Research article

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Revealing photonic Lorentz force as the microscopic origin of topological photonic states

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Abstract: Charged particles like electrons moving in a magnetic field encounter Lorentz force, which governs the formation of electronic topological edge states in quantum Hall effect systems. Here we show that photons transporting in magneto-optical materials and structures also encounter a physical effect called photonic Lorentz force via the indirect interaction with the magneto-optical medium assisted effective magnetic field. This effect can induce half-cycle spiral motion of light at the surface of a homogeneous metallic magneto-optical medium and inhomogeneous magneto-optical photonic crystals, and it governs the intriguing one-way transport properties of robustness and immunity against defects, disorders, and obstacles. Thus, photonic Lorentz force serves as the fundamental microscopic origin of macroscopic photonic topological states, much the same as classical Lorentz force does to electronic topological states.

Keywords: topological photonic states; microscopic origin; magneto-optical photonic crystals.

1 Introduction

Lorentz force [1], considered as one of the essential foundations of classical electrodynamics, originates from the magnetic field $\vec{B}$ working on a particle of charge q (e.g. electron) moving with a velocity $\vec{v}$. This force results in the drifting or localized cyclotron motions of charges, and stands in the core of a wide range of intriguing

macroscopic and microscopic electromagnetic phenomena such as Hall effect, quantum Hall effects, and many instruments including electron microscopes, cyclotron accelerators, and cosmic ray detectors. Electronic Lorentz force (ELF) is used to draw a comprehensive semi-classical physical picture of the quantum Hall effect (Figure 1A). Electrons inside the quantum Hall effect system undergo localized cyclotron motions rapidly around the magnetic flux due to the ELF effect. Once moving close to the boundary, the electrons will be bounced off the rigid edge and thus skip forward along the edge. The edged electrons are not affected by the impurities and immune to the backscattering and form a topologically protected one-way edge current. This semi-classical picture well discloses the microscopic origin of electronic topological state and qualitatively explains many unique transport behaviors of quantum Hall effect systems [2].

Recently, inspired by the analogy between electrons and photons [3, 4], plenty of studies have been carried out on topological photonic states in different systems [5–15]. An outstanding means to create topological photonic states is a magneto-optical system immersed in a magnetic field [16, 17]. Most of previous works mainly rely on numerical calculations of dispersion diagrams (i.e. band structure) to predict topological photonic states, and the concepts of topological physics and mathematics (e.g. Chern number, Berry phase and curvature, etc.) are borrowed to understand these states [18–26]. However, little of them touch fundamental questions about what causes photons originally radiating from a point source isotropically now only to propagate one way along the edge and why these one-way transporting photons are immune to backscattering, strong defect, and disorder. Therefore, the microscopic origin and physics picture of topological photonic states are still obscure nowadays even after so many theoretical, numerical, and experimental studies have been performed [27–33].

Here, we report that there exists a physical effect that we call photonic Lorentz force (PLF), which causes a cyclotron motion of electromagnetic waves and photons in magneto-optical systems, being much the same as ELF does to electrons. This PLF can well explain the microscopic origin of topological photonic states and help to

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Figure 1: Physical pictures of topological electronic states and topological photonic states.

(A) Quantum Hall effects in the electron gas system. Illustration of the cyclotron motions of electrons. $\vec{v}$ represents electron velocity and $\vec{F}_E$ corresponds to ELF. (B) Topological photonic states in the magneto-optical system. The red and blue arrows indicate the directions of energy flow and PLF, respectively. The green wavy arrows represent the evanescent electromagnetic waves.

draw comprehensive physics picture of their transport behaviors in different magneto-optical devices. As shown in Figure 1B, the energy radiated from a source at the edge is divided into two parts. On the one hand, one part radiates into the magneto-optical medium with attenuated amplitude because of the metallicity of magneto-optical material. Moreover, such electromagnetic wave is deflected to the right- or left-hand side due to the PLF effect, leading to a unidirectional swirling motion of electromagnetic waves within a thin layer near the edge. During the swirling motion, the energy goes into air and most of it is towed back into the magneto-optical medium by the PLF effect. On the other hand, the other part leaks into air because there is no rigid boundary to limit energy radiation. In other words, when energy fluxes propagate to the right- or left-hand side, they are always accompanied with a little energy leaking into air.

2 Transport behaviors of electromagnetic waves

The transport behaviors of electromagnetic waves and photons can be predicted by solving Maxwell's equations, in terms of electromagnetic field and energy flux (Poynting vector). We derive the energy flux expression in the infinite stationary homogeneous magneto-optical medium (yttrium ion garnet whose dielectric constant is ϵ), and the detailed derivation process is supplied in the Supplementary Material. For simplicity, only the transverse electric (TE) mode is considered (with non-zero E_z , H_x , and H_y).

We first consider the case of no external magnetic field applied (i.e. $\mu=1$). According to $\vec{k} \times \vec{E} = \omega \mu \vec{H}$, for a

plane wave transporting in $\vec{k}(k_x, k_y, 0)$, we get $H_x = \frac{E_z}{\omega} k_y$ and $H_y = -\frac{E_z}{\omega} k_x$. Based on the definition of the Poynting vector $\vec{S} = \vec{E} \times \vec{H}^*$, we get

$$\vec{S} = \frac{E_z^2}{\omega} [(k_x \vec{x} + k_y \vec{y})]. \quad (1)$$

Then, according to [34], the momentum of the photon wavepacket can be simplified to be $\vec{P} = \vec{D} \times \vec{B}$ in an infinitely uniform magneto-optical medium, so that

$$\vec{P} = \frac{\epsilon E_z^2}{\omega} [(k_x \vec{x} + k_y \vec{y})]. \quad (2)$$

Further, according to $\vec{F} = \frac{d\vec{P}}{dt}$, we get

$$\vec{F} = 2i\omega e^{2i\omega t} \cdot \frac{\epsilon E_z^2}{\omega} \left[(k_x \vec{x} + k_y \vec{y}) \right]. \quad (3)$$

Equations (1–3) show that $\vec{S}$, $\vec{P}$, and $\vec{F}$ are parallel to the wave vector $\vec{k}$, meaning that the transport behaviors of light are consistent with those in the homogeneous non-magnetic dielectric media.

However, when the magneto-optical medium is immersed into a static magnetic field, strong gyromagnetic anisotropy is induced to produce a permeability tensor. Based on Maxwell's equations for the TE mode, we get

$$\vec{S} = \frac{E_z^2}{\omega} \frac{\mu_r^2}{\mu_r^2 - \mu_k^2} \left[(k_x \vec{x} + k_y \vec{y}) + i \frac{\mu_k}{\mu_r} (k_y \vec{x} - k_x \vec{y}) \right]. \quad (4)$$

$$\vec{P} = \frac{E_z^2}{\omega} \epsilon \mu_r \left[(k_x \vec{x} + k_y \vec{y}) + i \frac{\mu_k}{\mu_r} (k_y \vec{x} - k_x \vec{y}) \right]. \quad (5)$$

$$\vec{F} = 2i\omega e^{2i\omega t} \cdot \frac{E_z^2}{\omega} \epsilon \mu_r \left[(k_x \vec{x} + k_y \vec{y}) + i \frac{\mu_k}{\mu_r} (k_y \vec{x} - k_x \vec{y}) \right]. \quad (6)$$

Obviously, all these Equations (4–6) consist of two parts, which are purely real and imaginary, respectively, i.e. $\vec{S} = \vec{S}_r + i\vec{S}_i$, $\vec{P} = \vec{P}_r + i\vec{P}_i$, and $\vec{F} = \vec{F}_r + i\vec{F}_i$. Simultaneously, because $(k_x \vec{x} + k_y \vec{y}) \cdot (k_y \vec{x} - k_x \vec{y}) = 0$, the purely real and imaginary parts are perpendicular to each other, i.e. $\vec{S}_r \perp \vec{S}_i$, $\vec{P}_r \perp \vec{P}_i$, and $\vec{F}_r \perp \vec{F}_i$. Also, $\vec{S}_r(\vec{S}_i)$ is proportional to $\vec{P}_r(\vec{P}_i)$ and $\vec{F}_r(\vec{F}_i)$, and when removing off the magnetic field [i.e. $H=0$ ($\mu=\mu_r=1, \mu_k=0$)], Equations (4–6) turn back to Equations (1–3), respectively. Therefore, it is perfectly reasonable that we can utilize the transport behaviors of energy fluxes $\vec{S}$ to directly and vividly characterize the transport properties of $\vec{P}$ and $\vec{F}$. Note, however, that the force $\vec{F}$ as defined in Equations (3) and (6) for photon and electromagnetic wave does not have the usual physical meaning of mechanical force for massive particles such as electrons, because all the transport behaviors of photons and electromagnetic waves can be well described by the space-time evolution of electromagnetic fields involved in photons and electromagnetics. Consequently, it is very rare to talk about the mechanical force imposed on photons by something; instead, it is very popular and well established to talk about the mechanical force imposed on massive particles by photons and electromagnetic waves. The PLF adopted and discussed in this paper is merely a useful and convenient conceptual tool developed to describe vividly the transport behavior of TPS photons and electromagnetic waves in magnetic-optical structures and devices, and essentially, it is not a true mechanical force.

Let $Q = \frac{E_z^2}{\omega} \frac{\mu_r^2}{\mu_r^2 - \mu_k^2}$ and $B(H) = \frac{\mu_k}{\mu_r}$; then Equation (4) can be rewritten as

$$\vec{S} = Q[(k_x \vec{x} + k_y \vec{y}) + iB(H)(k_y \vec{x} - k_x \vec{y})]. \quad (7)$$

This equation consists of two parts [$\vec{S}_r = Q(k_x \vec{x} + k_y \vec{y})$ and $\vec{S}_i = QB(H)(k_y \vec{x} - k_x \vec{y})$], and

$$\vec{S}_r \perp \vec{S}_i. \quad (8)$$

We propose that the real part $\vec{S}_r$ corresponds to the usual Poynting vector representing the transport direction of energy fluxes and quantity of light, while the imaginary part $\vec{S}_i$ describes PLF causing the transport behavior of light resembling the cyclotron motions of electrons. This phenomenon originates from the anti-symmetric permeability tensor induced by the magnetic field.

We define $B(H)$ as a function of the magnetic field to illustrate the rationality of comparison between $B(H)$ in PLF and B_z in ELF. In Figure 2, at the frequencies of $f_1 = 4.480$ and $f_2 = 9.464$ GHz, the intensities of $B(H)$ show almost perfect linearity as the magnetic field increases, meaning $B(H) \approx \beta H$, where β is a constant. In this case, Equation (7) is expressed as

$$\vec{S} \approx Q[(k_x \vec{x} + k_y \vec{y}) + i\beta H(k_y \vec{x} - k_x \vec{y})]. \quad (9)$$

It means that PLF is almost proportional to the quantity of H , which is similar to that in the well-known ELF. Besides, we choose $H = 1600$ G and find that $B(H)$ is equal to +1 and −1 at f_1 and f_2 , respectively. These two cases at f_1 and f_2 can be analogous to the positive and negative charge in electric systems, respectively. Then Equation (7) is simplified to be $\vec{S} = Q[(k_x \vec{x} + k_y \vec{y}) \pm i(k_y \vec{x} - k_x \vec{y})]$, indicating that the values of $\vec{S}_r$ and $\vec{S}_i$ are equal, and thus a perfect cycle motion occurs accordingly. It should be noted that we focus only on a narrow frequency range to achieve the linearity for the sake of simplifying the formula to make a better analogy with the electronic system. Actually, the linearity is not a necessary requirement here.

Furthermore, for ELF, we are concerned about a charge (i.e. electron) moving in a two-dimensional x - y plane

Figure 2: Relations of $B(H)$ - H at different frequencies.

Relation of $B(H)$ with H at: (A) $f_1 = 4.480$ GHz, (B) $f_2 = 9.464$ GHz. The black dotted lines are the reference lines.

with a velocity $\vec{v} = (v_x, v_y, 0)$ acted by a magnetic field $\vec{B} = (0, 0, B_z)$ along the z direction. Then magnetic force equation is

$$\vec{F}_E = q\vec{v} \times \vec{B} = q \begin{vmatrix} \vec{x} & \vec{y} & \vec{z} \\ v_x & v_y & 0 \\ 0 & 0 & B_z \end{vmatrix} = qB_z(v_y\vec{x} - v_x\vec{y}), \quad (10)$$

where $\vec{F}_E$ is the ELF and it is perpendicular to $\vec{v} = v_x\vec{x} + v_y\vec{y}$ always because of $(v_x\vec{x} + v_y\vec{y}) \cdot (v_y\vec{x} - v_x\vec{y}) = 0$, so

$$\vec{v} \perp \vec{F}_E. \quad (11)$$

Interestingly, there exists a surprising similarity between Equations (8) and (11), which both describe the cyclotron motions of photons and electrons in the magnetic field, respectively. For brevity, the physical effect named as PLF and written as $\vec{F}_i$ is proportional to the quantity $\vec{S}_i$ and unanimously causes the cyclotron motion of photons. This effect is similar to the classical Lorentz force $\vec{F}_E$, while the energy fluxes characterized by $\vec{S}_r$ is proportional to $\vec{k}$, and it is similar to the velocity $\vec{v}$ of electrons.

Moreover, many classical concepts for ELF can apply to PLF equally well. Here we define a left-handed law to intuitively judge the PLF direction against the transport

direction of energy fluxes. Flatten the left hand, and let the magnetic field pass through the palm. If the four fingers indicate $\vec{S}_r$, the thumb perpendicular to the four fingers represents $\vec{S}_i$. Besides, a positive $B(H)$ corresponds to the thumb pointing to $\vec{S}_i$, while a negative $B(H)$ corresponds to the opposite direction. Nonetheless, a very important difference between PLF and ELF is that $\vec{S}_i$ is “virtual”. Thus, photon does not directly act with the magnetic field, but rather acts with an effective magnetic field created by magneto-optical materials. As photons carry on momentum proportional to $|\vec{k}|$, they will encounter PLF $\vec{F}_p(\vec{F}_i) \propto \vec{S}_i$ and change their transmission trajectory. Their motions are regarded as the superposition of the fast circular motion and the slow drifting motion, which is very similar to that of electrons subjected by ELF.

3 Model verification of photonic Lorentz force

To confirm the above exotic transport properties dominated by PLF, we consider electromagnetic wave radiated from a point source oscillating at $f_1 = 4.480$ GHz

Figure 3: Numerical simulation results in an infinite uniform magneto-optical medium.

(A) E_z -field for $H=0$, (B) E_z -field for $H=1600$ G; the magenta dotted arrow represents the vortex direction of electromagnetic waves. (B1, B2) H_x - and H_y -field corresponding to (B). In all these cases the point source denoted by red shining stars radiates at $f_1 = 4.480$ GHz. The red and blue arrows represent $\vec{S}_r$ and $\vec{S}_i$, respectively.

[lower than the resonance frequency $f_0 = 6.509$ GHz and thus $B(H) = +1$] in an infinite uniform magneto-optical medium. We construct the synthetic picture by superimposing the streamline diagram (reflecting explicitly the energy flow direction) on the E_z -field pattern. Figure 3A shows that when $H=0$, one gets $\vec{S}_i = 0$ and $\vec{F}_p = 0$, so electromagnetic waves radiate uniformly in all directions, and $\vec{S}_r$ are parallel to each $\vec{k}$ point of the wave-front everywhere. Contrarily, Figure 3B shows that when $H=1600$ G, according to the left-handed law, $\vec{S}_r$ is subjected to the right-hand side PLF which is proportional to $\vec{S}_i$ (being perpendicular to $\vec{S}_r$ everywhere) while radiating outward. The PLF causes a rightward deflection of an electromagnetic wave and eventually forms a clockwise vortex electromagnetic field to radiate outward (Figure 3B). This physical picture is further supported by the distributions of H_x and H_y in Figure 3B1 and B2. The above physical pictures never occur in usual uniform nonmagnetic media.

We proceed to examine the case of an interface between the air and magneto-optical medium. Figure 4A1 and A2 show that when $H=0$, most energy radiates into the lower dielectric medium and E_z -field distribution is

symmetrical about the normal of the interface. However, when $H=1600$ G, $\vec{S}_i$ is perpendicular to $\vec{S}_r$ everywhere within the magneto-optical medium, which obeys the left-handed law. Therefore, the electromagnetic wave deflects to the left-hand side due to PLF, leading to an asymmetrical E_z -field distribution. Because of the secondary radiation (or reflection) of energy vortices from the lower magneto-optical medium, its mixture with the direct spherical radiation from the source will cause the asymmetric E_z -field pattern and the presence of both $\vec{S}_r$ and $\vec{S}_i$ in the upper air area near the interface (Figure 4B1). Although $\vec{S}_r$ and $\vec{S}_i$ are no longer perpendicular to each other here, PLF still exists to act upon photons in the vicinity of the interface and cause its deflection to the right-hand side, and this effect can be seen as the residual protrusion of the strong PLF within the magneto-optical medium. To some extent, energy flux still forms a clockwise cyclotron motion of the electromagnetic wave surrounding the source. In the far-field region far above the interface in air, only the direct spherical radiation field survives and $\vec{S}_i$ disappears, leading to a radiation pattern nearly identical to the case of the dielectric medium.

We further examine the case of a single magneto-optical rod. As shown in Figure 5A1 and A2, the rod has a

Figure 4: Numerical simulation results in semi-infinite systems.

(A1) $H=0$, (A2) $|E|$ at different y_i ($i=1, 2, 3, 4, 5$) in (A1), (B1) $H=1600$ G, (B2) $|E|$ of different y_i ($i=1, 2, 3, 4, 5$) in (B1). In all these cases the source radiates at $f_1 = 4.480$ GHz. The red and blue arrows represent $\vec{S}_r$ and $\vec{S}_i$, respectively.

Figure 5: Numerical simulation results.

(A1, A2) Single magneto-optical rod in air at $f_1 = 4.480$ GHz. (B1, B2) Single magneto-optical rod in air at $f_2 = 9.464$ GHz. The magenta arrows represent the transport direction of electromagnetic waves.

diameter of 11.6 cm, larger than the excited wavelength (7 cm), so strong Mie scattering occurs. Figure 5A1 shows that when $H=0$, electromagnetic wave radiation exhibits a regular dipole response, and $\vec{S}_i=0$ everywhere, so that no PLF exists in this nonmagnetic case. When $H=1600$ G (Figure 5A2), the radiation pattern is dramatically different, and a clear quadrupole resonance mode is dominantly excited within the rod. The four poles, together with the whole E_z -field, rotate counterclockwise as time elapses, leading to the vortex-like field distribution and the four-blade windmill-like motion of an electromagnetic wave. These peculiar field transportation characteristics are completely different from the well-known radial quadrupole radiation pattern in the nonmagnetic medium, which are well explained by PLF.

To see what happens when $f > f_0$, we choose $f_2 = 9.464$ GHz [$B(H) = -1$] where the magneto-optical medium shows metallic behavior. The results for a rod diameter of 19.35 cm much larger than the excited wavelength (3 cm) are shown in Figure 5B1 and B2. When $H=0$, the E_z -field exhibits a hexapole shape and radiates uniformly (Figure 5B1). Contrarily, when $H=1600$ G, Figure 5B2 shows a clockwise octapole windmill of energy

vortex rotating along the rod edge with prominent light tails. Different from the case of f_1 [where $B(H) = +1$], now a stronger energy flux is localized along the rod edge, and more interestingly, its rotation direction is inverted from counterclockwise to clockwise since $B(H)$ changes from positive to negative.

The presence and action of PLF in various magneto-optical structures indicate a fruitful frontier of electromagnetics that has not yet been unveiled. Importantly, this fundamental concept of PLF can beautifully reveal the microscopic origin and construct a comprehensive picture of topological photonic states. We further consider an interface between the air and magneto-optical medium at frequency $f > f_0$ [$B(H) < 0$]. As shown in Figure 6A, due to the strong metallic property, energy fluxes can only penetrate a certain depth with attenuation. According to the left-handed law, PLF points to the opposite direction of the thumb, induces a counterclockwise light vortex, and causes an energy flux to deflect to the left-hand side. Thus under the combined action of PLF and metallic property, a one-way edge state is created under the condition of $B(H) = -1$ (Figure 6B). The energy streamline diagram in Figure 6B agrees well with the schematic diagram in Figure 6A.

Figure 6: Topological photonic states in the semi-infinite system at f_2 . (A) Schematic diagram. (B) Simulation results of the E_z -field with $\vec{S}_r$ and $\vec{S}_i$.

4 Formations of topological photonic states

Next, we use the physical concept of PLF to construct the basic physical images for topological photonic states and understand their unique transmission behaviors in much more complicated magneto-optical photonic crystal (MOPC) [16, 17]. As MOPC structures possess much better effective metallicity provided by photonic bandgap, topological photonic states show much lower transport dissipation and loss as compared with the metallic magneto-optical systems in Figure 6. In Figure 7A1, a point source oscillating at $f_3 = 7.765$ GHz ($f_3 > f_0 = 3$ GHz) is located close to the edge of a honeycomb MOPC. The energy is strongly localized at the edge and propagates rightward. Since f_3 locates outside the light cone, the topological photonic state does not couple with air modes and electromagnetic energy is well confined at the zigzag edge to propagate unidirectionally. In this open system here and the air-magneto-optical system in Figure 6, both topological photonic states originate from the combination

Figure 7: Topological photonic states in different MOPCs.

(A1) open honeycomb MOPC, (B1) square MOPC waveguide, and (C1) square MOPC waveguide with an obstacle. The diagrams include the $|E|$ distributions (A1, B1, C1), schematic diagrams (A2, B2, C2), and energy flux distributions (A3, B3, C3). The black arrows represent the transport direction, the blue arrows indicate the direction and intensity of the energy vortex around the magneto-optical rod, and the dark blue layers are the perfect electric conductor. The index 1–4 in (C1) means four critical magneto-optical rods (Rods 1–4) discussed in the main text.

action of effective metallicity and PLF. However, the physical picture of Figure 7A1–A3 is different from that of Figure 6, and we can refer to Figure 5B2 for better understanding this difference. Figure 5B2 shows a single magneto-optical rod with a clockwise windmill energy vortex, while Figure 7A2 shows the collective behaviors of the microscopic vortexes of Figure 5B2, forming the macroscopic one-way transport. There exist two transport channels, i.e. the main and secondary channels. The majority of energy flux transports rightward along the zigzag edge, forming the main channel, while the minority of energy radiating backward is completely towed back to propagate forward via a counterclockwise loop along the inner edges of the whole hexagon (see Figure 7A1), forming the secondary channel.

The square MOPC waveguides can also achieve topological photonic states within the bandgap [16]. However, these topological photonic states are within the light cone to cause light leakage into air, so a waveguide consisting of a square MOPC and a metal cladding constructs to form a lossless channel for topological photonic states. The working frequency is $f_a = 4.3$ GHz ($f_a < f_0 = 6.509$ GHz). According to the left-handed law, as shown in Figure 7B1–B3, the energy flux rotates counterclockwise with a windmill shape around the magneto-optical rods, similar to that in Figure 5B. The physics of the topological photonic states in the one-way waveguide can be described by the physical picture as depicted in Figure 7B2, where the topological photonic state originates from the overall coupling effects of the counterclockwise energy vortex around each rod and the transporting modes in the waveguide. The majority of energy propagates rightward along the main channel, i.e. the waveguide. The energy radiating backward gradually deflects to transport forward and form the secondary channel under the action of PLF and bandgap. Rigorous simulations (Figure 7B1 and B3) completely confirm this physical PLF-based analysis. Besides, the energy radiating into air is reflected back by the metal cladding, and it enters into the main or secondary channel to propagate rightward. Thus, the physical picture delineated by the PLF acts well to explain the presence of topological photonic states in the square MOPC.

Finally, we handle the most intriguing feature of topological photonic states, i.e. their robustness and immunity against backscattering induced by an obstacle. We insert an obstacle into the waveguide (Figure 7B1) and show the simulation result (Figure 7C1). The physical image is depicted (Figure 7C2) based on PLF. The critical step for the robust transport around the obstacle is how the electromagnetic wave passes through the left 90°

sharp corner, climbs across the sharp tip of the obstacle, and passes through the right 90° sharp corner. According to Figure 7C2 and C3, the energy transport still follows the main and secondary channels denoted by the thick and thin red arrows, respectively (Figure 7C2). In the main channel, the incoming wave first transports along the straight waveguide in the form of a topological photonic state (Figure 7B1–B3). When it hits the left sharp corner, the energy flux is strongly scattered by the outermost magneto-optical rod at the corner (Rod 1). Due to the PLF, the major part of the energy flux is scattered downward, towed rightward, hits the surface of the obstacle, and is scattered leftward, upward, and downward simultaneously. The leftward (or downward) energy flux is then towed downward (or rightward) by the PLF once again. The electromagnetic wave repeats such a downward counterclockwise half-cycle spiral motion repeatedly, and forms the main channel of energy transport. Such a spirally skipping major energy flux eventually hits the left magneto-optical rod closest to the lower end of the obstacle metal slice (Rod 2). Then, it continues its half-cycle spiral motion (now rightward) under the action of PLF and hits its neighboring right MO rod (Rod 3). Next, it climbs upward following the half-cycle spiral motion also under the action of PLF, and passes through the right 90° sharp corner and Rod 4. Finally, it transports rightward along the waveguide in the form of a topological photonic state. In Figure 7C1–C3, the main channel essentially follows the trajectory formed by the metal obstacle and its left and right nearest neighboring rows of magneto-optical rods.

The secondary energy flux channel starts from the upward secondary scattering of the electromagnetic wave when the main energy flux passes around Rod 1 and hits the left surface of the obstacle. Due to the PLF effect, this secondary energy flux begins its long trip of half-cycle spiral motion along the trajectory essentially the same as the main channel. In Figure 7C2 and 4C3, another source contributing to the main energy flux comes from the secondary downward scattering wave when the incoming electromagnetic wave hits the rod right before Rod 1. This secondary energy flux pours into the main channel, becomes part of it, and increases its overall intensity. In simple words, the robust transport of the topological photonic state across the metal obstacle originates from the half-cycle spiral skipping motion of the electromagnetic wave under the action of PLF. The physical pictures of cycling motion drawn from Figure 7 perfectly echo those illustrated in Figure 1 for both electrons and photons under the actions of ELF and PLF, respectively.

5 Conclusions

In summary, we have revealed that photons transporting in magneto-optical materials and structures will encounter PLF via the indirect interaction of photons with the effective magnetic field assisted by the magneto-optical medium, similar to the situation for electrons in quantum Hall effect condensed matter systems. This effect can induce half-cycle spiral motion of light at the surface of homogeneous metallic magneto-optical medium and inhomogeneous MOPCs, and govern the intriguing one-way transport properties of robustness and immunity against backscattering induced by defects, disorders, and obstacles. By digging deeply into Maxwell's equations for magneto-optical materials with an external magnetic field, the energy flux of the electromagnetic wave can be divided into the purely real and purely imaginary parts $\vec{S}_r$ and $\vec{S}_i$, which are always perpendicular to each other ($\vec{S}_r \perp \vec{S}_i$) irrespective of the propagation direction of the electromagnetic wave. The imaginary part of the electromagnetic energy flux plays an important physical role in the transport of topological photonic states in the magneto-optical medium. This point has never been noted in classical electrodynamics, electromagnetics, and optics, and is in contrast to conventional wisdom that the imaginary part of electromagnetic energy flux can be neglected. The PLF is an indirect interaction upon photons via the effective magnetic field assisted by the magneto-optical medium, different from the case of classical Lorentz force originated from the magnetic field acting directly on charged particles such as electrons. Furthermore, we define the left-handed law to intuitively judge the direction of PLF against the transport direction of light.

We further deeply explore the physical phenomena in infinite, semi-infinite, single-rod systems, respectively, and construct much clarified microscopic physical images for topological photonic states in different MOPC structures based on the action of PLF. These pictures can well explain all relevant peculiar macroscopic transport properties of light in topological photonic state systems, and further confirm the PLF as being the microscopic origin of these peculiar behaviors. In some sense, the microscopic picture can well match the semi-classical physical picture for electronic topological states in the quantum Hall effect system and the critical role of Lorentz force in forming these topological photonic states. On the other hand, the fundamental similarity and difference between electrons and photons indicate that there exists a closer connection between the charge source (electrons) and radiation (photons) than previous general understanding

as expressed in standard textbooks of classical electromagnetics and electrodynamics. Revealing the physical effect of PLF may offer useful hints to explore deeply consequent nontrivial physical significances in topological photonic systems, and in a broader aspect of science, help to disclose vast new physics such as PLF in the old science discipline of electrodynamics, electromagnetics, and optics.

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