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Study of Weakly nonlinear Mass transport in Newtonian Fluid with Applied Magnetic Field under Concentration/Gravity modulation

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Abstract: In the present paper, a weakly nonlinear stability analysis is used to analyze the effect of time-periodic concentration/gravity modulation on mass transport. We have considered an infinite horizontal fluid layer with constant applied magnetic flux salted from above, subjected to an imposed time-periodic boundary concentration (ITBC) or gravity modulation (ITGM). In the case of ITBC, the concentration gradient between the plates of the fluid layer consists of a steady part and a time-dependent oscillatory part. The concentration of both walls is modulated. In the case of ITGM, the gravity field consists of two parts: a constant part and an externally imposed time periodic part, which can be realized by oscillating the fluid layer. We have expanded the infinitesimal disturbances in terms of power series of an amplitude of modulation, which is assumed to be small. Ginzburg-Landau equation is derived for finding the rate of mass transfer. Effect of various parameters on the mass transport is also discussed. It is found that the mass transport can be controlled by suitably adjusting the frequency and amplitude of modulation.

Keywords: Newtonian fluid, Applied magnetic field, Concentration modulation, Gravity modulation, Ginzburg - Landau equation

1 Introduction

The transport phenomena such as heat and mass transfer, play a vitally important role in human life. Convection and diffusion are responsible for temperature fluctuations and transport of pollution in air, water and soil. The ability to understand, predict and control to the transport phenomena is essential for many industrial applications such as aerodynamic shape design, oil recovery from an underground reservoir or multiphase/multicomponent flows in furnaces, heat exchange, and chemical reactors.

Many researchers have investigated the effect of temperature/gravity modulation on the onset of convection in a viscous fluid layer due to its wide range of applications in industries as well as technological areas such as biotechnology, microelectronics, nanotechnology and polymer science. The subject of mass transfer studies relative motion of some chemical species concerning other, driven by a concentration gradient. The linear theory of mass transfer has three basic features characterized as the mass transfer rate does not depend on the direction of mass exchange between phases, the Sherwood number does not depend on concentration and the mass transfer does not influence the hydrodynamics. The development of mass transfer theory closely follows as in heat transfer with the pioneering works of Lewis and Whitman [1] and Sherwood [2]. Bird *et al.* [3] jointly studies heat transfer, mass transfer, and momentum transfer and considered as a new discipline at that time.

Chandrasekhar [4] and Bejan [5] both provides a comprehensive review of the fundamentals of stability problem with uniform heating in porous media. The effect of modulation on the stability of the flow between rotating cylinder was investigated experimentally by Donnelly [6]. Venezian [7] investigated the thermal analog of Donnelly's experiment, using free-free surfaces and compared his results with them. Rosenblat and Herbert [8] have investigated the linear stability problem for free-free surfaces using low-frequency modulation and found an asymptotic solution. Researchers have solved many problems concerning how a time-dependent boundary temperature af-

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fect the onset of Rayleigh-Bénard convection. Davis [9] has reviewed most of the investigations relevant to these problems. Rosenblat and Tanaka [10] have used Galerkin procedure to study the linear problem for more realistic boundary conditions. Besides investigating the linear stability, Roppo *et al.* [11] have also carried out the weakly nonlinear analysis of the problem. A similar problem has been considered earlier by Gershuni and Zhukhovitskii [12] for a temperature profile obeying the rectangular law. Yih and Li [13] have investigated the formation of convective cells in a fluid between two horizontal rigid boundaries with time-periodic temperature distribution using Floquet theory.

Finucane and Kelly [14] have carried out an analytical experimental investigation on the results of Rosenblat and Herbert [8]. Aniss *et al.* [15] have worked out a linear problem of the convection parametric instability in the case of a Newtonian fluid confined in a Hele-Shaw cell and subjected to the periodic vertical motion. In their asymptotic analysis, they have investigated the influence of the gravity modulation on the instability. Further, Bhatia and Bhadauria [16, 17] and Bhadauria and Bhatia [18] have studied the linear stability problem for sawtooth, step-function, and day-night profiles. Recently, Bhadauria *et al.* [19] have investigated the local nonlinear stability analysis of modulated double-diffusive stationary convection for coupled stress liquid. Kumar *et al.* [20] have shown the effect of gravity modulation on double diffusive convection of couple stress liquid and found that on increasing Pr , Ra_S , Le and Couple stress parameter increases the rate of heat and mass transfer.

The first investigation on the effect of gravity modulation of fluid layer heated from below was reported by Gershuni and Sani [21]. Gershuni *et al.* [22] and Kamotini *et al.* [23] have applied a perturbation technique to analyze horizontal gravity modulation phenomena. Biringen and his coworkers [24, 25] have used the finite difference method to study the gravity modulation effect in 2D and 3D rectangular cavities under normal-gravity and zero gravity conditions. Experiments on the response of Rayleigh-Bénard convection to gravity modulation were carried out by Roggers *et al.* [26]. Wadih and Roux [27] presented a study on the instability of the convection in an infinitely long cylinder with gravity modulation oscillating along the vertical axis. Clever *et al.* [28] used Galerkin method after different range of parameters for the oscillatory convection in a gravitationally modulated fluid layer.

All the above study did not describe the effect of magnetic field. Although the magnetic field has the property that controls the flow formation externally and uses in many experimental and scientific types of equipment. Many re-

searchers considered the study of flow formation in different boundary condition under the inflation of magnetic field [29–37]. Srivastava *et al.* [38] investigate Soret effect on magneto-convection fluid flow in an anisotropic porous layer. Gupta *et al.* [39] also reported the effect of magnetic field on chaos in couple stress liquid in the porous layer. Swamy [40] have examined the effect of gravity modulation on dielectric fluid. Recently, Gupta *et al.* [41] and Gupta [42] presented the weakly nonlinear study of mass transport in a rotating fluid layer in presence of concentration modulation.

To the best of authors' knowledge, till date, there is no nonlinear study available which performed the effect of concentration modulation in presence of a magnetic field. It is with these motives that a weakly nonlinear analysis in an infinite horizontal fluid layer with the applied magnetic field has been made under concentration/ gravity modulation. Using the Ginzburg – Landau equation the effect of an applied magnetic field on mass transport is also studied. In this paper, we have investigated the effect of concentration/ gravity modulation on the convective mass transport of viscous incompressible Newtonian fluid between two horizontal parallel plates. The amplitude and frequency of the concentration and gravity modulations are externally controlled parameters, and hence the rate of mass transfer can be delayed or advanced by proper tuning of these parameters. The problem has potential application in achieving major enhancement of concentration and momentum in the material processing and related areas.

2 Mathematical Formulation

We consider viscous, incompressible and electrically conducting fluid confined between two parallel infinite horizontal plates $z = 0$ and $z = d$ at a distance d apart salted from above. We have chosen a Cartesian frame of reference in which the origin lies on the lower plate and the z -axis vertically upwards. In the present study, we have not considered the interface deformation, and the densities of the fluid and melt are assumed equal so that convection due to density variation neglected and only solutal convection taken in account. Coriell *et al.* [43] and Hurlé *et al.* [44] reported some conditions for unmodulated problem under which the effect of thermally driven convection and interface deformation are negligible. Wheeler *et al.* [45] have considered the Navier- Stokes equations and conservation equation for solutal convection in their study. They have neglected the convection due to a density change and considered only solutal convection. The Boussinesq approxi-

mation is employed which assumes that the density varies linearly with solute concentration and that is included only in body force term in the Navier-Stokes equations. A uniform magnetic field of strength B_0 is applied perpendicular to both the plates. Under these assumptions the governing equations can be written as (Gupta *et al.* [39], Gupta *et al.* [41] and Wheeler *et al.* [45]):

$$\nabla \cdot \vec{q} = 0 \quad (1)$$

$$\rho_0 \left[\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla) \vec{q} \right] = -\nabla p + \rho \vec{g} + \mu \nabla^2 \vec{q} - \sigma \mu_e^2 B_0^2 \vec{q} \quad (2)$$

$$\frac{\partial S}{\partial t} + (\vec{q} \cdot \nabla) S = \kappa_s \nabla^2 S \quad (3)$$

$$\rho = \rho_0 [1 + \beta_s (S - S_0)] \quad (4)$$

where $\vec{q}$ is the velocity, p the pressure, ρ is the density, g is the gravitational acceleration, μ is the viscosity, σ is the electrical conductivity, μ_e is magnetic permeability, B_0 is the strength of applied magnetic field, S is the solute concentration, κ_s is the mass diffusivity of the fluid, β_s is the mass expansion coefficient.

2.1 Time-periodic Concentration Modulation

In this section, we present the concentration modulation problem. The external driving force is modulated harmonically in time by varying the concentrations of lower and upper horizontal boundary. Accordingly, we have considered the concentration modulation at the boundary

$$S = S_0 - \frac{\Delta S}{2} \left[1 - \epsilon^2 \delta_1 \cos(\Omega t) \right] \quad \text{at} \quad z = 0 \quad (5)$$

$$S = S_0 + \frac{\Delta S}{2} \left[1 + \epsilon^2 \delta_1 \cos(\Omega t + \varphi) \right] \quad \text{at} \quad z = d \quad (6)$$

where ϵ represent small amplitude of modulation, Ω the frequency of modulation, φ the phase angle. We consider three types of concentration modulation, viz.,
Case(a) : symmetry (in-phase modulation (IPM), $\varphi = 0$),
Case(b) : asymmetry (out-phase modulation (OPM), $\varphi = \pi$) and
Case(c) : only upper wall concentration is modulated while the lower one is held at constant concentration (UPM) ($\varphi = -i \propto$)

2.2 Basic State

The basic state of the fluid is quiescent and is given by

$$\begin{aligned} \vec{q}_b &= (0, 0, 0), \quad p = p_b(z, t), \quad S = S_b(z, t), \\ \text{and} \quad \rho &= \rho_b(z, t) \end{aligned} \quad (7)$$

Which satisfy the following equations

$$\frac{\partial S_b}{\partial t} = \kappa_s \frac{\partial^2 S_b}{\partial z^2} \quad (8)$$

$$\frac{\partial p_b}{\partial z} = \rho_b g \quad (9)$$

and pressure $p_b(z, t)$ balances the buoyancy force.

The solution of Eq. (8) subject to the boundary conditions (5) and (6) is given as

$$\frac{\partial S_b}{\partial z} = \frac{\Delta S}{d} \left[1 + \epsilon^2 \delta_1 f(z, t) \right] \quad (10)$$

where

$$\begin{aligned} A(\lambda) &= \frac{\lambda}{2} \left(\frac{e^{-i\varphi} - e^{-\lambda}}{e^{\lambda} - e^{-\lambda}} \right), \quad \lambda^2 = \frac{-i\Omega d^2}{\kappa_s}, \\ f(z, t) &= \text{Re} \left[\left(A(\lambda) e^{\frac{\lambda z}{d}} + A(-\lambda) e^{-\frac{\lambda z}{d}} \right) e^{-i\Omega t} \right] \end{aligned}$$

and Re stand for real part.

Now, we consider the infinitesimal perturbations techniques to the basic state solution in the form

$$\vec{q} = \vec{q}_b + \vec{q}', \quad S = S_b + S', \quad p = p_b + p' \quad \text{and} \quad \rho = \rho_b + \rho' \quad (11)$$

where prime indicates that the quantities at perturbed state.

Using Eq. (11) into Eq. (1)-(4) and using the basic state solution, we obtained the governing equations in perturbed form as follows

$$\rho_0 \left[\frac{\partial \vec{q}'}{\partial t} + (\vec{q}' \cdot \nabla) \vec{q}' \right] = -\nabla p' + \rho' \vec{g} + \mu \nabla^2 \vec{q}' - \sigma \mu_e^2 B_0^2 \vec{q}' \quad (12)$$

$$\left[\frac{\partial S'}{\partial t} + (\vec{q}' \cdot \nabla) S' \right] + w' \frac{\partial S_b}{\partial z} = \kappa_s \nabla^2 S' \quad (13)$$

$$\rho' = \rho_0 \beta_s S' \quad (14)$$

In order to eliminate the pressure term from Eq. (12), we take the curl of Eq. (12). Since, We are assuming all physical quantities to be independent of y , due to the study being restricted to rolls with the y -coordinate as their axis, introducing the stream function as $u = \frac{\partial \psi}{\partial z}$, $w = -\frac{\partial \psi}{\partial x}$. Eqs. (12)-(13) may be written in dimensionless form by using following non-dimensional quantities

$$(x, y, z) = (x^*, y^*, z^*)d, \quad t = (t^*) \frac{d^2}{\kappa_s}, \quad (S) = S^* \Delta S$$

$$\text{and} \quad \psi = (\psi^*) \kappa_s,$$

as (for simplicity we drop the asterisks):

$$\frac{1}{Sc} \frac{\partial}{\partial t} (\nabla^2 \psi) = Ras \frac{\partial S}{\partial x} + \nabla^4 \psi - Ha^2 \nabla^2 \psi + \frac{1}{Sc} \frac{\partial(\psi, \nabla^2 \psi)}{\partial(x, z)} \quad (15)$$

$$\frac{\partial S}{\partial t} - (1 + f\epsilon^2 \delta_1) \frac{\partial \psi}{\partial x} = \nabla^2 S + \frac{\partial(\psi, S)}{\partial(x, z)} \quad (16)$$

where $f = Re \left[A(\lambda) e^{\lambda z} + A(-\lambda) e^{-\lambda z} \right] e^{-i\omega t}$.

$Sc = \frac{\nu}{\kappa_s}$ is the Schmidt number (ratio of diffusion in momentum to diffusion in solute), $Ras = \frac{\alpha_s g(\Delta S) d^3}{\nu \kappa_s}$ is the Solute Rayleigh number, $Ha = \mu_e B_0 d \sqrt{\frac{\sigma}{\nu}}$ is the Hartmann number.

3 Non-linear Stability Analysis

we will consider the slow time variations at the scale $\tau = \epsilon^2 t$, then Eq. (15) and (16) becomes

$$\left[\frac{\epsilon^2}{Sc} \frac{\partial}{\partial \tau} (\nabla^2) - \nabla^4 + Ha^2 \nabla^2 \right] \psi - Ras \frac{\partial S}{\partial x} = \frac{1}{Sc} \frac{\partial(\psi, \nabla^2 \psi)}{\partial(x, z)} \quad (17)$$

$$-(1 + f\epsilon^2 \delta_1) \frac{\partial \psi}{\partial x} + \left[\epsilon^2 \frac{\partial}{\partial \tau} - \nabla^2 \right] S = \frac{\partial(\psi, S)}{\partial(x, z)} \quad (18)$$

Eqs. (17) and (18) can be written in matrix form as

$$\begin{bmatrix} \frac{\epsilon^2}{Sc} \frac{\partial}{\partial \tau} (\nabla^2) - \nabla^4 + Ha^2 \nabla^2 & -Ras \frac{\partial}{\partial x} \\ -(1 + f\epsilon^2 \delta_1) \frac{\partial}{\partial x} & \epsilon^2 \frac{\partial}{\partial \tau} - \nabla^2 \end{bmatrix} \begin{bmatrix} \psi \\ S \end{bmatrix} = \begin{bmatrix} \frac{1}{Sc} \frac{\partial(\psi, \nabla^2 \psi)}{\partial(x, z)} \\ \frac{\partial(\psi, S)}{\partial(x, z)} \end{bmatrix} \quad (19)$$

We introduce a small perturbation parameter ϵ that shows deviation from the critical state of convective mass transfer, then the variable for a weakly nonlinear state may be expanded as power series of ϵ as in Eq. (19)

$$\left. \begin{aligned} \psi &= \epsilon \psi_1 + \epsilon^2 \psi_2 + \dots \\ S &= \epsilon S_1 + \epsilon^2 S_2 + \dots \\ Ras &= Ras_0 + \epsilon^2 Ras_2 + \dots \end{aligned} \right\} \quad (20)$$

where Ras_0 is the critical solute Rayleigh number at which convective mass transfer sets without modulation, ϵ is the expansion parameter.

Now substituting Eq. (20) in Eq. (19) and comparing like powers of ϵ on both sides we get solution of different orders:

Solution for the first order case, we have

$$\begin{bmatrix} -\nabla^4 + Ha^2 \nabla^2 & -Ras_0 \frac{\partial}{\partial x} \\ -\frac{\partial}{\partial x} & -\nabla^2 \end{bmatrix} \begin{bmatrix} \psi_1 \\ S_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (21)$$

The solution of the lowest order system, can be written as

$$\left. \begin{aligned} \psi_1 &= A(\tau) \sin(\kappa_c x) \cdot \sin(\pi z) \\ S_1 &= \frac{\kappa_c}{\delta^2} A(\tau) \cos(\kappa_c x) \cdot \sin(\pi z) \end{aligned} \right\} \quad (22)$$

where $\delta^2 = \kappa_c^2 + \pi^2$ is the total wave number. The expression of the critical solute Rayleigh number can be obtained from Eq. (21) and is given by

$$Ras_0 = \frac{\delta^6 + Ha^2 \delta^4}{\kappa_c^2} \quad (23)$$

which is a similar result obtained by Gupta *et al.* [39] analogous to thermal convection for Couple stress parameter $C = 0$. When $Ha = 0$, it recovers the result obtained by Chandrasekhar [4] analogous to thermal convection for Chandrasekhar number $Q = 0$.

3.1 Amplitude Equation (Ginzburg-Landau equation for mass transport of stationary Instability)

At second order, we have

$$\begin{bmatrix} -\nabla^4 + Ha^2 \nabla^2 & -Ras_0 \frac{\partial}{\partial x} \\ -\frac{\partial}{\partial x} & -\nabla^2 \end{bmatrix} \begin{bmatrix} \psi_2 \\ S_2 \end{bmatrix} = \begin{bmatrix} R_{21} \\ R_{22} \end{bmatrix} \quad (24)$$

where

$$R_{21} = \frac{1}{Sc} \left[\frac{\partial \psi_1}{\partial x} \frac{\partial}{\partial z} (\nabla^2 \psi_1) - \frac{\partial \psi_1}{\partial z} \frac{\partial}{\partial x} (\nabla^2 \psi_1) \right] \quad (25)$$

$$R_{22} = \frac{\partial \psi_1}{\partial x} \frac{\partial S_1}{\partial z} - \frac{\partial \psi_1}{\partial z} \frac{\partial S_1}{\partial x} \quad (26)$$

The solution of the second order system can be written as

$$\left. \begin{aligned} \psi_2 &= 0 \\ S_2 &= \frac{\kappa_c^2}{8\pi\delta^2} A^2(\tau) \sin(2\pi z) \end{aligned} \right\} \quad (27)$$

and we get

$$\left. \begin{aligned} R_{21} &= 0 \\ R_{22} &= \frac{\kappa_c^2 A^2(\tau)}{2\delta^2} \pi \sin(2\pi z) \end{aligned} \right\} \quad (28)$$

Solution for the third order case we have

$$\begin{bmatrix} -\nabla^4 + Ha^2 \nabla^2 & -Ras_0 \frac{\partial}{\partial x} \\ -\frac{\partial}{\partial x} & -\nabla^2 \end{bmatrix} \begin{bmatrix} \psi_3 \\ S_3 \end{bmatrix} = \begin{bmatrix} R_{31} \\ R_{32} \end{bmatrix}$$

where,

$$R_{31} = \left[\frac{\delta^2}{Sc} \frac{dA(\tau)}{d\tau} - \frac{\kappa_c^2}{\delta^2} A(\tau) Ras_2 \right] \sin(\kappa_c x) \sin(\pi z) \quad (29)$$

and

$$\begin{aligned} R_{32} &= \left[-\frac{\kappa_c}{\delta^2} \frac{dA(\tau)}{d\tau} + \frac{\kappa_c^3}{4\delta^2} A^3(\tau) \cos(2\pi z) + \delta_1 \kappa_c f A(\tau) \right] \\ &\quad \cos(\kappa_c x) \sin(\pi z) \end{aligned} \quad (30)$$

Now, using the solvability condition for the third order solution and we get Ginzburg-Landau equation for stationary instability in the form

$$\left(\frac{1}{Sc(Ha^2 + \delta^2)} + \frac{1}{\delta^2} \right) \frac{dA(\tau)}{d\tau} - F(\tau)A(\tau) + \frac{\kappa_c^2}{8\delta^2} A^3(\tau) = 0 \quad (31)$$

where $F(\tau) = R^* + 2\delta_1 F(z, t)$ when $F(z, t) = \int_0^1 f \sin^2(\pi z) dz$ and $R^* = \frac{Ras_2}{Ras_0}$.

The horizontal local Sherwood number Sh (which measures rate of mass transport) is given by

$$Sh = 1 + \frac{\kappa_c^2}{4\delta^2} A^2(\tau) \quad (32)$$

4 Analytical solution for unmodulated case

In the case of unmodulated system, the Ginzburg-Landau equation obtained in Eq. (31) can be written as

$$A_1 A'_u(\tau) - A_2 A_u(\tau) + A_3 A_u^3(\tau) = 0, \quad (33)$$

where $A_u(\tau)$ is amplitude of convective mass transport for unmodulated case. The coefficients A_1 , A_2 and A_3 are given by

$$A_1 = \frac{1}{Sc(Ha^2 + \delta^2)} + \frac{1}{\delta^2}, \quad A_2 = \frac{Ras_2}{Ras_0} \quad \text{and} \quad A_3 = \frac{\kappa_c^2}{8\delta^2}.$$

The solution of Eq. (33) is given by

$$A_u(\tau) = \frac{1}{\sqrt{\frac{A_2}{2A_3} + C_1 \exp\left[\frac{-2A_2}{A_1}\right]}}, \quad (34)$$

where C_1 is a parameter and can be obtained by suitable initial conditions. The horizontal local Sherwood number for unmodulated case can be obtained by using the value of $A_u(\tau)$ in the place of $A(\tau)$ in Eq. (32).

5 Time-periodic Gravity Modulation

In this section we study the effect of Gravity modulation on the onset convection under the influence of time-periodically varying gravity force $g \equiv (0, 0, -g(t))$ acting on it, where $g(t) = g_0(1 + \epsilon^2 \delta_2 \cos \Omega t)$ with g_0 the constant gravity in an otherwise unmodulated system, ϵ the small amplitude of modulation, ω the frequency of modulation and τ the time. A uniform adverse concentration gradient $\Delta S = (S_l - S_u)$ where $S_l < S_u$ is maintained between the lower and upper boundary.

At the third order solution, we obtained the following ex-

pressions as obtained in Eqs. (29) and (30)

$$R_{31} = \left[\frac{\delta^2}{Sc} \frac{dA(\tau)}{d\tau} - [Ras_2 + Ras_0 \delta_2 \cos(\omega\tau)] \frac{\kappa_c^2}{\delta^2} \right] \sin(\kappa_c x) \sin(\pi z) \quad (35)$$

$$R_{32} = \left[-\frac{\kappa_c}{\delta^2} \frac{dA(\tau)}{d\tau} + \frac{\kappa_c^3}{4\delta^2} A^3(\tau) \cos(2\pi z) \right] \cos(\kappa_c x) \sin(\pi z) \quad (36)$$

Now by Eqs. (29), (30), (35), (36) and solve, we get Ginzburg-Landau equation for stationary instability in the form

$$\left(\frac{1}{Sc(Ha^2 + \delta^2)} - \frac{1}{\delta^2} \right) \frac{dA(\tau)}{d\tau} - F(\tau)A(\tau) + \frac{\kappa_c^2}{8\delta^2} A^3(\tau) = 0 \quad (37)$$

where $F(\tau) = R^* + \delta_2 \cos(\omega\tau)$

6 Results and Discussion

In this paper, we study the combined effect of concentration and gravity modulation on mass transport of a Newtonian fluid with an applied magnetic field. A weakly nonlinear stability analysis has been performed to investigate the effect of concentration/gravity modulation on mass transport. The effect of modulations on the system has been assumed to be of order $O(\epsilon^2)$. Means, we consider only small amplitude modulations. Such assumption will help us in obtaining the amplitude equation of diffusion in rather simple and elegant manner and is much easier to obtain than in the case of the Lorenz model (Chaotic system). It is important to make nonlinear stability analysis if one wants to study mass transport because linear stability analysis does not provide any information about mass transport. Linear stability analysis established the stability of the system for different parameters appeared in the model.

The parameters that appears in present study and influence the mass transport are Sc , Ha , δ_1 , δ_2 and ω . The first is related to the property of the fluid and rest concern the external mechanism that is Hartmann number (Ha), concentration and gravity modulation for controlling the mass transport. The values of amplitude of modulations (δ_1 and δ_2 corresponding to concentration and gravity modulation respectively) are considered to be between 0 and 0.8 since we are studying the effect of small amplitude modulation on the mass transport. Further, as the effect of low frequencies on diffusion in solute concentration as well as the mass transport is maximum; therefore the modulations of concentration and gravity are assumed to be of low frequency. For all numerical computations the common value of the parameters are taken as

$\delta_1 = 0.5$, $\delta_2 = 0.5$, $\omega = 2$, $Ha = 2$ and $Sc = 0.05$ (In present paper, we have assumed that diffusion of solute concentration is high compared to diffusion in linear momentum, so the range of Sc has been chosen to be less than 1.).

The effect of concentration modulation on mass transport is depicted in Figs. 1-3, where graphs for the Sherwood number Sh are drawn with respect to slow time τ . From these figures, it found that the effect of concentration modulation on mass transport is destabilizing and so that Sherwood number is more in the modulated case than in the absence of modulation. In Fig. 1, the variation of Sherwood number Sh with slow time performed for in-phase modulation (IPM) at different values of chosen parameters. In Fig. 1(a) and 1(b), we have shown the effect of amplitude of concentration modulation (δ_1) and frequency of concentration modulation (ω) respectively on Sherwood number (Sh). We can see from the Fig.1(a) and 1(b) that on increasing the value of δ_1 and ω , the value of Sh does not change, it means that amplitude of modulation (δ_1) and frequency of modulation (ω) has negligible effect on mass transport, which is a similar result obtained by Gupta *et al.* [41] and Gupta [42] for frequency of modulation and amplitude of modulation in case of in-phase concentration modulation. From Fig.1(c) and 1(d), we examine the effect of Hartmann number (Ha) and Schmidt number (Sc) on Sherwood number (Sh) respectively. It observed that diffusion takes place more rapidly for increasing value of Hartmann number as well as for Schmidt number. However, the effect of Ha and Sc on mass transport diminish at higher values of time τ .

Fig. 2 displays the effect of chosen parameters on mass transport for out-phase modulation (OPM). In Fig.2(a), we examine the effect of amplitude of concentration modulation (δ_1) on Sherwood number (Sh). We observe that on the increase the value of an amplitude of concentration modulation (δ_1) increases the value of Sherwood number (Sh), which indicates that diffusion occurs and hence the rate of mass transfer increases. It means that effect of an amplitude of modulation on mass transport is destabilizing, a similar result investigated by Gupta *et al.* [41] for out phase concentration modulation. Effect of frequency of concentration modulation (ω) on the system is stabilizing (see fig.2(b)) as mass transport decreases on increasing the value of the frequency of concentration modulation. From Fig. 2(c) illustrate the effect of Hartmann number on mass transport rate in the case of out-phase modulation (OPM) while other parameters at their common values. As Ha increase the values of Sh increases. We know that Lorentz force opposes the fluid motion but we find that mass transport is still in progress because of the concentration at upper plate is much heavier than the lower plate. The effect

Fig. 1: Variation of Sherwood number Sh with time τ for in phase concentration modulation for different value of (a) δ_1 , (b) ω , (c) Ha , (d) Sc .

Fig. 2: Variation of Sherwood number Sh with time τ for out phase concentration modulation for different value of (a) δ_1 , (b) ω , (c) Ha , (d) Sc .

Fig. 3: Variation of Sherwood number Sh with time τ for only upper wall concentration is modulated while the lower one is held at constant concentration ($\varphi = -i \infty$) for different value of (a) δ_1 , (b) ω , (c) Ha , (d) Sc .

Fig. 4: Variation of Sherwood number Sh with time τ for gravity modulation for different value of (a) δ_2 , (b) ω , (c) Ha , (d) Sc .

of Schmidt number (Sc) on Sherwood number (Sh) in the case of out-phase modulation (OPM) while other parameters at their common values is illustrated in Fig. 2(d). With an increase in Sc from 0.01 through 0.05 to 0.2, there is clear boosts in Sherwood number i.e. mass transport is strongly accelerated. It is evident that the increase in Sc concentration diffusivity decreases or kinematic viscosity increases in both the cases linear momentum increases and hence mass transport increases. From figs. 2(c) and 2(d), indicates that the effect of these parameters is to advance the diffusion, i.e., a rate of mass transfer increases. Hence, Hartmann number Ha and Schmidt number Sc has destabilizing effect for out phase modulation.

Fig. 3 depict the fact that rate of mass transport (Sh) in the case of upper plate modulation (UPM) variations with δ_1 , ω , Ha and Sc are similar to what was found in Figs. 2(a)-(d). The results on Sh in Figs. 2 and 3 are as

$$Sh^{IPM} < Sh^{UPM} < Sh^{OPM}$$

Fig.4 describes the effect of gravity modulation for different parameters on mass transport rate. From Fig. 4(a), we have established the effect of an amplitude of gravity modulation δ_2 on the rate of mass transfer (Sh). It found that amplitude of modulation δ_2 has a destabilizing effect on the system as mass transport increases on increasing value of an amplitude of modulation. Fig. 4(b) shows that rate of mass transfer (Sh) decreases when we increase the value of the frequency of the gravity modulation (ω), i.e., a frequency of gravity modulation has to stabilize effect on the system. In Fig. 4(c), we depict the variation of Sh for different values of Ha . It shows that diffusion takes place more rapidly with time when we increase the value of Hartmann number, and also it increases the rate of mass transfer. So, Hartmann number has the destabilizing effect. From Fig. 4(d), we found that increasing value of Schmidt number (Sc) increases the rate of mass transport and hence Sc destabilizes the system, a similar result were found by Gupta *et al.* [41].

7 Conclusions

In this paper, we demonstrated weakly nonlinear analysis of mass transport by perturbation technique to obtain a closed form expression for the problem of weakly nonlinear mass transport of the Newtonian fluid of an incompressible and electrically conducting viscous fluid within two infinite horizontal fluid layer. A uniform magnetic field applied perpendicular to both the plates and salted from above with subjected to an imposed

time-periodic boundary concentration (ITBC) or gravity modulation (ITGM). The results obtained in this study are as follows:

1. The Hartmann number and the Schmidt number both have destabilized effect on the system.
2. The effect of increasing value of ω is to decrease the value Sherwood number.
3. The effect of increasing amplitude of modulation is to increase the value of mass transfer in both cases (concentration modulation and gravity modulation).

The results of this work for concentration modulation and gravity modulation can be summarized from the Figs. 1-4 as follows.

1. $Sh^{IPM} < Sh^{UPM} < Sh^{OPM}$
2. $Sh/\delta_{1,2}=0 < Sh/\delta_{1,2}=0.2 < Sh/\delta_{1,2}=0.5 < Sh/\delta_{1,2}=0.8$
3. $Sh/\omega=1 > Sh/\omega=2 > Sh/\omega=3 > Sh/\omega=4$
4. $Sh/Ha=0 < Sh/Ha=2 < Sh/Ha=5 < Sh/Ha=10$
5. $Sh/Sc=0.01 < Sh/Sc=0.05 < Sh/Sc=0.2 < Sh/Sc=0.5$

Note

Conflict of Interest: The authors declare that they have no conflict of interest.

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Nomenclature

Latin Symbols	
A	Amplitude of stream function
B_0	Applied magnetic field
g	Gravitational acceleration
g_0	Reference value of gravitational acceleration
Ha	Hartman number
p	Atmospheric pressure
$\vec{q}$	Velocity field ($u, 0, w$)
Ras	Solute Rayleigh number
S	Solute Concentration
Sc	Schmidt number
Sh	Sherwood number
t	Time
Greek Symbols	
β_s	Coefficient of concentration expansion
κ_s	Concentration diffusivity
δ_1	Amplitude of concentration modulation

δ_2	Amplitude of gravity modulation
ϵ	Small amplitude of modulation
φ	Phase angle
ρ_0	Characteristic density
ρ	Density
κ_c	Wave number
μ	Dynamic viscosity
μ_e	Magnetic permeability
ν	Kinematic viscosity
σ	Electrical conductivity
ω	Frequency of modulation
λ	Wavelength
ψ	Stream function
Other Symbols	
b	Basic state
$*$	Non-dimensional parameter

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