Research article

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Topological plasmonic edge states in a planar array of metallic nanoparticles

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Abstract: Photonic topological insulators (PTIs) are electromagnetic structures with highly robust unidirectional edge states, originating from their nontrivial bulk band topology. Here, we propose a plasmonic PTI that supports highly confined one-way edge states capable of transporting deep subwavelength optical frequency plasmons through arbitrary paths without back-scattering. The structure consists of a simple planar array of coupled plasmonic nanoparticles arranged in a perturbed honeycomb lattice that exhibits nontrivial band topology. The operation frequency of the emergent edge states is shown to be independent of the lattice constant, allowing for the miniaturization of the structure. As a high-frequency PTI with a simple and planar design, this structure is compatible with well-established integrated nanofabrication technologies and may find application in planar, compact, and topologically robust integrated nanophotonic devices.

Keywords: plasmonics; photonic topological insulators; nanoparticle arrays; edge states.

1 Introduction

Photonic topological insulators (PTI) are a class of photonic structures that bring the exciting properties of electronic topological insulators in solid-state electronic systems to photonics and optics [1–8]. While insulating in the bulk, these structures support unidirectional and topologically robust

edge modes at their boundaries with considerable immunity against backscattering. This opens up a new direction for realization of novel devices such as robust delay lines [4] and topological lasers [5, 9]. Since their first introduction in a two-dimensional (2D) time-reversal-symmetry (T -symmetry) broken magnetic photonic crystal [6], various photonic structures with nontrivial band topology have been proposed, including arrays of coupled ring resonators [4], bi-anisotropic metamaterials [7], and conventional photonic crystals [8]. However, these proposals are limited to either bulky dielectric waveguide structures or 3D anisotropic complex metamaterials with operation frequencies limited to the microwave regime, restricting their application in integrated and optical frequency nanophotonic devices, where effective routing of light can be challenging [10–12].

In this work, we address this challenge by using plasmonic structures in the form of a coupled metallic nanoparticle array. Arrays of coupled plasmonic nanoparticles are a class of plasmonic systems studied for their extensive applications [13], such as subdiffraction waveguiding [14], and are potential candidates for use as waveguides in dense integrated nanophotonic systems. Nontrivial topology in coupled plasmonic nanoparticle arrays has been previously realized in 1D plasmonic nanoparticle arrays using alternating spaced nanoparticles and quasi-1D zigzag chains [15–17]. These systems, similar to the Su-Schrieffer-Heeger [18] model, exhibit highly localized edge states at their ends, which are robust against perturbations in the array. However, since these edge modes are 0D (localized), they cannot deliver the most exciting promise of PTIs, namely, robust transmission using unidirectional edge states.

Here, we propose a PTI using a planar, deformed honeycomb array of coupled metallic nanoparticles. This is the first example of nontrivial topology in coupled plasmonic nanoparticle arrays beyond 1D chains with 0D localized edge states. As a T -invariant PTI, this ultracompact plasmonic structure supports one-way topological edge states at its boundaries that can guide light through arbitrary paths without back-scattering. Compared to the recently reported realizations regarding topological plasmonic systems [19–21], the structure proposed here has the important advantages of deep subwavelength

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operation at optical frequencies as well as having a simple planar design. This makes this system compatible with integrated nanofabrication techniques, paving the way for utilization of the exotic photonic topological phases in highly integrated nanophotonic devices.

2 Plasmonic array design and bulk band structure

The proposed structure is shown in Figure 1. As shown in this figure, the structure is composed of hexagonal clusters (meta-molecules) of six plasmonic nanospheres, arranged in a triangular lattice with a lattice constant of a , with $\vec{a}_1$ and $\vec{a}_2$ as the lattice vectors.

Here, the coupled plasmonic nanoparticle structure is modeled using the multipole expansion method [22]. When the size of the nanoparticles is small compared to the minimum of their center-to-center distance $d = (R_1, R_2)$ satisfying $r_s/d \leq 1/3$, the nanoparticles can be adequately modeled as point dipoles $\mathbf{p}_i$ [23] coupled through dipolar interactions. Assuming linear and nonmagnetic media, the induced dipole moment $\mathbf{p}_i$ of the nanosphere located at $\mathbf{r}_i$ is linearly proportional to the electric field at $\mathbf{r}_i$. In the absence of external excitations, this electric field is sum of the contributions of all other dipole moments in the array:

$$\mathbf{p}_i = \alpha(\omega) \sum_{j \neq i} G(\Delta \mathbf{r}_{ij}) \mathbf{p}_j, \quad (1)$$

where $\Delta \mathbf{r}_{ij}$ is a vector connecting the i th nanosphere to the j th, $\alpha(\omega)$ is the polarizability of an individual nanosphere, and $G(\mathbf{r})$ is the dyadic dipole-dipole interaction tensor, $G(\mathbf{r}) = 1/4\pi\epsilon_0(k^2 + \nabla\nabla)e^{ikr}/r$ [24]. Since in our problem, all of the dipoles and the corresponding $\Delta \mathbf{r}_{ij}$ values lie on a single (xy) plane, the in-plane and out-of-plane

components of $G(\mathbf{r})$ tensor are decoupled. Focusing on the transverse modes of the lattice with $\mathbf{p}_i = p_i \hat{z}$. The corresponding out-of-plane component of $G(\mathbf{r})$ would be equal to $G^\perp(\mathbf{r}) = 1/4\pi\epsilon_0 \left(k^2 + \frac{ik}{r} - \frac{1}{r^2} \right) e^{ikr}/r$. This expression takes into account full retardation effects. Here, we intend the system to operate in a deep subwavelength regime, where the quasi-static approximation can be applied, leading to the simplification of Green's function as $G_q^\perp(\mathbf{r}) = -1/4\pi\epsilon_0 r^3$.

The polarizability of an individual nanosphere, $\alpha(\omega)$ in Eq. (1), can be obtained as $\alpha(\omega) = 4\pi r_s^3 \epsilon_0 (\epsilon(\omega) - \epsilon_g) / (\epsilon(\omega) + 2\epsilon_g)$, where ϵ_g is the permittivity of the background medium in which the nanoparticles are embedded [25]. For the permittivity of the nanoparticles, $\epsilon(\omega)$, we use the Drude model, $\epsilon(\omega) = \epsilon_\infty - \omega_p^2 / (\omega^2 + i\omega\gamma)$, where ω_p is the plasma frequency, $1/\gamma$ is the relaxation time, and ϵ_∞ is the limit of the permittivity, as frequency tends to infinity. In order for the eigenvalue problem to be Hermitian, we do not consider losses in the formulation of the eigenvalue problem. Effects of absorption and radiation losses will be considered in the numerical calculations of the edge states and would be discussed in the last section of the paper.

2.1 Eigenvalue equation

By substituting the quasi-static transverse Green's function into Eq. (1), we obtain an eigenvalue equation as

$$\frac{1}{\tilde{\alpha}(\omega)} p_i = \sum_{j \neq i} G_q^\perp(\Delta \mathbf{r}_{ij}) p_j, \quad (2)$$

where $\tilde{\alpha}^{-1}(\omega) = 4\pi\epsilon_0 / \alpha(\omega)$ is the eigenvalue. This eigenvalue equation can be cast into a concise matrix form by using a vector of the dipole moments $\mathbf{p}_n = [p_n^1 \dots p_n^6]^T$ of the

Figure 1: The plasmonic nanoparticle array in real and reciprocal space.

(A) The proposed plasmonic nanosphere array. The primitive lattice is a triangular lattice with basis vectors $\vec{a}_1$ and $\vec{a}_2$ with $|\vec{a}_{12}| = a$. Each unit cell has six nanospheres (labeled from 1 to 6) arranged in a hexagonal cluster (called a *meta-molecule*). (B) The Brillouin zone corresponding to the rhombic unit cell of a perfect honeycomb lattice [highlighted at the left bottom of (A)], shown as a gray area, compared to the smaller Brillouin zone corresponding to the hexagonal unit cell adopted in this paper.

nanospheres of the unit cell located at the position $\mathbf{R}_n$ as the eigenvector:

$$\frac{1}{\tilde{\alpha}(\omega)} \mathbf{p}_n = \sum_{m \neq n} \mathcal{H}(\Delta \mathbf{R}_{m,n}) \mathbf{p}_m, \quad (3)$$

where the Hamiltonian $\mathcal{H}(\Delta \mathbf{R}_{m,n})$ is a 6×6 matrix with its elements $\mathcal{H}_{ij}(\Delta \mathbf{R}_{m,n})$ defined as

$$\mathcal{H}_{ij}(\Delta \mathbf{R}_{m,n}) = \begin{cases} 0 \\ G_q^\perp(\Delta \mathbf{R}_{m,n} + \mathbf{r}_{ij}) \end{cases}, \quad (4)$$

where $\mathbf{r}_{ij}$ represents the vector pointing from i th to j th nanosphere in an individual unit cell.

The band structure of an infinite lattice can be obtained by applying the Bloch boundary conditions to the $\mathcal{H}$ and obtaining the Bloch Hamiltonian. Since the quasi-static Green's function contains only the near-field interaction term and declines with distance as $1/r^3$, we use the tight-binding model and keep the nearest-neighbor interactions only. The resulting Hamiltonian would read

$$H_{\mathbf{k}} = \begin{pmatrix} 0 & M_1 & 0 & M_2 e^{i\mathbf{k} \cdot \mathbf{a}_1} & 0 & M_1 \\ M_1 & 0 & M_1 & 0 & M_2 e^{i\mathbf{k} \cdot \mathbf{a}_2} & 0 \\ 0 & M_1 & 0 & M_1 & 0 & M_2 e^{i\mathbf{k} \cdot (\mathbf{a}_2 - \mathbf{a}_1)} \\ M_2 e^{-i\mathbf{k} \cdot \mathbf{a}_1} & 0 & M_1 & 0 & M_1 & 0 \\ 0 & M_2 e^{-i\mathbf{k} \cdot \mathbf{a}_2} & 0 & M_1 & 0 & M_1 \\ M_1 & 0 & M_2 e^{-i\mathbf{k} \cdot (\mathbf{a}_2 - \mathbf{a}_1)} & 0 & M_1 & 0 \end{pmatrix}, \quad (5)$$

where $M_1 = G_q^\perp(R_1) = \frac{-1}{R_1^3}$ and $M_2 = G_q^\perp(R_2) = \frac{-1}{R_2^3}$ are the intra-cell and intercell interaction strengths between nearest-neighbor nanoparticles, respectively. The key parameter here is $t = (R_1/R_2)^3$, which indicates the intercell-to-intra-cell interaction strength ratio and can be adjusted by shrinking or expanding the hexagonal meta-molecules inside each unit cell. In order to obtain the band structure, the resulting eigenvalue equation $H_{\mathbf{k}} \psi(\mathbf{k}) = \tilde{\alpha}^{-1}(\omega) \psi(\mathbf{k})$ is solved for $\omega(\mathbf{k})$. The band structures for three cases of $R_1 < R_2$ (shrunk meta-molecules), $R_1 = R_2$ (honeycomb) and $R_1 > R_2$ (expanded meta-molecules) are shown in Figure 2, where the material and lattice parameters are set to $\varepsilon_\infty = \varepsilon_g = 1$, $a = 50$ nm, and $r_s = a/10$.

As shown in Figure 2A, for the case of $R_1 < R_2$, the band structure is gapped. The gap closes at $R_1 = R_2$ and

reopens again when $R_1 > R_2$. In the symmetric case of $R_1 = R_2$ with equal intercell and intracell interactions (Figure 2B), a doubly degenerate Dirac cone is formed at the center of the Brillouin zone, Γ . The existence of Dirac cones in a tight-binding model of a honeycomb lattice is well known [26, 27], and the two Dirac cones superimposed at Γ in Figure 2B are in fact the two Dirac cones of a honeycomb lattice at $\mathbf{K}$ and $\mathbf{K}'$ points of the bigger Brillouin zone (see Figure 1B) folded back at Γ as a result of choosing a hexagonal unit cell instead of the rhombic unit cell of a perfect honeycomb lattice. The frequency of the Dirac point ω_D is the surface plasmon resonance frequency of a single nanosphere $\omega_D = \omega_p / \sqrt{2\varepsilon_g + \varepsilon_\infty}$, which, in contrast to photonic crystals, is independent of the lattice constant.

It should be noted that the mechanism of the band gap formation here is different from the Bragg interferences in wavelength-scale photonic crystals and relies on the resonance of individual nanoparticles. Similar low-frequency band gaps in deep subwavelength resonant electromagnetic and acoustic metamaterials have been previously reported in the literature [28, 29], in which the band gap

frequency is determined by the resonance frequency of an individual resonator, instead of the specific arrangement of the resonators in the array [29]. Here, to investigate the effect of the lattice constant on the band gap frequency, the transmission spectra in arrays of shrunken meta-molecules with various lattice constants are numerically calculated. In this calculation, the coupled dipole method is used to analyze a 30×30 array of meta-molecules, in which the source dipole and destination nanosphere are 20 unit cells apart, and transmission is defined as the ratio of their dipole moments $|P_d/P_s|$. The results are shown in Figure 3. As the results of Figure 3 illustrate, in this sub-wavelength regime, the band gap, the frequencies over which transmission is minimum, remains unchanged for various lattice constants, as expected from the analytical model.

Figure 2: Bulk band structures and band inversion.

Bulk band structure of the unit cells for various values of t calculated using the tight-binding method, and their corresponding lattices. (A) $a/R_1 > 3$ or $t < 1$; the band structure is gapped. (B) $a/R_1 = 3$ or $t = 1$; a plasmonic double Dirac cone is formed at Γ . The 3D band structure showing the Dirac cones is shown in the inset. (C) $R_1 > R_2$ or $t > 1$. A band gap is opened in the shrunk and expanded cases with $t \neq 1$ [(B) and (C)]. The yellow area shows the band gap and the lowest and highest frequency bands are shown as dashed lines. The bands are inverted around Γ in (C) compared to (A), signaling a topological phase transition. (D–F) Corresponding lattice structures with the nearest-neighbor interaction strengths shown schematically by lines between the nanospheres.

Figure 3: The transmission spectra for a lossless 30×30 array of meta-molecules with $R/a = 3.15$ and $f_p = 1500$ THz, calculated using the coupled dipole method. It can be observed that the band gap (range of frequencies over which transmission is minimum) remains independent of the lattice constant.

3 Topological phase transition and edge states

Now, we show that the two gapped phases of the system with $R_1 < R_2$ and $R_1 > R_2$ are topologically distinct and a

topological phase transition happens at $R_1 = R_2$. Starting from the gapped phase with shrunk meta-molecules ($R_1 < R_2$), the system can be further deformed toward the $R_1 \ll R_2$ limit without closing the gap, where the meta-molecules in different unit cells are effectively decoupled with a vanishing overlap among their plasmonic eigenmodes. The Hamiltonian $H_{\mathbf{k}}$ is therefore topologically equivalent to a $\mathbf{k}$ -independent Hamiltonian of an individual meta-molecule [$H(\mathbf{k})$ in the $M_2 = 0$ limit], and the system is in a trivially gapped phase.

Going to the lattice with expanded meta-molecules ($R_1 > R_2$), the gap is first closed at $R_1 = R_2$ and opens again when $R_1 > R_2$. In order to discuss the topology in this case, we investigate the eigenstates of the bands looking for a band inversion.

In Figure 4A, the z -component of the electric fields E_z corresponding to the four relevant eigenstates $|2\rangle, \dots, |5\rangle$ of an individual unit cell is depicted. $|p_{x,y}\rangle$ and $|d_{xy,x^2-y^2}\rangle$ eigenstates are of odd and even parities, respectively, and are reminiscent of the $|p_{x,y}\rangle$ and $|d_{xy,x^2-y^2}\rangle$ electronic orbitals in atoms. By projecting the eigenstates of the band structure for $R_1 > R_2$ and $R_1 < R_2$ at Γ onto the p and d eigenstates of an individual unit cell, it is observed

Figure 4: The eigenstates and pseudo-spin states of a single unit cell. (A) The electric fields E_z of the p - and d -like eigenstates of an individual unit cell. (B) Pseudo-spin states $p_{\pm}$ and $d_{\pm}$ as linear combinations of the p - and d -like eigenstates. The phase difference between adjacent dipole moments is shown.

that the bands become *inverted* in the vicinity of Γ (Figure 2A and C) for the expanded case, signaling a topological phase transition taking place at the critical point $R_1 = R_2$ where the gap is closed.

The key difference of our plasmonic system with spin-1/2 electronic systems is its bosonic nature compared to the fermionic electronic systems, where the time-reversal (TR) operator $\mathcal{T}$ satisfies $\mathcal{T}^2 = -1$, leading to the Kramers doubling. This guarantees the existence of at least one pair of gapless helical edge states at the boundaries as long as TR-symmetry is not broken [30]. In our photonic/plasmonic system, $\mathcal{T}^2 = +1$ and topological protection has to rely on other symmetries. Here, an equivalent Kramers theorem can be constructed based on the present C_{6v} symmetry [8] with the pseudo-spin states defined as $|p_{\pm}\rangle = |p_x\rangle \pm i|p_y\rangle$ and $|d_{\pm}\rangle = |d_{xy}\rangle \pm i|d_{x^2-y^2}\rangle$. As a result, (pseudo-)TR-symmetry relies on the C_{6v} symmetry of the crystal and breaking this symmetry would be equivalent to the presence of magnetic impurities in the corresponding electronic topological insulators.

By projecting the Hamiltonian in Eq. (5) to the 4D subspace spanned by $[p_+, p_-, d_+, d_-]$, one can perturbatively obtain an effective 4×4 Hamiltonian up to second order in $\mathbf{k}$ around Γ for the two pseudo-spin eigenstates $p_{\pm}$ and $d_{\pm}$:

$$H_{\mathbf{k} \rightarrow (0,0)} = \begin{pmatrix} H_+(\mathbf{k}) & \mathbf{0} \\ \mathbf{0} & H_-(\mathbf{k}) \end{pmatrix}, \quad (6)$$

Figure 5: The Chern number C_+ associated with the pseudo-spin up Hamiltonian. The Berry curvature corresponding to a $C_+ = +1$ is shown in the inset.

where $H_+(\mathbf{k})$, $H_-(\mathbf{k})$ are massive Dirac Hamiltonians with additional kinetic energy terms corresponding to the two pseudo-spin states:

$$H_+(\mathbf{k}) = a_0 t (-k_x \sigma_1 + k_y \sigma_2) / 2 + (1 - t + a_0^2 t k^2 / 2) \sigma_3 \quad (7)$$

and

$$H_-(\mathbf{k}) = a_0 t (k_x \sigma_1 + k_y \sigma_2) / 2 + (1 - t + a_0^2 t k^2 / 2) \sigma_3, \quad (8)$$

where σ_1 , σ_2 , and σ_3 are the Pauli matrices [31], and a transformation $k_x \rightarrow -k_y$, $k_y \rightarrow k_x$ is made. This Hamiltonian is equivalent to the Bernevig-Hughes-Zhang [32] Hamiltonian of an HgTe quantum well. The total Chern number corresponding to this quantum spin Hall state is zero as the total Berry curvature of the bands of both pseudo-spins sums to zero [33], but the pseudo-spin up and down states (corresponding to $H_{\pm}(\mathbf{k})$ Hamiltonians) possess non-zero Chern numbers separately. In Figure 5, we show the Chern number C_+ corresponding to $H_+(\mathbf{k})$ for various $t = (R_1/R_2)^3$ values, calculated using the method proposed in [34]. It can be observed that for $t > 1$, we have $C_+ = +1$. Similarly, for the pseudo-spin down Chern number, we obtain a $C_- = -1$ for $t > 1$. This corresponds to a non-zero spin Chern number of $C = C_+ - C_- = 2$ [33]. Therefore, a pair of topologically protected edge states exist at the boundary of the two regions where the mass parameter $t - 1 = (R_1/R_2)^3 - 1$ changes sign [32], that is, at the boundary of the regions with expanded and shrunken meta-molecules.

3.1 Edge states spectra

Figure 6A shows the band diagram of a broad ribbon consisting of 40 expanded hexagons (topological) covered with 20 shrunken hexagons (trivial) on its both sides with a zigzag boundary. The bulk band structures

Figure 6: Edge states at the interface of trivial/topological lattices.

Band structure of a strip of 40 unit cells with (A) $t=1.3$ (topological), covered on both sides with 20 hexagons with $t=0.7$ (trivial). The gap is closed by the unidirectional edge states (red bands). (B) Band structure when $t=1.6$ and $t=0.4$ in the topological and trivial regions, respectively. The edge states become gapped around Γ . (C) Band structure of the lattice with the same parameters when transition from $t=1.6$ to $t=0.4$ occurs adiabatically along six unit cells. (D) Confinement of the edge states to the boundary.

of the expanded and shrunken arrays have a common band gap. The gap of the ribbon's band structure is closed by two bands (red and green) that are localized at the boundary of the trivial and topological regions (Figure 6D). Each of the two edge bands corresponding to pseudo-spin up and down states crosses the whole band gap with its group velocity not changing sign, showing its helical or pseudospin-locked unidirectional character.

As a result of the abrupt change in lattice deformations at the boundary of the two regions and breaking of the C_{6v} symmetry, a small gap appears at Γ for the edge states. This gap becomes more pronounced by increasing the contrast in interhexagon and intrahexagon interaction strengths between the two sides of the boundary (Figure 6B). The two pseudo-spin states are mixed at this point because of breaking the pseudo-TR symmetry, which causes possible back-scattering of the edge states. On the other hand, for values of t with larger deviations from 1 on the two sides of the boundary, the common bulk band gap is wider, leading to a stronger lateral confinement of the edge states, which is desirable. One can avoid this tradeoff by gradually changing t at the boundary so that the transition from the bulk trivial to the bulk topological region takes place over multiple unit cells [35]. The edge band structure of such a crystal is shown in Figure 6C. The edge bands have become more linear and their gap is now unnoticeable, while the main band gap has not narrowed down.

3.2 Robust helical edge states

The one-way character of the edge states can be observed by their reflection-free propagation through twisted paths. Here, we numerically show the existence of helical edge states at the boundary of two topologically distinct plasmonic lattices using fully fledged coupled dipole calculations. The resulting linear system of equations is then solved using the biconjugate gradient stabilized algorithm [36]. The parameters for the calculations are set as follows: the plasma frequency is set to $f_p = 1030$ THz, which gives a Dirac point frequency of 593 THz in the visible range, equal to that of gold nanoparticles embedded in glass. The lattice constant is set to $a = 50$ nm and $r_s = a/10$. The nanosphere array shown in Figure 7 is composed of two distinct regions with $a/R_1 = 3.1$ and $a/R_1 = 2.92$ on the upper and lower sides, respectively. By placing a circularly polarized dipole source with a frequency of $0.99\omega_D$ at the center of a unit cell close to the boundary, right/left propagating modes with pseudo-spin up/down can be excited (Figure 7A and B). As shown in the figure, no reflection occurs even at the sharp bends. In order to avoid reflections at the truncation limits of the lattice domains in Figure 7, we have introduced moderate amounts of losses to the last few unit cells to attenuate the excitations before reaching the lattice truncation point (not shown in the figure).

In the presence of losses, the system becomes non-Hermitian with complex eigenfrequencies, which can

Figure 7: Helical edge states confined to the nanospheres at the boundary of a topological and a trivial plasmonic lattice. The magnitude of the dipole moment of each nanosphere $|p|$ is shown in the figures. (A) The upper part of the array is a trivial lattice with $a/R_1 = 2.92$ and the lower part is a topological lattice with $a/R_1 = 3.1$ (size of the spheres is not to scale). (B and C) Rightward and leftward propagation of the edge states with up and down pseudo-spin. (D) The same edge state as in (B and C) in the presence of absorption losses.

affect the topological properties of the system [37]. Therefore, it is important to see if the edge states retain their properties predicted for the Hermitian counterpart and how they are affected. Radiative losses can be modeled by adding a radiation reaction term to the polarizability $\alpha(\omega)$ in the following way [38]:

$$\frac{1}{\alpha} \rightarrow \frac{1}{\alpha} - i \frac{2}{3} k^3. \quad (9)$$

By taking this term into account, as long as we are in the deep subwavelength regime, no noticeable effect will be observed on the edge states. This term becomes important only for systems operating at higher frequencies. For absorption losses, we have included in Figure 7D moderate amounts

of losses in the Drude model by setting $\gamma/2\pi = 1$ THz. It can be observed that the expected edge state still exists but is attenuated as it propagates along the boundary. Therefore, similar to the previously reported results [20, 39, 40], adding losses to the model only limits the propagation length of the excitations and does not have a detrimental effect on the edge states and their unique properties.

4 Conclusion

To conclude, we proposed a plasmonic PTI based on a discrete array of plasmonic nanoparticles. Emulating the quantum spin Hall phase observed in electronic HgTe quantum wells, this structure supports 1D helical edge states that can transport light confined down to the sub-wavelength scales without back-scattering even at sharp bends. In the coupled dipole description of the system, the frequency of the topological band gap depends only on the dipole resonance frequency of the nanospheres; consequently, the frequency of the edge states is independent of the lattice constant in contrast to similar topological photonic crystals, allowing for tuning the length scale of the structure. In fact, here, the band structure is formed as a result of hybridization of the deep subwavelength localized surface plasmon resonances of individual nanoparticles instead of the Bragg interference of scattering photons. As a planar nanoparticle array, the proposed structure is compatible with well-established integrated nanofabrication technologies. Overall, this structure brings the robustness of PTIs to compact and planar plasmonic systems, paving the way for their application in robust integrated nanophotonic devices.

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