Research Article

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Band gap manipulation of viscoelastic functionally graded phononic crystal

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Abstract: In this study, band gaps of SH-waves (horizontally polarized shear waves) propagating in a thermal-sensitive viscoelastic matrix are investigated. Metallic films acting as heat sources are periodically embedded into the matrix, which establishes a periodically inhomogeneous thermal field. The homogenous matrix is therefore transformed into functionally gradient phononic crystals (PCs). A three-parameter solid model is employed to describe the viscoelasticity of the present matrix. By virtue of a transfer matrix method incorporated within a laminated model, the dispersion equation of SH-waves is finally obtained, from which the band gaps are determined. The transmission spectra of a finite-periodic PC are also solved to validate the band gaps. In numerical examples, the influences of incident angles of SH-waves and viscoelasticity of matrix on band gaps are discussed first. Then the research focuses on the means to tune the band gaps by manipulating the inputted powers of heat sources. Numerical examples demonstrate that such a strategy is effective and convenient in tuning the positions and widths of band gaps. A viscous parameter, i.e., the ratio of initial-state to final-state storage moduli, significantly affects the band locations and bandwidths, while the locations of low-order band gaps hardly move with the incident angle of SH-waves. Band gaps of several orders are expected to locate in lower-frequency domain, and the total bandwidth becomes larger as the inputted heat flux increases. This paper lays theoretical foundation to manufacture viscoelastic functionally graded PCs which can be used in frequency-selective devices.

1 Introduction

Phononic crystals (PCs), exhibiting forbidden band gaps, are artificial periodic structures made up of at least two types of materials with different mechanical and physical properties. The forbidden band gaps of PCs mean that elastic wave with specific frequency is not allowed to propagate through the periodic structures. This unique ability endows PCs with a bright prospect to be applied in the area of noise suppression [1], filter [2,3], shock insulation [4,5], acoustic transducers [6], and so on. For promoting engineering applications of forbidden band gaps, many effective active adjusting and manipulating methods based on altering mechanical properties, sizes of crystal lattice, and topological structures have attracted a great deal of attention [7–11].

Unlike most nanomaterials, whose mechanical properties are homogeneous in macroscale [12], functionally graded materials (FGMs), first applied in aerospace structures, are deliberately manufactured to have mechanical properties with continuous spatial variations [13,14]. Recently, the concept of FGM has been introduced into the carbon nanotube-reinforced composites and some nonlinear mechanical characteristics were studied [15,16]. Nowadays, FGMs are applied widely in various structures as members or coatings [17]. PCs constituted by FGMs are expected to possess less interfacial defect and have attracted the attention of many researchers in the last 10 years. Wu et al. [18] investigated one-dimensional PCs made of isotropic FGMs by applying spectral finite elements and transfer matrix method. Their results showed that the desired forbidden band gaps can be designed by appropriate selection of structure parameters. Band gaps and wave transmission of in-plane wave propagation in layered functionally graded phononic crystals (FGPCs) were investigated by Fomenko et al. [19] with the consideration of solid diffusion in homogenous isotropic materials. Bednarik et al. [20] studied the longitudinal wave propagation along the thickness of a plate consisting of FGMs. By using a plane

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wave expansion method, Su et al. [21] studied the influence of FGM parameters on the band gaps of one-dimensional PCs, and the results showed that different FGM properties can remarkably alter the band gaps. Lan et al. [22] considered a laminate structure consisting of piezoelectric and piezomagnetic materials with graded interlayer of which properties gradually change from piezoelectric material to piezomagnetic material or from piezomagnetic material to piezoelectric material. Investigation on SH-wave dispersion curves in that paper revealed that material parameters, including piezoelectric constants, piezomagnetic constants, and elastic constants, have a complex influence on the band gaps of SH-waves. Furthermore, Yan et al. [23] investigated in detail the influence of nonlocal interface imperfections and nanoscale size on the band structures of SH-waves in PCs with FGM interlayers, using a meshless radial basis function collocation method based on the Eringen nonlocal elasticity theory. However, they did not take the viscoelasticity of the components into account. Apart from studies about the band gaps of SH-waves, Guo et al. [24] studied the influence of FGM interlayers on dispersion curves of in-plane wave in one-dimensional piezoelectric and piezomagnetic PCs.

Bian et al. [25] investigated the band gap tuning of P-waves and SH-waves with arbitrary incidence angle based on homogenous thermal fields. Most recently, Bian et al. [26] have come up with a type of FGPC realized by a homogenous thermal-sensitive material loaded with periodic thermal fields, whose band gaps of SH-waves were designed to be tuned by manipulating periodic thermal fields. Studies demonstrate that band structures can be tuned effectively and conveniently according to such a strategy. In our further investigation, a phenomenon is found that the thermal-sensitive materials can own a notable viscoelasticity, which means the storage modulus of materials is strongly dependent on the frequency of wave propagation [27–31]. Previous research shows that the band structures of PCs are considerably influenced by frequency-dependent storage modulus. Among these research studies, Zhao and Wei [32,33] investigated the influence of material viscosity on the band structures of 1D and 2D PCs based on an analytical method. Zhu et al. [34] applied a boundary element method and an experimental method to study the defect modes and band gaps of viscoelastic PCs. Li et al. [35] further investigated the influence of viscoelastic adhesive layers on the band structures of SH-waves. It can be found from the aforementioned studies that a proper forbidden band gap can be obtained by adjusting the viscous parameters of constituents or adhesive layers. The results of these

studies lay foundation for the research of the present paper. Although there have been many studies on viscoelastic PCs, the effects of viscoelasticity on the band structures of thermal FGPCs are rarely investigated. This motivates the study of the viscoelastic effects of the thermal-sensitive FGMs on the band gaps of PCs.

Therefore, the present study focuses on the manipulation of band gaps of SH-waves in FGPCs considering the frequency-dependent storage modulus. The three-parameter solid model is employed to describe the viscoelasticity of the thermal-sensitive matrix. Periodic thermal fields, owing to the thin metallic films periodically embedded into the matrix and acting as heat sources, turn the homogenous matrix into an FGPC. A transfer matrix method incorporated within an approximate laminated model is employed to theoretically predict the band gaps of SH-waves, which is validated by comparison with the transmission spectra of a finite-periodic PC. Influences of the incident angles of SH-waves and viscoelasticity of the matrix on the band gaps are discussed first in numerical examples. Tuning the band gaps by manipulating the inputted powers of periodic heat sources is studied in detail, and its effectiveness and convenience are demonstrated. The aim of the present study is to provide guidelines for designing viscoelastic FGPCs.

2 Models and analytical solutions

2.1 Thermal-sensitive and viscoelastic matrix

A three-dimensional structure composed of a homogenous matrix and a series of thin metallic films is illustrated in Figure 1. The metallic films are periodically embedded into the matrix, which serve as heating sources and construct periodic thermal field. The matrix selected in the present work is an epoxy matrix, which obviously possesses viscoelastic characteristics, and whose moduli strongly depend on both temperature and vibration frequency. As is known to all, only the storage modulus will be involved in determining wave band gaps in the viscoelastic material, so the storage modulus of an Epoxy Resin 20-3440-032 (presented by BUEHLER Company) is first measured by the dynamic mechanical analysis (DMA Q800) using a frequency f of 1 Hz, while its temperature ranges from 20 to 90°C. It shall be mentioned that this temperature range covers the glass transition temperature of the present epoxy, which is about 45–46°C. The experimental results are displayed

Figure 1: Schematic diagram of the laminated structure composed of viscoelastic epoxy and periodically distributed metallic films.

in the study of Bian et al. [26], where the following formulas are also presented:

$$E^*(f = 1 \text{ Hz}) = \begin{cases} -73.3T + 3065 & 20^\circ \text{C} \leq T \leq 41.7^\circ \text{C} \\ -6.33 \times 10^{-2}T + 11.6 & 41.7^\circ \text{C} \leq T \leq 90^\circ \text{C} \end{cases} \text{MPa.} \quad (1)$$

Second, the behavior of viscoelastic characteristics of the present epoxy is described by the three-parameter solid model, which consists of the Kelvin and spring models in series connection or the Maxwell and spring models in parallel connection. According to this model, the storage modulus in the frequency domain satisfies,

$$E^*(f) = \frac{E_\infty + \tau^2 f^2 E_0}{1 + \tau^2 f^2}, \quad (2)$$

where τ , E_0 , and E_∞ denote the relaxation time, initial-state ($f \rightarrow \infty$), and final-state ($f \rightarrow 0$) storage moduli, respectively. Clearly, $E^* = E_\infty$ if $E_0 = E_\infty$, which indicates

the viscoelastic material now transforms into an elastic material. In the following study, E_∞ approximates to take value of $E^*(f = 1 \text{ Hz})$ as presented in equation (1).

Finally, another elastic parameter of epoxy, i.e., Poisson's ratio (ν), is assumed to be independent of temperature and frequency.

2.2 Thermal field

When the matrix is heated by the periodically embedded metallic films, periodic thermal fields will be established within the matrix, as shown in Figure 2, where each metallic film is so thin that its thickness and therefore its heat capacity can be omitted. It plays the role of a heater with heat flux q_0 . The periodic thermal field is determined by solving the one-dimensional equation of heat conduction, $d^2\Delta T/dx^2 = 0$. Within one half of a periodic domain, the temperature rise (ΔT) can be expressed as

$$\Delta T = \frac{q_0}{2\lambda} \left(\frac{a}{2} - x \right) + \Delta T_0 \left(0 \leq x \leq \frac{a}{2} \right), \quad (3)$$

where ΔT_0 is the pre-known temperature rise at $x = a/2$. λ is the thermal conductivity of epoxy, which is also assumed to be independent of temperature and frequency in the present study, and is taken as 0.15 W/(m K) hereafter.

2.3 Dispersion equation of SH-waves

In the absence of body forces, the dynamic equilibrium equations of an isotropic, homogeneous, viscoelastic solid are [25] as follows:

Figure 2: Simplified model of one-dimensional thermal conduction.

$$\mu^* \nabla^2 \mathbf{u} + (\lambda^* + \mu^*) \nabla (\nabla \cdot \mathbf{u}) = \rho \ddot{\mathbf{u}}, \quad (4)$$

where ρ is the mass density and μ^* and λ^* represent the Lamé constants, both of which are frequency dependent as the viscoelastic solid is involved in. For wave propagation of SH-waves, the solution of equation (4) can be expressed as:

$$u_2 = (\psi_+ e^{ik_T x \cos \theta} + \psi_- e^{-ik_T x \cos \theta}) e^{ik_0 z - i\omega t}, \quad (5)$$

where ψ_+ and ψ_- denote the amplitudes of the incident and reflected SH-waves, respectively, k_T and $k_0 (=k_T \sin \theta)$ are the transverse wave number and the apparent wave number, respectively, θ represents the incident angle of the transverse wave propagation, and ω is the circular frequency.

As described above, the storage modulus of the matrix will alter as its temperature rises. In view of this mechanism, the homogeneous matrix will turn into an inhomogeneous and periodic one, or a functionally graded PC, once it is heated by periodic metallic films. Since wave propagation in inhomogeneous solids cannot be solved directly, we resort to an approximate laminated model, according to which the inhomogeneous solid is divided into many fictitious thin layers and each layer approximates to a homogenous one. Equation (5) can now be applied to each sheet. Calculation difficulty due to this model can be overcome by employing a transfer matrix method. For this purpose, we rewrite the mechanical qualities in the following matrix form:

$$\begin{Bmatrix} u_2^{(j)} \\ \tau_{xy}^{(j)} / (i\mu^* k_T) \end{Bmatrix} = [M_j] \begin{Bmatrix} \psi_+ e^{ik_T x \cos \theta} \\ \psi_- e^{-ik_T x \cos \theta} \end{Bmatrix}^{(j)} e^{ik_0 z - i\omega t} \quad \text{with} \quad (6)$$

$$[M_j] = \begin{bmatrix} 1 & 1 \\ W_T^{(j)} \cos \theta_j & -W_T^{(j)} \cos \theta_j \end{bmatrix}$$

where the superscript “ j ” represents a quality taking the value of the j th fictitious laminate (the first laminate denotes the closest laminate to the incident wave). $W_T^{(j)} = \frac{(\rho c_T)^{(j)}}{(\rho c_T)^{(1)}}$ is the ratio of mechanical impedance in the transverse direction, and $c_T = \sqrt{\mu^* / \rho}$ is the velocity of the transverse wave. It should be noted that the Snell theorem is satisfied here, which means all apparent wave numbers, $k_0 = k_T \sin \theta_j$, must take the same value.

The continuity of displacement u_2 and stress τ_{xy} at the fictitious interfaces $x_j = j \cdot a/n$ (n denotes the number of fictitious laminates in laminated mode) leads to

$$\begin{Bmatrix} \psi_+ e^{ik_T a \cos \theta} \\ \psi_- e^{-ik_T a \cos \theta} \end{Bmatrix}^{(n)} = [T] \begin{Bmatrix} \psi_+ \\ \psi_- \end{Bmatrix}^{(1)}, \quad (7)$$

where $[T] = \prod_{j=n}^1 [M_{j+1}]^{-1} [M_j] [N_j]$ with $[N_j] = \text{diag} [e^{ik_T^{(j)} a/n \cos \theta_j}, e^{-ik_T^{(j)} a/n \cos \theta_j}]$ and $[M_{n+1}] = [M_1]$.

The Bloch theorem must be satisfied in a periodic structure, which requires the displacement and stress to be

$$u_2(x + a) = e^{ika} u_2(x), \quad \tau_{xy}(x + a) = e^{ika} \tau_{xy}(x), \quad (8)$$

where κ denotes the Bloch wave vector. In conjunction with equations (5), (7), and (8), the following dispersion equation is obtained:

$$|[T] - e^{ika} [I_2]| = 0, \quad (9)$$

where $[I_2]$ is a second-order unit matrix. From the dispersion curves, one can find some special frequencies with no wave vectors exiting. The so-called band gaps of elastic waves propagating in an infinite-periodic PC appear in such domains.

2.4 Transmission spectra

Band gaps mentioned above can also be characterized by the transmission waves through a finite-periodic PC. For this purpose, the coefficient of transmission power is calculated, which is [25]

$$\alpha = \frac{(\rho c_T \cos \theta)^R}{(\rho c_T \cos \theta)^E} \left| \frac{\psi^R}{\psi^E} \right|^2, \quad (10)$$

where the superscripts E and R represent the quantities in the substrate and detector, respectively. ψ is the amplitude of the incident or transmitted wave. Equation (10) indicates that the value of α will definitely vary with different substrates and detectors. However, the coefficient of transmission power will be impressively small in band gaps than in other frequency domains, whatever the substrates and detectors are.

3 Results and discussion

In the following numerical examples, the lattice constant (a) of the present PC is taken as 20 mm. At the midpoint of every unit cell ($x = a/2$), the pre-known temperature rise is $\Delta T_0 = 0$ and the ambient temperature is supposed to be 20°C. Poisson's ratio takes the value of 0.3, and the relaxation time the value of $\tau = 1 \times 10^{-5}$ s. For FGPCs discussed here, each unit cell is evenly discretized into 40 fictitious layers in the laminated model, to assure the numerical results have a high precision.

Figure 3: Band gaps of a normal incident SH-wave characterized by (a) dispersion curves and (b) transmission spectra.

Band gaps of a normal incident SH-wave (i.e., the incident angle $\theta = 0$) characterized by dispersion curves in an infinite-periodic PC and transmission spectra in a finite-periodic PC with five unit cells are first calculated and displayed in Figure 3, where the ratio of initial-state modulus to final-state modulus, $\zeta = E_0/E_\infty$, is taken as 10. The heat flux of each metallic film is 0.8 mW/mm^2 . As is expected, several band gaps emerge in Figure 3(a), and in the same frequency domain, the coefficients of transmission power, α , are extremely small, as shown in Figure 3(b). The good agreement of locations and widths of band gaps in dispersion curves with those in transmission spectra verifies the present model.

Next, the band gaps of two oblique incident SH-waves are studied. The incident angles are supposed to be 45° and 60° in Figure 4(a) and (b), respectively. The rest of the parameters are the same as those adopted in Figure 3. From the comparison among Figures 3(a), 4(a) and (b), it is seen that the locations of band gaps, especially that of the first or lowest band gap, move upward hardly as the incident angle increases from 0 to 60° , which indicates that low-order band gaps are valid for SH-waves at incident angles with a very wide range (a very important characteristic for a device in practice). Therefore, numerical examples will focus on the normal incident SH-waves in the following discussion.

According to the viscoelastic model employed here, a bigger modulus ratio, ζ , represents a more viscous material. In Figure 5, the band gaps of a normal incident SH-wave propagating in materials with different values of viscosity are demonstrated, where the heat flux is 0.4 mW/mm^2 . As the value of ζ increases, the material transitions from an elastic one ($\zeta = 1$) to a high viscous

one; meanwhile, the locations of the first-order band gaps move upward evidently and the widths of these band gaps are enlarged gradually. Beyond the enlargement of band gaps, the slope of the first dispersion curve means the phase velocity also becomes more complex with the transition of material from an elastic one to a high viscous one. In other words, the viscosity of a material will significantly alter the band gaps of SH-waves, which shall be paid enough attention in calculating the band gaps with high accuracy.

As shown above, the modulus ratio, actually the relaxing time as well, has an effect on the structures of bands gaps. However, they cannot be tuned once a certain viscoelastic material is chosen. For addressing the need to manipulate the band gaps of PCs conveniently, the method of tuning the storage modulus of a thermal-sensitive matrix by tuning powers of heat sources and therefore thermal fields is applied. The results of such a strategy are illustrated in Figure 6, where the modulus ratio $\zeta = 10$. Due to the inhomogeneous thermal field, the storage moduli of an epoxy matrix vary gradually within a unit cell. The band structures of normal incident SH-waves in the present FGPCs are clearly tuned by inputted powers of heat sources. As the inputted heat flux for each metallic film increases from 0.2 to 0.4 mW/mm^2 , the first-order band gap becomes lower and wider (from $48,118\text{--}52,137 \text{ Hz}$ to $39,284\text{--}48,544 \text{ Hz}$), and it becomes still lower but narrower when the heat flux increases to 1.0 mW/mm^2 ($1,681\text{--}4,858 \text{ Hz}$) and 1.2 mW/mm^2 ($1,516\text{--}3,348 \text{ Hz}$). Meanwhile, more band gaps emerge in the same frequency domain. The total bandwidth, i.e., the sum of widths of all band gaps in a certain frequency domain,

Figure 4: Band gaps of two oblique incident SH-waves: (a) 45° and (b) 60°.

Figure 5: Band gaps of normal incident SH-waves propagating in materials with different values of viscosity: (a) $\zeta = 1$, (b) $\zeta = 6$, and (c) $\zeta = 12$.

is 4,019 Hz for 0.2 mW/mm², 9,260 Hz for 0.4 mW/mm², 47,395 Hz for 1.0 mW/mm², and 48,109 Hz for 1.2 mW/mm², respectively. Obviously, the total bandwidth has an about 12-fold increase from 0.2 to 1.2 mW/mm². A reasonable explanation is that the storage modulus varies smoothly if the peak temperature within a unit cell is below 41.7°C (the slope of the storage modulus–temperature curve jumps at this point). Once temperatures in part of a unit cell exceed 41.7°C (the heat flux is 0.68 mW/mm² correspondingly), the storage modulus varies sharply, which indicates that the storage moduli in different domains have more differences and therefore more band gaps emerge. From these figures, it is also found that the slopes of dispersion curves decrease as the power of heat flux increases, which implies that the phase velocity of SH-waves decreases as the storage modulus decreases.

The results in Figure 6 reveal that the band gaps of SH-waves are effectively tuned by the inputted heat flux of each metallic film. Some more detailed results are presented in Figure 7, where each red line indicates the width and position of a complete band gap. As is expected, once the inputted heat flux is greater than 0.68 mW/mm², the number of band gaps will dramatically increase and the widths of most band gaps become narrower. When the heat flux increases from 0.8 to 2.0 mW/mm², the band gaps of several orders shift to lower-frequency zone and bandwidth broadens gradually, especially when the heat flux is larger than 1.4 mW/mm². For instance, the first band gap locates in the frequency domain of 1,413–2,626 Hz when the inputted heat flux is 1.4 mW/mm². However, the location of the first band gap shifts to the frequency domain of 1,271–1,970 Hz when the

Figure 6: Band gaps of normal incident SH-waves with the heat flux of (a) 0.2 mW/mm^2 , (b) 0.4 mW/mm^2 , (c) 1.0 mW/mm^2 , and (d) 1.2 mW/mm^2 .

Figure 7: Bandwidths and locations of band gaps with different heat fluxes.

inputted heat flux changes to 2.0 mW/mm^2 . The same law is also suitable for the second and third band gaps. In other words, band gaps of SH-waves propagating in a thermal-sensitive matrix can be tuned effectively and conveniently by manipulating the inputted powers of heat sources across a broad range.

4 Conclusion

Employing a transfer matrix method, the band gaps of SH-waves propagating in a thermal-sensitive viscoelastic

matrix are analytically studied. Periodically inhomogeneous thermal field is established by periodically embedded heat sources, which further results in the periodic and gradient variation of the storage modulus. A functionally graded PC is therefore formed. To overcome the theoretical difficulty due to the inhomogeneity, an approximate laminated model is introduced in the process of obtaining the dispersion equation. Band gaps of such an FGPC are theoretically verified by the transmission spectra of a finite-periodic PC at first.

1. From the numerical examples shown, it can be clearly found that the locations of low-order band gaps hardly move whatever the incident angles of SH-waves are. Such a characteristic will be helpful to acquire PCs with high stability.
2. The two parameters of viscoelasticity, i.e., the ratio of initial-final moduli and the relaxation time, have significant effects on the structures of band gaps. The locations of the first-order band gaps move upward significantly, and the total bandwidths of these band gaps are enlarged gradually with the increase in the modulus ratio.
3. A strategy based on manipulating the inputted powers of heat sources to tune the band gaps of SH-waves propagating in a thermal-sensitive matrix turned out to be effective and convenient. With the increase in the inputted heat flux, the band gap location of several orders moves to the lower-frequency

domain and the total bandwidth within the same frequency domain enlarges drastically.

The results presented in this study are expected to provide guidelines for the design of viscoelastic FGPCs.

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