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MHD slip flow of chemically reacting UCM fluid through a dilating channel with heat source/sink

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Abstract: The present article explores the effects of uniform heat source and first order destructive chemical reaction on an upper convected Maxwell fluid through an expanding or contracting channel with the porous slip condition at the upper plate. It is assumed that the fluid is sucked or injected through the upper plate. The temperature and concentration at the plates is maintained constant. Using suitable similarity transformations, nonlinear coupled ODEs are developed from the governing PDEs. The subsequent ODEs are converted into a first order system and integrated via shooting method. The effect of various prominent parameters on heat, flow and mass transfer characteristics are studied in detail through graphs and tables. The present results suggest that the presence of chemical reaction and heat source yields in the reduction of concentration and of the enhancement of temperature the fluid. It is also observed that the wall expansion shows an increasing effect on the radial velocity component, but the slip parameter exhibits an opposing effect. The viscous case has been studied as a special case where the present results are found to be close to the earlier ones. The flow of such nonlinear viscoelastic fluids has important applications in separation processes like petroleum and medical industries.

Keywords: Upper convected Maxwell fluid, porous slip, mixed convection, shooting method, chemical reaction

1 Introduction

The expanding and contracting boundaries have applications in many practical situations such as of filtration, the regression of grains in the course of combustion of solid propellant, binary gas diffusion, artificial dialysis, the flow of blood, circulation of blood and air during the respiration process, separation processes of other membranes. Berman [1] considered the 2-D flow of a Newtonian fluid through stationary walls with same permeability and obtained an analytical solution. This study has been extended by Majadinali et al. [2] by coupling the injection and suction with wall motion. He studied the role of expansion wall ratio of a viscous laminar flow between two weakly permeable surfaces with same permeability which enables the fluid to flow in the course of expansion or contraction. The solutions were obtained by using both analytical and numerical procedures and the results were compared graphically. Majadinali and Zhou [3] extended the problem for large and moderate suction or injection flows in contracting or expanding channels with a membrane closing one end of the channel. For the same problem, Boutros et al. [4] developed Lie group solutions for Re, wall expansion and compared these solutions with the numerical solutions. Asghar et al. [5] developed an exact solution by using the laws of conservation and symmetry methods to reduce to the PDE to second order ODE and hence find solutions to the problems like Majadinali et al. [2]. Later, Asghar et al. [6] accomplished a solution with Adomain Decomposition Method for small Reynolds number and wall expansion ratios and compared the results with [2] and [4]. Hence a wide range of solution procedures were developed. In many metallurgical and chemical engineering processes 1st order chemical reaction has an important role, Subramanyam et al. [7] employed a parameter to study the generative and destructive chemical reactions in expanding porous channels. The presence of temperature and concentration gradients in the channel induces the convection process in the channel. So authors [8–17] have considered the combined effects of chemical reactions like autocatalysis and porous slip conditions along with mixed convection to understand the flow properties.

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The flow of non-Newtonian fluids have attracted many researchers as most of the real world fluids are non-Newtonian. A large number of models were developed to analyze the properties like viscoelasticity, time independent and time dependent viscosities. Jambal et al. [18] studied the heat transfer and flow in circular ducts and parallel plates for a power law fluid by taking both dissipation due to the viscosity and the heat conduction of the fluid and analyzed the flow characteristics by using finite difference method. Odelu and Naresh [19] have studied the chemical reaction effects on a micropolar fluid confined in between an expanding or contracting porous channel. Chen and Zhu [20] have obtained an analytical solution for Couette-Poiseuille flow of Bingham fluids confined in a porous channel with velocity slip on the boundaries. Srinivasacharya et al. [21] obtained the profiles of velocity and temperature distributions for a streamlined flow of a couple stress fluid in porous expanding or contracting channel by employing quasilinearization method. Makinde and Chinyoka [22] considered a dusty fluid to study the transient MHD flow and heat transfer in a parallel channel with the Navier slip condition at the boundaries and obtained solution by using semi implicit finite difference scheme. Abbas and Sheikh [23] studied the effects of homogenous - heterogeneous reactions on a ferrofluid with flow above a flat plate with a stagnation point by assuming a slip condition on the boundary. They followed the non-linear slip condition suggested by Thompson and Troian [24] which is a function of shear rate at the boundary and their results revealed that the velocity and concentration increase with slip parameters. HAM solutions for a micropolar fluid between expanding/contracting porous plates with different permeabilities has been obtained by Si et al. [25]. Later, Xinhui et al. [26] extended the problem by considering the slip velocity based on Beavers and Joseph slip condition [27] but with an assumption of equal permeability at the plates. Shojaeian and Ali [28] studied the entropy generation and convective heat transfer of non-Newtonian and Newtonian fluids confined in between parallel plates by considering boundary conditions due to slip effect. Ahmed et al. [29] have analyzed the laminar nanofluid flow through the dilating permeable channel and used similarity transformations to reduce the flow field equation into non-linear ODE. The solution has been obtained by using Galarkin, Runge Kutta Fehlberg schemes and a comparison between the methods has been presented.

A systematic framework for the rheological behaviour of rate type of fluids has been developed by Oldroyd [30] and he later studied some steady flows of general elastic viscous fluids [31]. Zahorski [32], in his book “Mechanics

of Viscoelastic fluids”, has discussed the tensor theoretical representations along with rheological experimental studies. He also explained the fundamentals of mechanical principles of viscoelastic fluids and its relation to continuum in modern mechanics. Nasar et al. [33] have noticed that the shear thinning rate of the viscosity might cause a reduction in the normal velocity of Oldroyd-b fluid through a stretching channel. Si et al. [34] analysed the asymmetric viscoelastic flow through an expanding or contracting porous channel and obtained a solution by using HAM. Bhatti and Abbas [35] analysed the slip effects on the creeping flow of a Jeffrey fluid under the action of an external magnetic field and discussed the effects for non-Newtonian and Newtonian cases. Kalpana and Sumit [36] extended this study for Jeffrey nanofluid and studied the effect of thermal radiation, viscous dissipation and magnetic field effects. By considering the Rivlin Ericksen tensors, grade fluids like second grade and third grade models have been developed [37]. Due to the importance of chemical reaction in many branches of chemical and biological engineering several authors [38–40] have incorporated the concentration equation along with the momentum equations. Hayat et al. [41] studied the magnetic field effects on a chemically reacting third grade fluid above a stretching sheet using HAM and analysed the velocity, skin friction, concentration and local Sherwood number profiles with respect to various parameters. Satya Narayana and Harish Babu [42] have observed that by enhancing the magnetic field strength, the longitudinal velocity can be decreased and it has an opposite effect with relaxation parameter of a Jeffrey fluid. They have also found that increasing heat source would enhance the temperature and chemical parameter decreases the concentration for a steady flow above a stretching sheet. Middleman [43] explained the implementation of upper convected derivative in the Maxwell model which provides a balance between physical reality and necessary mathematical simplicity for the numerical solution of practical problems. Alizadeh-Pahlavan and Kayvan [44] inspected the effects of magnetic and elastic forces for an unsteady flow of UCM fluid over a stretching sheet and observed that they have decreased effects on the flow. The effects of various parameters on temperature and velocity profiles in a MHD UCM fluid flow over a stretching sheet in the presence of a heat sink or source were analysed by Mukhopadhyay [45] by using a shooting technique. Vajravelu et al. [46] analysed the effect of chemical reaction on the MHD flow of an UC Maxwell fluid above a porous surface by using a Keller box numerical technique and observed that the order of the chemical reaction increases the diffusion boundary layer thickness. Hayat and Awis [47] considered a 3-D flow of UC

Maxwell fluid above a stretching sheet and inspected the effect of the Deborah number on the velocity components. Bilal et al. [48] investigated 3-D flow of an UCM fluid with nanoparticles and studied the effects of elasticity on flow, heat and boundary properties. Nandy [49] studied the effect of unsteady, velocity slip and Maxwell parameters on Nusselt number, heat, flow, skin friction, concentration, Sherwood numbers and showed that they have considerable effect for an UCM fluid over a permeable shrinking surface with nanoparticles. Choi et al. [50] have analyzed the 2-D flow of upper convected Maxwell fluid in a porous surface channel using both 4th order Runge Kutta integration scheme and power series. Their observation suggested that at a constant Reynolds number the longitudinal velocity at the center line decreases for the higher Deborah number and it has a flattening effect on axial velocity for high Reynolds numbers for a fixed Deborah number. Hayat and Abbas [51] found HAM solutions for the 2-D boundary layer flow of a UC Maxwell fluid in a porous channel. Hayat et al. [52] have considered the action of radiation on MHD UCM fluid flow through a porous medium and obtained solution using HAM. Beg and Makinde [53] have analysed the species transfer of a UCM fluid flow through a Darcian high permeability channel at high Deborah numbers by using 6th order Runge Kutta method.

In the present article, a mixed convective flow of UCM fluid confined in a contracting or expanding porous channel with slip conditions is explored. The effect of heat source and chemical reaction on the concentration, heat transfer and the velocities is studied. The effect of various parameters on the temperature, concentration and velocity profiles is analysed and presented in the form of graphs. The axial velocity for the Newtonian case has been studied and the obtained results are close to the ones in the literature [2, 4, 6].

Nomenclature

t	Time (s)
V_1	Suction/injection velocity at upper wall (m/s)
a(t)	Distance from the origin to the upper wall (m)
B_0	Magnetic flux density (Tesla)
T	Temperature (Kelvin)
T_2	Temperature at the upper wall
T_1	Temperature at the lower wall
C	Concentration (moles)
C_1	Concentration at the lower wall
C_2	Concentration at the upper wall
u	Velocity component in x-direction (m/s)
P	Fluid pressure (Pa)

g	Acceleration due to gravity (m/s^2)
v	Velocity component in y-direction (m/s)
c	Specific heat at constant temperature ($\text{J kg}^{-1}\text{K}^{-1}$)
Q_H	Heat source/sink
k_r	Chemical reaction rate (s ⁻¹)
D	Mass diffusivity (m^2/s)
k	Thermal conductivity (Watt/m/K)
σ_1	Dimensionless constant which depends on the pore size of the permeable material
k_1	Permeability of the porous medium (Darcy)
$\dot{n}_A$	Mass transfer rate
Wi	Wiessenberg number $\frac{\beta V_1}{a}$
Re	Permeation Reynolds number, $\frac{\rho V_1 a}{\mu}$
Ha	Hartmann number,
Ec	Eckert number,
Gr	Grashof number (thermal),
Gc	Grashof number (solutal),
Sh	Sherwood Number, $\frac{\dot{n}_A}{av(C_2 - C_1)}$
Pr	Prandtl number,
Sc	Schmidt number,
K_r	Non dimensional chemical reaction parameter, $\frac{k_r a^2}{v}$
H_s	Non dimensional heat source/sink parameter, $\frac{Q_H a^2}{k}$
Sl	Non dimensional Slip parameter $\frac{\sqrt{k_1}}{a\sigma_1}$

Greek Notations

μ	Fluid viscosity (mPa/sec)
ρ	Fluid density (Kg/m^3)
λ	Dimensionless y coordinate,
ξ	Dimensionless axial variable,
ν	Kinematic viscosity (m^2/s)
σ	Electric conductivity (Seimens)
β	Maxwell parameter (s)
α	Wall expansion ratio,
β_T	Thermal expansion coefficient (K^{-1})
β_C	Solutal expansion coefficient (Mol^{-1})

2 Problem formulation

Consider an unsteady flow of upper convected Maxwell fluid in an expanding or contracting semi-infinite porous channel where the plates are placed at $Y = 0$ and $Y = a$. One end of the channel is closed by a stretching sheet or membrane which stretches or contracts as the distance between the plates, $a(t)$ changes. The X and Y axes are taken along and perpendicular to the axial flow direction with a constant magnetic field in the Z- direction as shown in the Fig. 1. It is assumed that the fluid is constantly sucked

Fig. 1: The schematic representation of UCM fluid through expanding/contracting plates.

or injected from the upper plate with a velocity V_1 , the temperature and concentration at the plates are constantly maintained at T_2 , C_2 and T_1 , C_1 . As convective heat transport in porous channels due to Ohmic heating and buoyancy forces have a major part in applications of heat exchangers and nuclear reactors, we have considered the effect of mixed convection in the present model. The governing equations for the system is given by the equations (1)–(5).

$$\nabla \cdot \vec{q} = 0 \quad (1)$$

$$\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla) \vec{q} = -(\nabla P + \nabla \cdot S) + \mu \nabla^2 \vec{q} - J \times B + \rho g (\beta_T (T - T_1) + \beta_C (C - C_1)) \quad (2)$$

$$S + \beta \left(\frac{dS}{dt} + \vec{q} \cdot \nabla S - (\nabla \vec{q})^T \cdot S - S \cdot \nabla \vec{q} \right) = \mu (\nabla \vec{q} + (\nabla \vec{q})^T) \quad (3)$$

$$\rho c \left[\frac{\partial T}{\partial t} + (\vec{q} \cdot \nabla) T \right] = k \nabla^2 T + \frac{J^2}{\sigma} + Q_H (T - T_1) \quad (4)$$

$$\frac{\partial C}{\partial t} + (\vec{q} \cdot \nabla) C = D \nabla^2 C - K_r (C - C_1) \quad (5)$$

By following Mukhopadhyay [45] and Hayat and Awis [47], the equations (6) – (10) are formulated which describe the continuity, momentum, energy and concentration of the current problem in a Cartesian coordinate system.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (6)$$

$$\rho \left[\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \beta \left(u^2 \frac{\partial^2 u}{\partial x^2} + v^2 \frac{\partial^2 u}{\partial y^2} + 2uv \frac{\partial^2 u}{\partial x \partial y} \right) \right] = -\frac{\partial P}{\partial x} + \mu \nabla^2 u - \sigma B_0^2 u + \rho g \beta_T (T - T_1) + \rho g \beta_C (C - C_1) \quad (7)$$

$$\rho \left[\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + \beta \left(u^2 \frac{\partial^2 v}{\partial x^2} + v^2 \frac{\partial^2 v}{\partial y^2} + 2uv \frac{\partial^2 v}{\partial x \partial y} \right) \right] = -\frac{\partial P}{\partial y} + \mu \nabla^2 v - \sigma B_0^2 v \quad (8)$$

$$\rho c \left[\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right] = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \sigma B_0^2 (u^2 + v^2) + Q_H (T - T_1) \quad (9)$$

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) - k_r (C - C_1) \quad (10)$$

Where P is pressure, β is the relaxation coefficient of Maxwell fluid, Q_H is heat source/ sink parameter, D is diffusivity coefficient and k_r is the chemical reaction parameter. The theory and experiments of Beavers and Joseph [27] demonstrates the importance of the velocity slip effect due to the porous nature of the boundary where the gradient of external velocity is proportional to the slip velocity. Later, this theory has been mathematically justified by Saffman [54] and Mikelic and Jager [55] through statistical and analytical methods to show that the approximate condition at the interface given by Beavers, Joseph [27] and Saffman [54] is satisfied by velocity field for a laminar flow of a Newtonian fluid above a porous surface. The slip velocity due to Beavers and Joseph's condition is considered in the boundary conditions (11) along with temperature and concentration at the plates.

$$\begin{aligned} T &= T_1, \quad C = C_1, \quad u = 0, \quad v = 0, \quad \text{at } y = 0 \\ T &= T_2, \quad C = C_2, \quad u = -\frac{\sqrt{K_1}}{\sigma_1} \frac{\partial u}{\partial y}, \quad v = -V_1, \quad \text{at } y = a \end{aligned} \quad (11)$$

Majdalini et al. [2] developed solutions for the generalised Proudman and Johnson's [56] equation by using the following similarity variables (12) for velocity and this work has been extended to study the temperature and concentration [19].

$$u = \frac{vX}{a^2} F', \quad v = -\frac{v}{a} F$$

$$T = T_1 + \frac{vV_1}{ac} \left(\phi_1 + \frac{x^2}{a^2} \phi_2 \right), \quad C = C_1 + \frac{\dot{n}_A}{av} \left(g_1 + \frac{x^2}{a^2} g_2 \right) \quad (12)$$

Using the above variables a similarity solution in time and space is derived for constant α 's which are specified by the value of initial value $\alpha = \frac{a\dot{a}}{v} = \frac{a_0\dot{a}_0}{v}$ where a_0 , $\dot{a}_0$ are the initial distance and wall expansion ratios. By following Asghar et al. [6], instead of λ and t , the function F is redefined to be a function of λ and $\alpha(t)$. By assuming α to be constant

or quasi constant in time, we have the condition $F_{\lambda t} = 0$. The relation between F and f is given by $F(\lambda) = \text{Re}f(\lambda)$. Thus we have developed nonlinear ODEs from the governing equations (13) – (17).

$$\begin{aligned} f^{iv}(1 - \text{Wi} \text{Re} f^2) &= -\alpha(\lambda f^{///} + 3f^{//}) + \text{Re}(f' f^{//} - f^{///} f) \\ &\quad - 2\text{Wi} \text{Re} f^{//}((f')^2 + f f^{//}) + H a^2 f^{//} \\ &\quad - \frac{Ec \text{Gr}}{\xi \text{Re}}(\phi_1' + \xi^2 \phi_2') - \frac{Sh \text{Gc}}{\xi \text{Re}}(g_1' + \xi^2 g_2') \end{aligned} \quad (13)$$

$$\begin{aligned} \phi_1^{//} &= -\alpha \text{Pr}(\phi_1 + \lambda \phi_1') - \text{Re} \text{Pr}(f \phi_1') - H a^2 \text{Re} \text{Pr}(f^2) \\ &\quad - 2\phi_2 - H_S \phi_1 \end{aligned} \quad (14)$$

$$\begin{aligned} \phi_2^{//} &= -\alpha \text{Pr}(3\phi_2 + \lambda \phi_2') + \text{Re} \text{Pr}(\phi_2 f' - f \phi_2') \\ &\quad - H a^2 \text{Re} \text{Pr}(f')^2 - H_S \phi_2 \end{aligned} \quad (15)$$

$$g_1^{//} = -\alpha \text{Sc}(g_1 + \lambda g_1') - \text{Re} \text{Sc} f g_1' + K_r g_1 - 2g_2 \quad (16)$$

$$g_2^{//} = -\alpha \text{Sc}(3g_2 + \lambda g_2') + \text{Re} \text{Sc}(2f' g_2 - f g_2') + K_r g_2 \quad (17)$$

The boundary conditions on the velocities, temperature and concentration in terms of $f(\lambda)$, $\phi_1(\lambda)$, $\phi_2(\lambda)$, $g_1(\lambda)$ and $g_2(\lambda)$ are given in (18).

$$\begin{aligned} f &= 0, \quad f' = 0, \quad \phi_1 = 0, \quad \phi_2 = 0, \quad g_1 = 0, \\ g_2 &= 0 \quad \text{at} \quad 0 \\ f &= 1, \quad f^{//} = -\frac{f(1)}{Sl}, \quad \phi_1 = 1/Ec, \quad \phi_2 = 0, \quad g_1 = 1/Sh, \\ g_2 &= 0 \quad \text{at} \quad 1 \end{aligned} \quad (18)$$

For practical purposes, the non-dimensional functions of skin friction, local Sherwood and Nusselt numbers [46] are given by

$$\begin{aligned} C_f &= \frac{\mu}{\rho V_1^2} \frac{\partial u}{\partial y} \quad Sh = \frac{-a}{(C_2 - C_1)} \frac{\partial C}{\partial y} \\ Nu &= \frac{-a}{(T_2 - T_1)} \frac{\partial T}{\partial y} \end{aligned} \quad (19)$$

3 Solution procedure

A nonlinear system of first order equations (21) is formulated from equations (13)-(17). Shooting method is implemented by reducing the BVP to IVP and an algebraic nonlinear system. To integrate the IVP till $\lambda=1$, we need sufficient boundary conditions at $\lambda=0$. Hence, we assume guess

slopes $as_1, as_2, as_3, as_4, as_5, as_6$ to form the initial conditions at the lower boundary (20).

$$\begin{aligned} z_1 &= 0; \quad z_2 = 0; \quad z_3 = as_1; \quad z_4 = as_2; \quad z_5 = 0; \\ z_6 &= as_3; \\ z_7 &= 0; \quad z_8 = as_4; \quad z_9 = 0; \quad z_{10} = as_5; \quad z_{11} = 0; \\ z_{12} &= as_6; \end{aligned} \quad (20)$$

$$\begin{aligned} z_1 &= f \quad z_5 = \phi_1 \quad z_7 = \phi_2 \quad z_9 = g_1 \quad z_{11} = g_2 \\ z_2 &= f' \quad z_3 = f^{//} \quad z_4 = f^{///} \\ z_4^{//} &= f^{iv} = (-\alpha(\lambda z_4 + 3z_3) + \text{Re}(z_2 z_3 - z_1 z_4) \\ &\quad - 2\text{Wi} \text{Re}(z_2^2 z_3 + z_1 z_3^2) + H a^2 z_3 \\ &\quad - \frac{Ec \text{Gr}}{\xi \text{Re}}(z_6 + \xi^2 z_8) - \frac{Sh \text{Gc}}{\xi \text{Re}}(z_{10} + \xi^2 z_{12})) / (1 - \text{Wi} \text{Re} z_1^2) \end{aligned}$$

$$\begin{aligned} z_5^{//} &= z_6 = \phi_1' \\ z_6^{//} &= \phi_1^{//} = -\alpha \text{Pr}(z_5 + \lambda z_6) - \text{Pr} \text{Re}(z_1 z_6) \\ &\quad - H a^2 \text{Pr} \text{Re}(z_1)^2 - H_S \phi_1 - 2z_7 \\ z_7^{//} &= \phi_2' = z_8 \\ z_7^{//} &= \phi_2^{//} = -\alpha \text{Pr}(z_7 + \lambda z_8) + \text{Pr} \text{Re}(2z_2 z_7 - z_1 z_8) \\ &\quad - H a^2 \text{Pr} \text{Re}(z_2)^2 - H_S z_7 \end{aligned}$$

$$\begin{aligned} z_9^{//} &= g_1' = z_{10} \\ z_{10}^{//} &= g_1^{//} = -\alpha \text{Sc}(z_9 + \lambda z_{10}) - \text{Sc} \text{Re} z_1 z_{10} + K_r z_9 - 2z_{11} \\ z_{11}^{//} &= g_2' = z_{12} \\ z_{12}^{//} &= g_2^{//} = -\alpha \text{Sc}(3z_{11} + \lambda z_{12}) + \text{Re} \text{Sc}(2z_2 z_{11} - z_1 z_{12}) \\ &\quad + K_r z_{11} \end{aligned} \quad (21)$$

The assumed slopes form a column vector 'as'. Due to the fast convergence of Newton Raphson scheme, the algebraic problem is formulated in vector form using the conditions at the upper boundary. This numerical problem is solved by using a suitable initial guess vector for 'as' and iterate the vector 'y' to get the values at the upper boundary by using R-K 4th order algorithm such that the algebraic problem is satisfied.

4 Results

The unsteady slip flow of mixed convective chemically reacting upper convected Maxwell fluid through an expanding or contracting channel with uniform heat source is investigated with numerical computations using a shooting approach. The effect of various prominent non dimensional parameters such as Heat source parameter (H_S) and non-dimensional chemical reaction parameter (K_r), Weissenberg number (Wi), slip parameter (Sl), wall expansion/contraction parameter (α), Hartmann number (Ha) on non-dimensional temperature and species concentration distributions, heat and mass transfer coefficients, the

Table 1: Effect of various parameters on skin friction, heat and mass transfer coefficients at $Ec = 1$; $Gr = 9$; $Re = 3$; $Ha = 2$; $Gc = 9$; $\gamma = 0.632\xi 5$; $Sh = 1$; $Sc = 0.95$; $Pr = 0.7$

Parameter	$\lambda=0$	$\lambda=1$	Skin Friction		Heat transfer coefficient		Mass transfer coefficient	
			$\lambda=0$	$\lambda=1$	$\lambda=0$	$\lambda=1$		
$Wi=0.1 \alpha=1 Hs=5 Kr=0.2$	Sl	0.1	5.3752	-3.1240	13.0219	-7.9759	1.7270	0.0665
		0.2	4.8436	-2.5091	12.6092	-8.0988	1.7046	0.0752
		0.3	4.4921	-2.0951	12.3536	-8.2134	1.6896	0.0812
		2	3.2254	-0.5480	11.5564	-8.8576	1.6348	0.1045
$Sl=0.1 Wi=0.1 Hs=5 Kr=0.2$	α	-1	5.1563	-4.2102	8.4504	-7.5839	1.0303	0.8186
		0	5.2149	-3.6701	10.5025	-7.8313	1.3474	0.4052
		1	5.3752	-3.1240	13.0219	-7.9759	1.7270	0.0665
		2	5.6556	-2.5891	16.0874	-8.0255	2.1687	-0.2019
$Wi=0.1 \alpha=1 Sl=0.1 Kr=0.2$	H_s	-4	5.6620	-3.6514	4.4078	-2.3290	1.7181	0.0711
		0	5.5750	-3.5295	6.4846	-3.9885	1.7196	0.0702
		2	5.5138	-3.4233	8.2399	-5.1772	1.7213	0.0692
		3	5.4758	-3.3490	9.4462	-5.9294	1.7227	0.0686
$Wi=0.1 \alpha=1 Sl=0.1 Hs=5$	Kr	0	5.3724	-3.1199	13.0237	-7.9756	1.7777	0.0186
		2	5.3988	-3.1544	13.0086	-7.9776	1.3476	0.4628
		4	5.4219	-3.1788	12.9989	-7.9785	1.0452	0.8463
		6	5.4422	-3.1966	12.9926	-7.9787	0.8253	1.1866
$\alpha=1 Sl=0.1 Hs=5 Kr=0.2$	Wi	0	5.3369	-3.2303	12.9592	-7.9934	1.7238	0.0678
		0.1	5.3752	-3.1240	13.0219	-7.9759	1.7270	0.0665
		0.15	5.4079	-3.0602	13.0614	-7.9649	1.7292	0.0657
		0.2	5.4519	-2.9889	13.1078	-7.9522	1.7384	0.0647

Table 2: Comparison values of axial velocity in the case of Newtonian fluid at $Re = 5$ and $\alpha = 0.5$

λ	Majdalani et al. [2]	Asghar et al. [4]	Boutros et al. [6]	Present
0	1.536002	1.559474	1.556324	1.55947443
0.05	1.531846	1.554822	1.551780	1.55482216
0.1	1.519377	1.540888	1.538164	1.54088861
0.15	1.498596	1.517743	1.515522	1.51774361
0.2	1.469505	1.485503	1.483935	1.48550349
0.25	1.432114	1.444331	1.443517	1.44433105
0.3	1.386445	1.394435	1.394421	1.39443543
0.35	1.332539	1.336071	1.336839	1.33607179
0.4	1.270464	1.269540	1.271006	1.26954084
0.45	1.200325	1.195188	1.197207	1.19518811
0.5	1.122275	1.113403	1.115778	1.11340290
0.55	1.036527	1.024617	1.027110	1.02461693
0.6	0.943364	0.929302	0.931656	0.92930265
0.65	0.843156	0.827971	0.829933	0.82797120
0.7	0.736373	0.721170	0.722523	0.72117004
0.75	0.623597	0.609480	0.610078	0.60948025
0.8	0.505538	0.493513	0.493322	0.49351360
0.85	0.383052	0.373909	0.373046	0.37390927
0.9	0.257149	0.251330	0.250109	0.25133045
0.95	0.129010	0.126461	0.125435	0.12646066
1	0	0	0	0

velocity components and the skin friction are studied in the domain $[0, 1]$ and the results are shown with the help of tables and graphs.

The slip velocity due to Beavers and Joseph condition [27] effects the overall flow, temperature and concentration across the channel. The Figure 2 depicts the slip parameter's effect on the velocity components. It is noticed that the axial component decreases towards the middle

and then it tends to increase towards the upper boundary, on the other hand the radial component gets reduced for increasing slip parameters. This is because of the slip velocity results in the reduction of the pressure gradient. The mass transfer rate and skin friction decrease at the lower plate and increase at the upper plate while the rate of heat transfer decreases at both the plates for higher slip parameter coefficients. Hence the heat transfer rates at the plates

(i)

Fig. 2: Slip parameter's effect on (iii) Axial velocity (iv) Radial velocity with $Ec = 2$; $Sh = 2$; $Re = 3.4$; $Pr = 0.7$; $Gr = 11.56$; $Wi = 0.05$; $Ha = 2$; $Gc = 11.56$; $Sc = 1.4$; $\alpha = 0.6325\xi$; $H_S = 2$; $Kr = 0.2$. $\alpha = 0.1$

can be enhanced by choosing the porous surfaces with less slip parameters for the given parametric values.

The results for various values of expansion wall ratio i.e. temperature, concentration, longitudinal and radial velocities are portrayed in the Figure 3. The solid represents the contraction ratio and the dot dashed line, i.e. $\alpha = 0$ represents the fixed boundary and the other two lines show the expansion of the channel. The temperature, concentration and the radial components are observed to be increasing for higher α . The axial velocity is found to be increasing towards the middle of the channel and decreasing towards the upper boundary for increasing α . From the Table 1, it is observed that the skin friction increases at both the plates with α whereas the heat and mass transfer coefficients increase at the lower boundary on the contrary it decreases at the upper plate. This shows that the temperature in the channel and heat transfer rate at the upper plate enhance during the expansion of the channel.

The presence of heat source/sink in the channel is illustrated in the Figure 4 with temperature profiles. The solid line represents the heat sink, the dot dashed line represents the absence of the heat source in the present model. It is clear to see that the temperature is increas-

(ii)

(iii)

(iv)

Fig. 3: Effect of wall expansion/ contraction parameter on (i) Temperature (ii) Concentration (iii) Axial velocity (iv) Radial velocity with $Ec = 2$; $Sh = 2$; $Pr = 0.71$; $Re = 3$; $Wi = 0.05$; $Ha = 3$; $Gr = 9$; $Gc = 9$; $Sc = 1.2$; $\alpha = 0.6325\xi$; $H_S = 1$; $Kr = 0.4$; $Sl = 1$.

Fig. 4: Influence of Heat source parameter on Temperature with $Ec = 2$; $Pr = 0.4$; $Sh = 2$; $Re = 4$; $\alpha = 4$; $Sl = 1$; $Ha = 3$; $Wi = 0.2$; $Gr = 16$; $Gc = 16$; $Sc = 1.5$; $\gamma = 0.6325$; $Kr = 4$.

Fig. 5: Influence of Hartmann number on Temperature with $Ec = 2$; $Pr = 0.7$; $Re = 4$; $Wi = 0.05$; $\alpha = 5$; $Gr = 16$; $Gc = 16$; $Sc = 1.5$; $\gamma = 0.6325$; $Sh = 2$; $H_s = 0.5$; $Kr = 2$; $Sl = 1$.

Fig. 6: Effect of chemical reaction parameter on Concentration with $Ec = 2$; $Pr = 0.4$; $Re = 5$; $Wi = 0.1$; $\alpha = 4$; $Ha = 3$; $Gr = 25$; $Gc = 25$; $Sc = 1.5$; $\gamma = 0.6325$; $Sh = 2$; $H_s = 1$; $Sl = 1$.

ing with heat source parameter. The increment in heat source parameter implies more heat production in the fluid and thus increasing the overall temperature. The heat and mass transfer coefficients increase at the lower and decrease at the upper boundaries, whereas the skin friction has an opposite pattern for higher heat source parametric values. The effect of Joule heating on the fluid due to the

(i)

(ii)

Fig. 7: The influence of Wi on (i) Radial velocity (ii) Axial velocity with $Ec = 2$; $Pr = 0.7$; $Re = 3.4$; $\alpha = 4$; $Ha = 3$; $Gr = 11.56$; $Gc = 11.56$; $Sc = 1.4$; $\gamma = 0.6325$; $Sh = 2$; $H_s = 2$; $Kr = 0.2$; $Sl = 1$.

applied external magnetic field is depicted in Figure 5. Increasing magnetic field increases the Lorentz force on the fluid [57], Joule heating of the fluid is due to the resistivity of the fluid which has a natural property to raise its temperature under external magnetic field. The absence of magnetic field is represented by the solid line. The gradual increment of the temperature of the electrically conducting fluid due to increasing Hartmann numbers.

The Figure 6 shows the impact of chemical reaction on the heat, mass and flow characteristics. The solid line in the Figure 6 represents the absence of chemical reaction in the present problem. It is evident from the figure that the concentration decreases for higher chemical reaction parameters. As the chemical reaction is of destructive nature, increasing the chemical reaction parameter would inflate the reduction of concentration of the species across the channel. Apart from the concentration, the effect of Kr on the boundary properties is also studied. As chemical reaction parameter increases, the skin friction also increases at the lower boundary, but decreases at the upper one. The heat transfer coefficient decreases at both the boundaries and the mass transfer coefficient follows an opposite pattern of skin friction. Therefore, the presence of destructive

chemical reaction decreases the concentration across the channel and increases the mass transfer rate at $\lambda = 1$.

The Figure 7 shows the significance of elastic forces on the radial and axial components. The solid line represents the absence of Weissenberg effect i.e. Newtonian fluid. The radial velocity is increasing for increasing Wi values. Due to the domination of viscoelastic forces on inertial forces for Reynolds number less than 5, it is observed that the longitudinal component increases towards the center and it declines up to the channel end. For increasing Weissenberg numbers, the enhancement of heat transfer and skin friction is noted at both boundaries while the mass transfer coefficients increase on the lower boundary and decrease at the upper one. The Newtonian case has been investigated and the attained non dimensional axial velocities are close to the existing ones [2, 4, 6]. The comparative results are shown in Table 2.

5 Conclusions

The unsteady heat transfer and MHD flow of a chemically reacting mixed convective UCM fluid through expanding or contracting porous channel with a uniform heat source is studied numerically. The study concludes the following points.

- The fluid temperature is enhanced with the influence of the Joule heating effect due to the magnetic field and also with the heat source.
- The concentration distribution decreases for increasing chemical reaction parameter, whereas it increases with wall expansion ratio.
- The axial velocity profiles of the slip parameter and the Weissenberg number follow opposite trend.
- The radial velocity is increasing with the wall expansion ratio, while it is reduced due to the slip flow.
- Maxwell fluid and wall expansion ratio parameters show similar effects on skin friction and mass transfer rate at boundaries.

These results have possible applications in science and technology such as biorheology, food processing, fluid transport in biological organisms, polymer industry and petroleum industries. This study can be extended by considering other properties like varying viscosities and thermal conductivities across the channel which is very natural in the applications like separation of crude oil.

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A Appendix

Shooting algorithm:

The system of equations are solved using the shooting algorithm. The algorithm consists of Runge Kutta 4th order and Newton Raphson functions. The initial value problem (20-21) is integrated till $\lambda = 1$. It can be clearly seen that there are 6 variables as1, as2...as6 in the initial vector (20) for RK integration and there are exactly 6 boundary conditions at '1'. The values at 1 which are obtained through RK4 method are substituted in these boundary conditions to obtain 6 nonlinear algebraic equations. However the implementation of the above idea needs initial vector for RK4 in terms numbers rather than symbols. Hence Newton Raphson is a good choice to solve the system of coupled nonlinear equations.

Basic steps:

1. Initialisation of the parameters: It contains 14 main parameters with two boundary parameters XSTART and XSTOP stated in the main code.
2. Initialisation of initial vector of newton raphson vector 'as' and therefore initial vector of RK4 in the function 'inCond(as)'.
3. The governing system of equations are stored in dEqs.
4. The boundary conditions are written in Residual function which is called in Newton Raphson (NR) program. The NR function uses the Jacobian function to find the accurate root.
5. The apt values of the assumed slopes are obtained through Newton Raphson program such that the boundary conditions are satisfied by calling Residual function. These values are stored in 'as' by replacing the original guess.
6. The final 'as' is substituted in the RK4 initial vector to obtain the accurate solution.

The basic code for shooting algorithm has been presented by Jaan Kiusalaas [58]. This code has been adopted and modified for coupled nonlinear equations with multiple functions of λ .