (LP mode bases in this paper). Then, the four mode bases are converted as another four parameters to make the physical meanings more intuitive, namely, M_1 , M_2 , θ_1 and θ_2 , which are the function of polarized angle ψ . As the coordinates change along with ψ , there should be a trace $f(\psi)$ in the four-dimensional space for a specific spatial state. By drawing the trend of four CPs (or their equivalent variables I_D , I_A , δ , γ) along with ψ , we are able to tell the corresponding spatial state at any polarization. Thus, the information of the spatial state is described totally. In the following section, we list some examples to verify the model and introduce how the proposed model works.

3 Result and discussion

3.1 Random l th order spatial states

Figure 3 shows an example of random spatial state where $|\mathbf{E}| = (E_1, E_2, E_3, E_4) = (2.4, 2.7, 3.1, 1.5)$ and $\alpha = (\alpha_1, \alpha_2, \alpha_3, \alpha_4) = (2.1, 0.2, 0.9, 3.1)$ or $1.72e^{2.47i}EH_{l-1,m}^{\text{even}} + 1.01e^{-1.21i}EH_{l-1,m}^{\text{odd}} + 1.01e^{1.43i}HE_{l+1,m}^{\text{even}} + 2.73e^{0.58i}HE_{l+1,m}^{\text{odd}}$ expressed in CV mode bases. Substituting $|\mathbf{E}|$ and α into Eq. (5) to obtain the four CPs, we acquire the trend of four CPs in Figure 3A. Further, substituting four CPs into Eq. (9), the trend of the DC component I_D , AC component I_A and amplitude axis δ are calculated in Figure 3B, as the function of polarizer angle ψ .

Figure 3D, E provides the simulation results at the polarized angle 0° , 45° , 90° , 135° and without polarizer. Notice that, the results are the same for the arbitrary radial order m because the radial order only affects the radial intensity distribution. Thus, Figure 3D, E just gives the examples when $m=1$ for presentation. As shown, the intensity patterns correspond well with the curves in Figure 3B.

Figure 3C provides a diagram to describe the decomposition of the hybrid state when $\psi=0$ and $l=1$. The intensity patterns can be regarded as the superposition of a DC term (doughnut pattern) with intensity of 4.4 and an

AC term ($2l$ lobes pattern) with an intensity of 6.6, whose amplitude axes of $\xi = \delta + k\pi = 1.1$ or 2.7 . And Figure 3F demonstrates the physical meanings of these parameters for an arbitrary higher order mode. The only difference between Figure 3F, C is that the number of lobes is $2l$ and the amplitude axes are $\xi = (\delta + k\pi)/l$ in Figure 3F.

The phase information is obtained by substituting the CPs into Eq. (10). And even without the complete calculation of phase, it is able to estimate the direction of the interference vortex. For example, when $\psi=0$, $\theta_2 - \theta_1 = -1.27$. Thus, the factor $\sin(\theta_2 - \theta_1)$ in Eq. (10) is negative which contributes a clockwise vortex in the interference patterns. When $\psi = \pi/4$, $\theta_2 - \theta_1 = 0.47$, where a counter-clockwise vortex exists in an interference pattern. We do not provide the interference patterns without polarizer because they connect with the polarization of reference beam. Thus, they make little sense and can be neglected. In this paper, the reference beam is a circularly polarized light wave with a fundamental mode to ensure the homogeneous intensity at each polarization.

We have generated such random states in the experiment and used the expanded Jones model to explain them. The experimental setup is the same as our previously reported article [50], except for the removed quarter-wave plate in this paper, as shown in Figure 4. The setup is a fiber Mach-Zehnder interferometric system. A beam with fundamental mode is emitted from the Tunable laser (KEYSIGHT, 8164B, N7786B, Santa Rosa, CA, USA), and split into two branches with a 5:5 ratio. The left branch provides the fundamental mode used as a reference beam, while the right branch provides the higher-order modes as a signal beam. The long period fiber grating (LPFG) in the right branch subjects the beam to a strong disturbance, so that the fundamental modes are able to bridge the gap of the propagation constant difference and become the higher-order modes. Meanwhile, the polarization controller (PC) provides a weak disturbance to make the couple among the degenerated modes with a single order, so that the amplitudes and phases of the four degenerated modes are adjusted. The disturbance from PC is able to fill a slight gap of propagation constant among the same order degenerated modes but is not strong enough for different order modes. Thus, different order modes will not couple with each other when adjusting PCs. Then, a $40\times$ objective is used to make the light spot appropriate and adjust the divergence of light from the right branch. Finally, the beams from the two branches join together after passing through a non-polarization beam splitter (NPBS). After passing through the NPBS, a polarizer is set to obtain the information of higher-order modes at each polarization. The final patterns are detected by a charge

Figure 4: Experimental setup for the generation and detection of high-order spatial states.

PC, polarization controller; SMF, single mode step index fiber; FMF, few modes step index fiber; LPFG, long period fiber grating; Col., collimator; NPBS, non-polarization beam splitter; Pol., polarizer; CCD, charge coupled device.

couple device (CCD, 400–1800 nm, FIND-R-SCOPE-VIS, 85700, Palatine, IL, USA).

The results are shown in Figure 5. Because a random higher-order state is desired, we just need to casually adjust the PC until the result is easy to read. Figure 5A1–A3 provided the result of a higher-order state consisting of $0.58e^{0.49i}TM_{01} + 0.35e^{3.12i}TE_{01} + 0.33e^{-2.14i}HE_{21}^{even} + 0.66e^{1.57i}HE_{21}^{odd}$. The detected patterns at the polarization angles 0° , 45° , 90° and 135° are given in Figure 5A2. Based on these intensity and interference patterns, we recover the amplitudes and phases of the four degenerated modes, which are the eight values to be measured. However, just seven of them are independent because only the phase difference makes sense. One of α_{1-4} may be defined as a constant and we define $\alpha_1=0$ in Figure 5. From each polarization, three parameters are measured, that is, I_D , I_A and δ , whose definitions are provided in Eq. (9) or Figure 3C, F. Each of the measured parameters corresponds to a equation to solve $|E_{1-4}|$ and α_{1-4} . As a result, at least three polarizations are necessary to completely define a higher-order spatial state. Nevertheless, the measuring process is not the main point of this paper, we are not going to really elaborate it. Instead, the measuring principle is given in Supplement 1. Using the recovered amplitudes and phases, the corresponding simulation results are provided in Figure 5A3. At the right side of Figure 5A2, the pattern without a polarizer is detected, but the intensity is attenuated because it exceeds the

range of a charged couple device (CCD) camera (if attenuating the pattern without a polarizer until it falls under the detection range of a CCD camera, the darkest pattern at some polarized angle will not be clear). We have not considered the pattern without the polarizer when recovering the signal. However, the final simulation result shown in Figure 5A3 corresponds well with the experiment result. It is worth mentioning that the intensity pattern at $\psi=135^\circ$ is very close to an LP mode but not. It still possesses some OAM components, reflected by the slight clockwise vortex of the interference pattern at $\psi=135^\circ$. The vortex component is small but still alive. In our model, the tiny vortex component is able to be considered and it makes the simulation closer to the real physical phenomena. Also, we provide several short videos (Supplement 2) to show the patterns with and without interference at each polarization. As can be seen, the trend of Figure 5A1 corresponds well with the experimental results provided by the video. Figure 5B1–B3 provides another random experimental result that consists of $0.45e^{0.86i}TM_{01} + 0.27e^{1.42i}TE_{01} + 0.72e^{-0.50i}HE_{21}^{even} + 0.46e^{-3.14i}HE_{21}^{odd}$. The analytical process is similar and the results are also well described by Figure 5B1. The complete patterns at each polarization are provided in Supplement 2.

Such random higher-order hybrid spatial states widely exist in fiber systems, but they are rarely reported because of the complexity in the maths. Current models with lacking dimensions are not able to describe these spatial

Figure 5: Corresponding relation between the expanded Jones model and mode patterns of random spatial states.

(A) $|\mathbf{E}| = (0.34, 0.75, 0.74, 0.88), \alpha = (0, 2.05, 1.08, 0.67)$ or $0.58e^{0.49i}TM_{01} + 0.35e^{3.12i}TE_{01} + 0.33e^{-2.14i}HE_{21}^{even} + 0.66e^{1.57i}HE_{21}^{odd}$ expressed in CV mode bases. (B) $|\mathbf{E}| = (0.92, 0.49, 0.57, 0.77), \alpha = (0, 2.58, -2.65, 2.03)$ or $0.45e^{0.86i}TM_{01} + 0.27e^{1.42i}TE_{01} + 0.72e^{-0.50i}HE_{21}^{even} + 0.46e^{-3.14i}HE_{21}^{odd}$ expressed in CV mode bases. (A1, B1) The trend of I_D, I_A and δ along with polarized angle ψ . (A2, B2) Experiment (Exp.) and (A3, B3) Simulation (Sim.) results at the polarized angle $0^\circ, 45^\circ, 90^\circ$ and 135° for $l=1, m=1$. Inten., intensity pattern; Infer., interference pattern.

states. Instead, just very limited special states have been reported [15–33]. The expanded Jones complex model is the model able to explain these common spatial states in fiber.

The convenience and superiority of the expanded Jones model have been discussed. To verify the reliability of the expanded Jones model, we still need to explore whether it works well in classical special states, such as pure CV modes, LP modes and OAM modes. As will be shown, these classical special states are just some special cases in the expanded Jones model.

3.2 Pure CV modes and LP modes

For pure CV modes and LP modes, their intensity patterns are always a $2l$ lobes shape at arbitrary polarization.

The DC component is always zero in these two cases. Figure 6A1–A3 shows a pure CV mode $HE_{l+1,m}^{even}$, where the AC component $I_A=1$ is satisfied everywhere, while the AC angle δ decreases linearly. Thus, the intensity pattern should be always a $2l$ lobes shape with the same intensity, but the intensity axes change linearly. The sudden changing in the $\delta \sim \psi$ figure is because we restricted δ in the range between $-\pi/2$ and $\pi/2$. Figure 6A2, A3 provide several intensity patterns of the first and the second orders, which are well matched with Figure 6A1–A3. The experimental results of these two kinds of spatial states have been widely reported [16, 40, 43], so we do not give the experimental results here. Instead, the second order cases are provided as the expansion. For l th orders, the noteworthy thing is that the amplitude axes of the $2l$ lobes patterns (AC terms) are located at δ/l but not at δ .

Figure 6: Corresponding relation between the expanded Jones model and mode patterns of a pure CV mode and a pure LP mode.

(A) $|\mathbf{E}| = (1, 0, 0, 1), \alpha = (0, 0, 0, \pi)$ or $\text{HE}_{l+1,m}^{\text{even}}$ expressed in CV mode bases. (B) $|\mathbf{E}| = (0, 1, 0, 0), \alpha = (0, 0, 0, 0)$ or $\frac{1}{2}(\text{EH}_{l+1,m}^{\text{odd}} + \text{EH}_{l+1,m}^{\text{odd}})$ expressed in CV mode bases. (A1, B1) The trend of I_D , I_A and δ along with polarized angle ψ . The patterns at the polarized angle $0^\circ, 45^\circ, 90^\circ$ and 135° when (A2, B2) $l=1, m=1$; (A3, B3) $l=2, m=1$. Inten., intensity pattern; Infer., interference pattern.

And, I_D , I_A and δ only depict the intensity information. For confirming phase information, we still need to discuss the factor γ by substituting the four CPs into Eq. (10) or Eq. (7). In this case, $\sin(\theta_2 - \theta_1) = 0$ occurs at any ψ angle, and γ is calculated as

$$\gamma = \begin{cases} 0 & \xi \in \left[\frac{-\psi - \pi/2}{l}, \frac{-\psi + \pi/2}{l} \right) \\ \pi & \xi \in \left[\frac{-\psi + \pi/2}{l}, \frac{-\psi + 3\pi/2}{l} \right) \end{cases} \quad (11)$$

The phase information is the typical binary phase distribution with π phase difference, which is exactly the phase characterization of CV modes. Therefore, both intensity and phase information correspond well with the expanded Jones complex space model.

Figure 6B1–B3 shows another example $\hat{y}\text{LP}_{l,m}^{\text{even}}$, the pure LP mode with y-polarization. It is a little different with the CV mode case, where the AC component changes sinusoidally and the AC angle is constant. Compared with CV modes ($2l$ lobes patterns, invariant intensity and linearly changing intensity axes), for LP modes, the intensity patterns are also always the $2l$ lobes shape, however, the intensity changes sinusoidally, and the intensity axes are invariant. Figure 6B2, B3 provide the patterns at several angles, corresponding well with Figure 6B1. Also, the phase information should be given, that is,

$$\gamma = \begin{cases} 0 & \xi \in \left[\frac{-\pi/2}{l}, \frac{\pi/2}{l} \right) \\ \pi & \xi \in \left[\frac{\pi/2}{l}, \frac{3\pi/2}{l} \right) \end{cases} \quad (12)$$

It is also a binary phase distribution, but the range no longer connects to ψ , compared with Eq. (11). That is the difference between CV modes and LP modes.

3.3 Pure arbitrary polarized OAM modes

For pure OAM modes, the intensity patterns should be a doughnut shape all times. Thus, the AC component should be zero at any polarized angle ψ . Figure 7A1–A3 gives the example of a pure elliptically polarized OAM_{+l} mode. In this case, the DC component changes sinusoidally and the AC angle δ makes no sense due to the zero AC component. Figure 7A2, A3 provides these intensity patterns at several polarized angles. Because the DC

component changes with ψ but never vanishes, it should be elliptical polarization. Meanwhile, the phase is calculated as

$$\gamma = l\xi - \theta_1. \quad (13)$$

As the initial phase, the particular value of θ_1 is not the concerned. The field has the vortex phase factor $e^{il\xi}$, which suggests the TC of $+l$. Thus, the interference patterns in Figure 7A2, A3 are always a perfect counter-clockwise vortex.

Figure 7B1–B3 provides another example of a pure linearly polarized OAM mode— $\hat{x}\text{OAM}_{-l}$. There are two difference with Figure 7A1–A3. One of the differences is that the DC component is zero when $\psi = \pi/2$, which

Figure 7: Corresponding relation between the expanded Jones model and mode patterns of an elliptically polarized OAM mode and a circular polarized OAM mode.

(A) $|\mathbf{E}| = (1, 1, 1, 1), \alpha = (0, \pi/4, \pi/2, 3\pi/4)$ or $0.77e^{1.18i} \text{EH}_{l+1,m}^{\text{even}} + 0.77e^{-0.39i} \text{EH}_{l+1,m}^{\text{odd}} + 1.85e^{-0.39i} \text{HE}_{l+1,m}^{\text{even}} + 1.85e^{1.18i} \text{HE}_{l+1,m}^{\text{odd}}$ expressed in CV mode bases.

(B) $|\mathbf{E}| = (1, 0, 1, 0), \alpha = (0, 0, -\pi/2, 0)$ or $\frac{1}{2}(\text{EH}_{l+1,m}^{\text{even}} + i\text{EH}_{l+1,m}^{\text{odd}} + \text{HE}_{l+1,m}^{\text{even}} - i\text{HE}_{l+1,m}^{\text{odd}})$ expressed in CV mode bases. (A1, B1) The trend of I_D , I_A and δ along with polarized angle ψ . The patterns at the polarized angle $0^\circ, 45^\circ, 90^\circ$ and 135° when (A2, B2) $l=1, m=1$; (A3, B3) $l=2, m=1$. Inten., intensity pattern; Infer., interference pattern.

indicates that vanishment occurs at $\psi = \pi/2$, and the maximum polarized component occurs at $\psi = 0$. Thus, it is an x-polarized beam. The other difference is that the phase is calculated as

$$\gamma = -l\xi + \theta_1, \quad (14)$$

where the $-l$ TC is hinted.

4 Conclusion

It is a four-dimensional problem to completely describe arbitrary spatial states with a single azimuthal order, because the system consists of four independent complex variables, that is, the amplitudes and phases of four degenerated modes. Researchers usually presented their own models to explain the spatial states they first discovered in recent years. However, when the researched situation is changed, their models are usually invalid, because their models just discuss low-dimensional subspace of four-dimensional complex space. In this paper, we propose a complete four-dimensional model, named the expanded Jones complex space model. The proposed expanded Jones model is based on the discussion of the combination of four degenerated modes with arbitrary amplitudes and phases. As a result, it can cover most of their reported results because it is easy to describe a low-dimensional space by using a higher dimensional model. The experiments provided in this paper verify its reliability. However, the limitation of this model is that it cannot analyze those spatial states consisting of more than one order, as for other theoretical models.

The brief procedure of the expanded Jones complex space model is provided here. If we are interested in an l th azimuthal order spatial state, we can always decompose them into four degenerated CV modes, LP modes or OAM modes with different amplitudes and phases. Based on the expressions in a specific mode bases, we are able to calculate the corresponding CPs. By drawing the numerical relation among CPs or the equivalent parameters along with the change of the polarized angle, the intensity and phase information at arbitrary polarization can be obtained intuitively. Notice that, in this paper, we just derive the CPs in the decomposition of LP mode bases. Indeed, CPs are not the same in different mode bases. Although the final derived CPs are different, their physical meanings are similar.

The expanded Jones complex space model concludes the l th higher-order spatial states well. It may be a helpful tool to simplify the analyses related to spatial modes.

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