Review Article

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Impact of double-diffusive convection and motile gyrotactic microorganisms on magnetohydrodynamics bioconvection tangent hyperbolic nanofluid

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Abstract: The double-diffusive tangent hyperbolic nanofluid containing motile gyrotactic microorganisms and magnetohydrodynamics past a stretching sheet is examined. By adopting the scaling group of transformation, the governing equations of motion are transformed into a system of nonlinear ordinary differential equations. The Keller box scheme, a finite difference method, has been employed for the solution of the nonlinear ordinary differential equations. The behaviour of the working fluid against various parameters of physical nature has been analyzed through graphs and tables. The behaviour of different physical quantities of interest such as heat transfer rate, density of the motile gyrotactic microorganisms and mass transfer rate is also discussed in the form of tables and graphs. It is found that the modified Dufour parameter has an increasing effect on the temperature profile. The solute profile is observed to decay as a result of an augmentation in the nanofluid Lewis number.

Keywords: magnetohydrodynamics, bioconvection, gyrotactic microorganisms, nanofluid, magnetic field, Keller box method, stretching sheet, double diffusion

1 Introduction

In fluid dynamics, bioconvection [1–3] occurs when microorganisms, which are denser than water, swim upwards. The upper surface of the fluid becomes thicker due to the assemblage of microorganisms. As a result, the upper surface becomes unstable and microorganisms fall down, which creates bioconvection. Bioconvection continues

to be explored widely because of its enormous applications in the field of pharmaceutical industry, purification of cultures, microfluidic devices, mass transport enhancement and mixing, microbial enhanced oil recovery and enzyme biosensors. Bioconvection systems could be categorized based on the directional motion of different species of microorganisms. In particular, gyrotactic microorganisms are the ones whose swimming direction is dependent on a balance between gravitational and viscous torques [4,5]. Oyelakin et al. [6] pondered the impact of bioconvection and motile gyrotactic microorganisms on the Casson nanofluid past a stretching sheet and observed that the microorganism profile decreases as a result of an increment in the Peclet number. Saini and Sharma [7] explored the effects of bioconvection and gyrotactic microorganisms on the nanofluid flow over a porous stretching sheet. It is noted that the Lewis number escalates the bioconvection process. Dhanai et al. [8] explored the impact of bioconvection on the fluid flow over an inclined stretching sheet and assessed that the microorganism density profile is enhanced with an improvement in the bioconvection Schmidt number. Mahdy [9] pondered the effects of motile microorganisms on the fluid past a stretching wedge and noted that a positive variation in the Peclet number leads to an augmentation in the microorganism profile. Avinash et al. [10] pondered the impact of bioconvection and aligned magnetic field on the nanofluid flow over a vertical plate and concluded that the heat transfer rate increases with an improvement in the Lewis number. Makinde and Animasaun [11] studied the effects of magnetohydrodynamics (MHD), bioconvection, nonlinear thermal radiation and nanoparticles on fluid past an upper horizontal surface of a paraboloid of revolution and found that the Brownian motion boosts the concentration profile. Khan et al. [12] studied the impact of MHD, gyrotactic microorganisms, slip condition and nanoparticles on the fluid flow over a vertical stretching plate; it was observed that the magnetic field suppresses the dimensionless velocity inside the boundary layer. Later, the effects of different features of the gyrotactic microorganisms on the fluid flow are analyzed in various investigations [13–15].

Nanotechnology has been considered the most substantial and fascinating forefront area in physics,

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engineering, chemistry and biology. The thermal conductivity of a nanofluid is greater than that of the base fluid. The thermal conductivity of the fluid is considered to be enhanced by the nanoparticles present in the fluid. Buongiorno [16] established a model to examine the thermal conductivity of nanofluids. Baby and Ramaprabhu [17] analyzed the heat transport of fluids using graphene nanoparticles. They reported that the thermal conductivity of hydrogen-exfoliated graphene is enhanced with an increment in the volume fraction of the nanoparticles. Khan and Gorla [18] pondered the mass transfer of the nanofluid flow over a convective sheet using the Keller box scheme and noted that the heat transfer rate is high in the dilatant fluids compared with that in the pseudoplastic fluids. Das [19] discussed the rotating flow of a nanofluid with respect to the constant heat source. A boost in the volume fraction of nanoparticles was observed to cause an increment in the thermal boundary layer thickness. Gireesha et al. [20] considered the Hall impact on a dusty nanofluid and concluded that the skin friction coefficient decreases due to an improvement in the Hall current.

The experimental and the theoretical scientific studies of the non-Newtonian liquids together with MHD have achieved a considerable attention of researchers because of their adequate applications in the field of aeronautics, chemical, mechanical, civil and bio-engineering. The fluid becomes electrically conducting under the effect of MHD like ionized gases, plasmas and liquid metals such as mercury. The impact of MHD and nonlinear thermal radiation on the Sisko nanofluid flow over a nonlinear stretching surface is premeditated by Prasannakumara et al. [21]. Rashidi et al. [22] pondered the MHD viscoelastic fluid together with the Soret and Dufour effects and observed that the velocity profile decreases with an improvement in the magnetic parameter. Kothandapani and Prakash [23] studied the effect of magnetic field on peristaltic tangent hyperbolic nanofluid past a asymmetric channel. Gaffar et al. [24] showed the tangent hyperbolic fluid flow over a cylinder together with the MHD and partial slip effects. Nagendramma et al. [25] analyzed the tangent hyperbolic fluid flow over a stretching sheet together with the MHD effect. Das et al. [26] investigated the impact of magnetic field, chemical reaction and double-diffusive convection on the Casson fluid flow past a stretching plate and noted that the skin friction coefficient decreases as a result of an augmentation in the Grashof number. Sravanthi and Gorla [27] examined the effect of the Maxwell nanofluid flow over an exponentially stretching sheet together with MHD, chemical reaction and heat source/sink.

Double-diffusion phenomena describe a form of convection driven by two different density gradients, holding

distinct rate of diffusion. Double-diffusive convection occurs in a variety of scientific disciplines such as oceanography, biology, astrophysics, geology, crystal growth and chemical reactions [28]. Nield and Kuznetsov [29] scrutinized the nanofluid past a porous medium along with the double-diffusive convection effect. The impact of double-diffusive convection on the fluid flow over a square cavity is analyzed by Mahapatra et al. [30]. Gireesha et al. [31] discussed the Casson nanofluid past a stretching sheet along with the MHD and double-diffusive convection. Rana and Chand [32] explored the effect of double-diffusive convection on viscoelastic fluid and deduced that a Rayleigh number increases with an improvement in the Soret parameter. Gaikwad et al. [33] have monitored the fluid flow above a stretching sheet together with double-diffusive convection and found that an augmentation in the Nusselt number takes place with an improvement in the Dufour parameter. Kumar et al. [34] inspected the influence of nanoparticles and double diffusion on viscoelastic fluid and monitored that an increase in the velocity field occurs with an increment in the Dufour Lewis number.

Convection is a process common to particles, gases and vapours. Convection occurs when a fluid is in motion and that motion carries with it a material of interest such as the particles or the droplets of an aerosol. There are two types of convection: free convection and forced convection. In free convection or natural convection, the fluid motion cannot led by external sources such as fans, pumps, and suction devices etc. Gravity is the main driving force in the case of free convection. Free convection has various environmental and industrial applications such as plate tectonics, oceanic currents, formation of microstructures during the cooling of molten metals, fluid flows around shrouded heat dissipation fins, solar ponds and free air cooling without the aid of fans. In forced convection, the fluid motion is generated externally with the help of pumps, fans, suction devices, etc. This mechanism has enormous applications in our daily life such as heat exchangers, central heating system, steam turbines and air conditioning. Mixed convection is the situation in which both free convection and forced convection are of comparable order. Mixed convection is of great interest to researchers due to its enormous applications in the industrial and engineering sectors. Ibrahim and Gamachu [35] found the numerical solution of the mixed convective Williamson nanofluid past a stretching sheet by the Galerkin finite element method. Shateyi and Marewo [36] adopted the spectral quasi-linearization method to achieve the numerical solution of the mixed convective magneto Jeffrey fluid flow over an exponentially stretching sheet together with the thermal radiation and observed that the fluid velocity improves with an augmentation in the buoyancy parameter.

Nalinakshi *et al.* [37] found the numerical solution of the mixed convective fluid past a vertical stretched plate using a nonlinear shooting method. El-Aziz and Tamer Nabil [38] gave the numerical solution for the problem of the MHD and Hall current effect on mixed convective fluid past a stretching sheet using the homotopy analysis method (HAM) and noted that a positive variation in the Hall current parameter leads to an increase in the velocity field. Beg *et al.* [39] employed an explicit finite difference scheme to yield the solution of the magneto mixed convection nanofluid flow over a stretchable surface under the effect of MHD and viscous dissipation. The numerical solution of the gravity-driven Navier–Stokes equation has been reported by Zhang *et al.* using a finite difference method [40]. Pal and Chatterjee [41] studied the impact of the Soret and Dufour effects along with nonlinear thermal radiation on the double-diffusive convective fluid past a stretchable surface and achieved the numerical solution for problem using the Runge–Kutta–Fehlberg method along with the shooting scheme. They noted that the velocity field increases with an enhancement in the Grashof number.

The aim of this study was to construct a mathematical model that describes a form of convection driven by two different density gradients, which have different rates of diffusion (double-diffusive convection). So far, no reviews have been reported on the non-Newtonian fluid past a stretching sheet embedded with nanoparticles, double-diffusive convection and motile gyrotactic microorganisms.

2 Mathematical formulation

Figure 1 displays the effect of tangent hyperbolic nanofluid past a stretching sheet with stretching velocity $u_w = ax$ along the x -axis. When the Reynolds number is assumed to be small, the induced magnetic field can be neglected compared with the applied magnetic field B_0 , which is applied transversely to the surface. T_w , γ_w , C_w and N_w denote the temperature, solute concentration, concentration of nanoparticles and density of the motile gyrotactic microorganisms at the wall, respectively, whereas T_∞ , γ_∞ , C_∞ and N_∞ denote the ambient temperature, solute concentration, concentration of nanoparticles and density of the motile gyrotactic microorganisms, respectively. The fluid has further been assumed to contain the gyrotactic microorganisms. The microorganisms present in the fluid move towards light. The “bottom heavy” mass of the microorganisms orients its body and enables them to move against the gravity g , which is called as gyrotactic phenomena. The presence of microorganisms is considered to be beneficial for the suspension of the

Figure 1: Geometry of the problem.

nanofluid. To maintain the stability of convection, the motion of microorganisms has been taken, independent of that of the nanoparticles. The double-diffusive fluid flow over a stretching sheet embedded with gyrotactic microorganisms has not been explored yet, and we want to rectify this problem in this study.

The governing equations include some important effects that have eminent involvement in the industries and engineering fields. The momentum equation includes bioconvection and MHD. MHD has been used in many engineering processes such as nuclear reactor, MHD power generation, in which heat energy is directly converted into electrical energy, Yamato-1 boat incorporating a superconductor cooled by liquid helium and microfluidics. A microorganism or microbe is an organism that is so small that it can be seen only through a microscope (invisible to the naked eye). The presence of microorganisms in the fluid becomes the core area of the research during the past decade. The presence of microorganisms in the base fluid causes a “stabilization” or “destabilization” in the motion of nanoparticles. The microorganisms have various applications in genetic engineering, wastewater engineering, agricultural engineering and chemical engineering. The temperature equation and concentration equations are embedded with nanoparticles and double-diffusive convection. Nanoparticles are used to enhance the thermal conductivity of the fluid and used in tissue engineering, mechanical engineering, nanomedicine, environmental engineering, etc. Double diffusion portrays the form of convection conducted by two different density gradients. There are various examples in environmental engineering such as Arctic Ocean study and Lake Kivu, in which magma, sand and materials of different densities are diffused with water. The same situation is applicable in our modelled problem, in which microorganisms and nanoparticles of different densities are diffused together in the fluid. The last governing equation tells us about the

impact of gyrotactic microorganisms present in the fluid. Various types of microorganisms such as algae, fungi, protozoa and bacteria are suspended in the fluid. These microorganisms swim in the fluid under the combination of gravitational and viscous torques (gyrotactic) in fluid flow. The gyrotactic microorganisms have enormous contribution to genetic engineering, microbial engineering and soil engineering. Under the usual boundary layer approximations, the equations of conservation of mass, momentum, thermal energy, solute, concentration of nanoparticles and gyrotactic microorganisms take the following forms [11–13,31,32,34]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$\begin{aligned} u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = & \nu(1-n) \frac{\partial^2 u}{\partial y^2} + \sqrt{2} \Gamma \nu n \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} \\ & + ((1-\phi_\infty) \rho_f g \beta (T - T_\infty) - g(\rho_p - \rho_f)(\phi - \phi_\infty) \\ & - g(\rho_m - \rho_f) \gamma (N - N_\infty)) - \frac{\sigma_1 B_0^2}{\rho_f} u, \end{aligned} \quad (2)$$

$$\begin{aligned} u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = & \alpha \frac{\partial^2 T}{\partial y^2} + \tau \left(D_B \left(\frac{\partial C}{\partial y} \right) \left(\frac{\partial T}{\partial y} \right) + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right) \\ & + D_{TC} \left(\frac{\partial^2 C}{\partial y^2} \right), \end{aligned} \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_s \frac{\partial^2 C}{\partial y^2} + D_{CT} \left(\frac{\partial^2 T}{\partial y^2} \right), \quad (4)$$

$$u \frac{\partial \phi}{\partial x} + v \frac{\partial \phi}{\partial y} = D_B \frac{\partial^2 \phi}{\partial y^2} + \frac{D_T}{T_\infty} \left(\frac{\partial^2 T}{\partial y^2} \right), \quad (5)$$

$$u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} + \frac{b W_c}{(\phi_w - \phi_\infty)} \frac{\partial}{\partial y} \left(N \frac{\partial \phi}{\partial y} \right) = D_m \frac{\partial^2 N}{\partial y^2}. \quad (6)$$

The subjected conditions at the boundary are as follows:

$$\left. \begin{aligned} u = u_w = ax, \quad v = 0, \quad T = T_w, \quad C = C_w, \\ \phi = \phi_w, \quad N = N_w \quad \text{at } y = 0, \\ u \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty, \quad \phi \rightarrow \phi_\infty, \\ N \rightarrow N_\infty \quad \text{as } y \rightarrow \infty. \end{aligned} \right\} \quad (7)$$

where the symbol “ ρ_f ” depicts the fluid density and “ ρ_p ” represents the density of nanoparticles. The similarity transformations [38] are as follows:

$$\left. \begin{aligned} \eta = \sqrt{\frac{a}{\nu}} y, \quad u = ax f'(\eta), \quad v = -\sqrt{a \nu} f(\eta), \\ \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \gamma = \frac{C - C_\infty}{C_w - C_\infty}, \\ \xi = \frac{\phi - \phi_\infty}{\phi_w - \phi_\infty}, \quad \chi = \frac{N - N_\infty}{N_w - N_\infty}. \end{aligned} \right\} \quad (8)$$

Invoking equation (8), equation (1) is automatically satisfied and equations (2)–(6) become:

$$\begin{aligned} ((1-n) + n \text{We} f'') f''' - (f')^2 + f f'' - M^2 f' \\ + \Lambda (\theta - \text{Nr} \xi - \text{Nc} \chi) = 0, \end{aligned} \quad (9)$$

$$\theta'' + \text{Pr} (f \theta' + \text{Nb} \theta' \xi') + \text{Nt} (\theta')^2 + \text{Nd} \gamma'' = 0, \quad (10)$$

$$\gamma'' + \text{Pr} \text{Le} f \gamma' + \text{Ld} \text{Pr} \theta'' = 0, \quad (11)$$

$$\xi'' + \text{Pr} \text{Ln} f \xi' + \frac{\text{Nt}}{\text{Nb}} \theta'' = 0, \quad (12)$$

$$\chi'' + \text{Lb} f \chi' - \text{Pe} (\chi' \xi' + \xi'' (\sigma + \chi)) = 0, \quad (13)$$

with the following boundary conditions:

$$\left. \begin{aligned} f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1, \quad \gamma(0) = 1, \\ \xi(0) = 1, \quad \chi(0) = 1 \quad \text{at } \eta = 0, \\ f'(\eta) \rightarrow 0, \quad \theta(\eta) \rightarrow 0, \quad \gamma(\eta) \rightarrow 0, \quad \xi(\eta) \rightarrow 0, \\ \chi(\eta) \rightarrow 0 \quad \text{at } \eta \rightarrow \infty. \end{aligned} \right\} \quad (14)$$

Distinct physical parameters arising after the conversion of PDEs into ODEs are as follows:

$$\left. \begin{aligned} \text{We} = \Gamma \chi \sqrt{\frac{2a^3}{\nu}}, \quad \text{Nb} = \frac{\tau D_B}{\nu} (C_w - C_\infty), \\ \text{Nt} = \frac{D_T}{T_\infty} \frac{\tau}{\nu} (T_w - T_\infty), \quad \text{Le} = \frac{\alpha}{D_s}, \\ \text{Ln} = \frac{\alpha}{D_B}, \quad \text{Pe} = \frac{b W_c}{D_m}, \quad \text{Ld} = \frac{\alpha}{D_m}, \\ \text{Nd} = \frac{\alpha D_{TC} (C_w - C_\infty)}{\nu (T_w - T_\infty)}, \quad \sigma = \frac{N_\infty}{N_w - N_\infty}, \\ G_T = \frac{\chi^3 (1 - C_\infty) \rho_f g \beta_T (T_w - T_\infty)}{\nu^2}, \\ \tau = \frac{\rho C_p}{\rho C_f}, \quad \text{Nr} = \frac{(\rho_p - \rho_f)(\phi_w - \phi_\infty)}{(1 - \phi_\infty) \rho_f \beta (T_w - T_\infty)}, \\ M = \frac{\sigma B_0^2}{a \rho_f}, \quad \text{Pr} = \frac{\nu}{\alpha}, \quad \text{Lb} = \frac{\alpha}{D_m}, \quad \Lambda = \frac{G_T}{\text{Re}_x^2}. \end{aligned} \right\} \quad (15)$$

The important quantities of interest like rate of shear stress C_f and heat as well as mass transfer rates Nu_x and Sh_x and $Sh_{x,n}$ and Nn_x are as follows:

$$\left. \begin{aligned} C_f &= \frac{2\tau_w}{\rho u_w^2}, \\ Nu_x &= \frac{xq_w}{k(T_w - T_{\infty})}, \\ Sh_x &= \frac{xq_m}{D_B(\phi_w - \phi_{\infty})}, \\ Sh_{x,n} &= \frac{xq_{mn}}{D_s(C_w - C_{\infty})}, \\ Nn_x &= \frac{xq_n}{D_n(N_w - N_{\infty})}, \end{aligned} \right\} \quad (16)$$

whereas expressions regarding τ_w , q_w , q_m , q_{mn} and q_n are as follows:

$$\left. \begin{aligned} \tau_w &= \mu(1-n) \frac{\partial u}{\partial y} \Big|_{y=0} + \mu \frac{n\Gamma}{\sqrt{2}} \left(\frac{\partial u}{\partial y} \right)^3 \Big|_{y=0}, \\ q_w &= -k \frac{\partial T}{\partial y} \Big|_{y=0}, \\ q_m &= -D_B \frac{\partial \phi}{\partial y} \Big|_{y=0}, \\ q_{mn} &= -D_s \frac{\partial C}{\partial y} \Big|_{y=0}, \\ q_n &= -D_n \frac{\partial \chi}{\partial y} \Big|_{y=0}. \end{aligned} \right\} \quad (17)$$

By substituting equation (17) into equation (16) and using the similarity transformation, the quantities defined in equation (17) are nondimensionalized as follows:

$$\left. \begin{aligned} \frac{1}{2} C_f Re_x^{1/2} &= (1-n)f''(0) - \frac{1}{2} n We (f''(0))^3, \\ Nu_x Re_x^{-1/2} &= -\theta'(0), \\ Sh_x Re_x^{-1/2} &= -\xi'(0), \\ Sh_{x,n} Re_x^{-1/2} &= -\gamma'(0), \\ Nn_x Re_x^{-1/2} &= -\chi'(0), \end{aligned} \right\} \quad (18)$$

where $Re_x = \frac{u_w x}{\nu}$.

3 Numerical scheme

The dimensionless system of equations (9)–(13) along with the boundary condition (14) should be handled with the help of the numerical scheme called the implicit finite difference

Figure 2: Mechanism of the present technique.

$$\left. \begin{aligned} f'(\eta) &= z_1, & z'_1 &= z_2, & \theta'(\eta) &= z_3, & \gamma' &= z_4, \\ \xi' &= z_5, & \chi' &= z_6 \end{aligned} \right\} \quad (19)$$

method (Keller box technique) [42,43] for distinguished parameters that emerged during numerical simulation of the problem. Such type of differential equations in this article can usually be solved with the help of other numerical techniques such as shooting method, HAM and bvp4c [27,31–34,44–49]. In this study, the standard Keller box method has been used. This numerical technique is quite effective and flexible to solve the parabolic-type boundary value problems of any order, is unconditionally stable and attains remarkable accuracy. The Keller box scheme is numerically more stable and converges using less iterations compared with other numerical techniques. Figure 2 shows the flow chart procedure of the Keller box method. By adopting the new variables z_1 , z_2 , z_3 , z_4 , z_5 and z_6 ,

The dimensionless equations (9)–(13) are transformed into first-order differential equations (ODEs) as follows:

$$\begin{aligned} ((1-n) + n We z_1) z_2 - z_1^2 + f z_2 - M^2 z_1 \\ + \Lambda(\theta - Nr \xi - Nc \chi) = 0, \end{aligned} \quad (20)$$

$$z_3' + \text{Pr}(fz_3 + \text{Nb } z_3 z_5) + \text{Nt } z_3^2 + \text{Nd } z_4' = 0, \quad (21)$$

$$z_4' + \text{Pr } \text{Le } fz_4 + \text{Ld } \text{Pr } z_3' = 0 \quad (22)$$

$$z_5' + \text{Pr } \text{Ln } z_5 + \frac{\text{Nt}}{\text{Nb}} z_3' = 0, \quad (23)$$

$$z_6' + \text{Lb } fz_6 - \text{Pe}(z_6 z_5 + z_5'(\sigma + \chi)) = 0. \quad (24)$$

The transformed boundary conditions are as follows:

$$\left. \begin{aligned} f(0) = 0, \quad z_1(0) = 1, \quad \theta(0) = 1, \quad \gamma(0) = 1, \\ \xi(0) = 1, \quad \chi(0) = 1 \quad \text{at } \eta = 0, \\ z_1(\eta) \rightarrow 0, \quad \theta(\eta) \rightarrow 0, \quad \gamma(\eta) \rightarrow 0, \quad \xi(\eta) \rightarrow 0, \\ \chi(\eta) \rightarrow 0 \quad \text{at } \eta \rightarrow \infty. \end{aligned} \right\} \quad (25)$$

Figure 3 portrays the mesh structure for central difference approximations. The stepping procedure for the selection of the nodes in the case of domain discretization is as follows:

$$\eta_0 = 0, \quad \eta_j = \eta_{j-1} + \eta_j, \quad j = 1, 2, 3, \dots, J, \quad \eta_J = \eta_{\max}.$$

The derivatives of equations (20)–(24) are approximated by employing the central difference at the midpoint $\eta_{j-1/2}$ given below:

$$\frac{f_j - f_{j-1}}{h_j} = \frac{(z_1)_j + (z_1)_{j-1}}{2}, \quad (26)$$

$$\frac{(z_1)_j - (z_1)_{j-1}}{h_j} = \frac{(z_2)_j + (z_2)_{j-1}}{2}, \quad (27)$$

Figure 3: One-dimensional mesh for difference approximations.

$$\frac{\theta_j - \theta_{j-1}}{h_j} = \frac{(z_3)_j + (z_3)_{j-1}}{2}, \quad (28)$$

$$\frac{\gamma_j - \gamma_{j-1}}{h_j} = \frac{(z_4)_j + (z_4)_{j-1}}{2}, \quad (29)$$

$$\frac{\xi_j - \xi_{j-1}}{h_j} = \frac{(z_5)_j + (z_5)_{j-1}}{2}, \quad (30)$$

$$\frac{\chi_j - \chi_{j-1}}{h_j} = \frac{(z_6)_j + (z_6)_{j-1}}{2}, \quad (31)$$

$$\begin{aligned} & \left[(1 - n) + n \text{We} \left(\frac{(z_2)_j + (z_2)_{j-1}}{2} \right) \right] z_2 \\ & - \left(\frac{(z_1)_j + (z_1)_{j-1}}{2} \right)^2 - M^2 \left(\frac{(z_1)_j + (z_1)_{j-1}}{2} \right) \\ & + \left(\frac{f_j + f_{j-1}}{2} \right) \left(\frac{(z_2)_j + (z_2)_{j-1}}{2} \right) \\ & + \Lambda \left(\frac{\theta_j + \theta_{j-1}}{2} \right) - \Lambda \text{Nr} \left(\frac{\xi_j + \xi_{j-1}}{2} \right) \\ & - \Lambda \text{Nc} \left(\frac{\chi_j + \chi_{j-1}}{2} \right) = 0, \end{aligned} \quad (32)$$

$$\begin{aligned} & \left(\frac{(z_3)_j + (z_3)_{j-1}}{h_j} \right) + \text{Pr} \left(\frac{(z_3)_j + (z_3)_{j-1}}{2} \right) \\ & \left(\frac{f_j + f_{j-1}}{2} \right) + \text{Nt} \left(\frac{(z_3)_j + (z_3)_{j-1}}{2} \right)^2 \\ & + \text{Pr } \text{Nb} \left(\frac{(z_3)_j + (z_3)_{j-1}}{2} \right) \left(\frac{(z_5)_j + (z_5)_{j-1}}{2} \right) \\ & + \text{Nd} \left(\frac{(z_4)_j + (z_4)_{j-1}}{2} \right) = 0, \end{aligned} \quad (33)$$

$$\begin{aligned} & \left(\frac{(z_4)_j + (z_4)_{j-1}}{h_j} \right) + \text{Pr } \text{Ld} \left(\frac{(z_3)_j + (z_3)_{j-1}}{h_j} \right) \\ & + \text{Pr } \text{Le} \left(\frac{f_j + f_{j-1}}{2} \right) \left(\frac{(z_4)_j + (z_4)_{j-1}}{2} \right) = 0, \end{aligned} \quad (34)$$

$$\begin{aligned} & \left(\frac{(z_5)_j + (z_5)_{j-1}}{h_j} \right) + \left(\frac{\text{Nt}}{\text{Nb}} \right) \left(\frac{(z_3)_j + (z_3)_{j-1}}{h_j} \right) \\ & + \text{Pr } \text{Ln} \left(\frac{f_j + f_{j-1}}{2} \right) \left(\frac{(z_5)_j + (z_5)_{j-1}}{2} \right) = 0, \end{aligned} \quad (35)$$

$$\begin{aligned}
& \left(\frac{(z_6)_j + (z_6)_{j-1}}{h_j} \right) + \text{Lb} \left(\frac{(z_6)_j + (z_6)_{j-1}}{2} \right) \\
& \left(\frac{f_j + f_{j-1}}{2} \right) - \text{Pe} \left(\frac{(z_6)_j + (z_6)_{j-1}}{2} \right) \\
& \left(\frac{(z_5)_j + (z_5)_{j-1}}{2} \right) - \text{Pe} \left(\frac{(z_5)_j + (z_5)_{j-1}}{h_j} \right) \\
& \left(\sigma + \left(\frac{\chi_j + \chi_{j-1}}{2} \right) \right) = 0,
\end{aligned} \quad (36)$$

$$\left. \begin{aligned}
f_j^{n+1} &= f_j^n + \delta f_j^n, \quad (z_1)_j^{n+1} = (z_1)_j^n + \delta (z_1)_j^n, \\
(z_2)_j^{n+1} &= (z_2)_j^n + \delta (z_2)_j^n, \quad (z_3)_j^{n+1} = (z_3)_j^n + \delta (z_3)_j^n, \\
(z_4)_j^{n+1} &= (z_4)_j^n + \delta (z_4)_j^n, \quad (z_5)_j^{n+1} = (z_5)_j^n + \delta (z_5)_j^n, \\
(z_6)_j^{n+1} &= (z_6)_j^n + \delta (z_6)_j^n, \quad \theta_j^{n+1} = \theta_j^n + \delta \theta_j^n, \\
\gamma_j^{n+1} &= \gamma_j^n + \delta \gamma_j^n, \quad \xi_j^{n+1} = \xi_j^n + \delta \xi_j^n, \\
\chi_j^{n+1} &= \chi_j^n + \delta \chi_j^n.
\end{aligned} \right\} \quad (37)$$

After linearization of the above-mentioned system of equations, the subsequent block-tridiagonal block structure:

$$A = \begin{bmatrix} [A_1] & [B_1] & & & \\ [C_2] & [A_2] & [B_2] & & \\ & & \ddots & \ddots & \\ & & & \ddots & \\ & & & & [C_{j-1}] & [A_{j-1}] & [B_{j-1}] \\ & & & & & [C_j] & [A_j] \end{bmatrix},$$

$$\delta = \begin{bmatrix} [\delta_1] \\ [\delta_2] \\ \vdots \\ \vdots \\ [\delta_{j-1}] \\ [\delta_j] \end{bmatrix}, \quad R = \begin{bmatrix} [R_1] \\ [R_2] \\ \vdots \\ \vdots \\ [R_{j-1}] \\ [R_j] \end{bmatrix}$$

or

$$[A][\delta] = [R], \quad (38)$$

where A is the $j \times j$ tridiagonal matrix of block size 11×11 , and δ and R are the column matrices of j rows. Now equation (38) has been tackled using the LU factorization method with lower triangular matrix L and upper triangular matrix U enumerated as follows:

$$L = \begin{bmatrix} [\alpha_1] & & & & \\ [\beta_2] & [\alpha_2] & & & \\ & & \ddots & & \\ & & & [\alpha_{j-1}] & \\ & & & [\beta_j] & [\alpha_j] \end{bmatrix},$$

$$U = \begin{bmatrix} [I] & [\xi_1] & & & \\ & [I] & [\xi_2] & & \\ & & \ddots & \ddots & \\ & & & [I] & [\xi_{j-1}] \\ & & & & [I] \end{bmatrix}.$$

To solve the problem numerically, the domain of the problem has been considered $[0, \eta_{\max}]$ instead of $[0, \infty)$, where $\eta_{\max} = 16$ and the step size is $h_j = 0.01$. All the numerical results achieved in this problem are subjected to an error tolerance of 10^{-5} .

Table 1 displays the comparison analysis of the given numerical scheme results with Ibrahim [40].

Table 1: Numerical comparison of the obtained results with Ibrahim [40] for various values of Pr

Pr	Ibrahim [40]	This study
0.00	1.0000	1.00000
0.25	1.1180	1.11802
1.00	1.4142	1.41411

4 Results and discussion

To discuss the outcomes, the behaviour of various pertinent parameters against the Nusselt number, the Sherwood number, motile density profile, velocity field, temperature field, mass fraction field and solute profile is monitored. Table 2 exhibits the behaviour of distinguished parameters on heat transfer at the boundary, mass fraction field and the motile microorganisms density profile for thermophoresis parameter (Nt) = 0.1, Prandtl number (Pr) = 6.2, Lewis number (Le) = 0.5, Dufour Lewis number (Ld) = 0.1 and mixed convection parameter Λ = 0.1. The heat transfer rate diminishes in the case of magnetic parameter M , Weissenberg number (We), modified Dufour parameter (Nd), power law index n , nanofluid Lewis number (Ln) and buoyancy ratio parameter (Nr), whereas an embellishment in the Nusselt number is seen for the Brownian motion parameter (Nb) and the bioconvection Rayleigh number (Nc). The Nusselt number has shown no variation in the

Table 2: Variation in $Nu_x Re_x^{-1/2}$, $Sh u_x Re_x^{-1/2}$ and $Nn_x Re_x^{-1/2}$ for different parameters when $Nt = 0.1$, $Pr = 6.2$, $Le = 0.5$, $Ld = 0.1$ and $\Lambda = 0.1$ are fixed

M	Nc	Nr	Nb	We	n	σ	Pe	Lb	Nd	Ln	$-\theta'(0)$	$-\xi'(0)$	$-\chi'(0)$
0.1	0.5	0.5	0.1	0.3	0.2	0.5	1	1	0.1	2	0.93786	1.48950	1.30837
0.2											0.93815	1.50424	1.32242
0.3											0.93841	1.51812	1.33562
	0.1										2.03841	2.71812	3.45016
	0.3										2.03853	2.72591	2.63562
	0.5										2.03865	2.73339	2.64400
		0.1									2.05487	3.12893	2.93505
		0.2									2.05483	3.12731	2.93360
		0.3									2.05480	3.12569	2.93214
			0.4								0.33911	7.72999	11.6733
			0.5								0.55922	7.70119	11.6306
			0.6								0.68789	7.69032	11.6146
				0.1							0.82446	4.07381	6.23334
				0.2							0.82438	4.06941	6.22659
				0.3							0.82431	4.06476	6.21943
					0.3						0.82367	4.03450	6.17277
					0.4						0.82271	3.98592	6.09800
					0.5						0.82134	3.90893	5.97975
						0.1					0.82438	4.06941	4.69593
						0.2					0.82438	4.06941	5.07859
						0.3					0.82438	4.06941	5.46126
							0.1				0.82438	4.06941	1.03179
							0.5				0.82438	4.06941	3.31479
							1				0.82438	4.06941	6.22659
								0.5			0.82438	4.06941	3.31479
								1			0.82438	4.06941	6.22659
								1.5			0.82438	4.06941	9.18276
									0.1		0.89710	2.21688	3.50681
									0.2		0.81434	2.20547	3.48796
									0.3		0.78569	2.18375	3.45366
										1	0.89710	2.21688	3.50681
										2	0.85497	3.26596	5.04071
										3	0.82438	4.06941	6.22659

case of microorganism concentration difference parameter σ , Peclet number (Pe) and bioconvection Lewis number (Lb). The mass fraction field depreciates in the case of M , Nc , nanofluid Lewis number (Ln) and buoyancy ratio parameter (Nr), but a positive variation is observed for Nb , We , Nd and n , whereas static behaviour is seen for σ , Pe and Lb . Furthermore, the number of motile microorganisms has been seen to increase in the case of positive variation in M , σ , Pe , Lb and Ln , but the situation is opposite in the case of Nr , Nc , n , We , Nd and Nb .

Figure 4 exhibits the effect of the magnetic parameter M on the velocity profile $f'(\eta)$. It has been found that an increase in M decreases the velocity profile. Actually, the resistive force called the Lorentz force is generated due to the application of the magnetic field to the electrically conducting fluid. As a result, the velocity

Figure 4: Effect of parameter M on the velocity profile.

of the fluid reduces. Figure 5 indicates the effect of n on the velocity field $f'(\eta)$. The parameter decides the viscosity of the fluid or how much viscous the fluid is.

Figure 5: Effect of parameter n on the velocity profile. Figure 7: Effect of parameter M on the temperature profile.

The fluid behaves like shear thinning for the case of $n < 1$, shear thickening for the larger values of $n > 1$ and Newtonian in the case of $n = 1$. The velocity of the fluid decreases in the case of $n > 1$, and as a result, the velocity field diminishes. Figure 6 depicts the effect of $f'(\eta)$ on We . The Weissenberg number is defined as the ratio of viscous forces to the inertial forces. This parameter is important to study the fluid flow behaviour. The Weissenberg number actually depicts the elastic nature of the fluid. It is noted that the higher values of the Weissenberg number indicate the solid nature of the fluid, while lower values of the Weissenberg number depict the liquid nature of the fluid. It is clear that an augmentation in the Weissenberg number leads to a reduction in the velocity of the fluid. Figure 7 highlights the variation in the temperature profile $\theta(\eta)$ against the various values of M and observed that an electric current in the presence of magnetic field generates a Lorentz force. This force resists the motion of the fluid; hence, additional heat is produced, which enhances the fluid temperature. Figure 8 highlights the behaviour of

Figure 8: Effect of parameter Pr on the temperature profile. Figure 6: Effect of parameter We on the velocity profile.

temperature field $\theta(\eta)$ against Pr . The Prandtl number is a dimensionless quantity, which is defined as the ratio of momentum diffusivity to thermal diffusivity and has important application in the study of boundary layer concept. The thermal diffusivity dominates in the case of $Pr \ll 1$, whereas momentum diffusivity dominates in the case of $Pr \gg 1$. It is observed that the fluids with small Prandtl number are free flowing liquids with high thermal conductivity and favourable choice for heat conducting fluids. The thermal conductivity of the fluid decreases with an augmentation in the value of Pr , and the heat transfer decelerates, which decreases the temperature of the flow field, and as a result, a decrease in the temperature is observed.

Figure 9 portrays the effect of Brownian diffusion parameter (Nb) on the temperature distribution $\theta(\eta)$. Brownian motion is actually the random motion of the particles suspended in the fluid. The temperature of the fluid increases as a result of the random collision of particles suspended in the liquid, which further leads to an expected improvement in the temperature profile $\theta(\eta)$.

Figure 9: Effect of parameter Nb on the temperature profile.

Figure 11: Effect of parameter Nd on the temperature profile.

Figure 10: Effect of parameter Nt on the temperature profile.

Figure 12: Effect of parameter Nb on the concentration profile.

Figure 10 explores the effect of the thermophoresis parameter (Nt) on the temperature distribution $\theta(\eta)$. In the thermophoresis process, smaller particles migrate from the region having high temperature to the region having low temperature, which ultimately causes an improvement in the fluid temperature.

Figure 11 shows the behaviour of temperature profile $\theta(\eta)$ against the different values of Nd . The situation in which heat and mass transfer happens simultaneously in a moving fluid affecting each other causes a cross-diffusion. The mass transfer caused by temperature gradient is called the Soret effect, whereas the heat transfer caused by concentration is called the Dufour effect. The Dufour number implies the effect of the concentration on the thermal energy flux in the flow. It is found that a variation in the modified Dufour number leads to a monotonic enhancement in the temperature field $\theta(\eta)$. Figure 12 highlights the effect of Nb on the mass fraction field. Brownian diffusion and thermophoresis parameters emerge as a result of an inclusion of nanoparticles into the fluid. Brownian diffusion and thermophoresis parameters help to understand the

motion of the nanoparticles in the fluid. It is verified that the higher values of Nb are the root cause to boost the random motion among the nanoparticles present in the fluid. This results in the decrease in the concentration of the fluid.

Figure 13 describes the effect of Nt on the mass fraction field. It is observed that increasing values of Nt push nanoparticles away from the warm surface. The density of the concentration boundary layer upsurges due to an augmentation in the value of Nt , which leads to an embellishment in the mass fraction field. Figure 14 portrays the effect of the nanofluid Lewis parameter (Ln) on the mass fraction field. The Lewis number is defined as the ratio of thermal diffusivity to the mass diffusivity, and it is the prominent factor to study the heat and mass transfer. It is observed that the concentration profile decreases due to the dependence of the Lewis number on the Brownian diffusion coefficient, which means that an augmentation in the Brownian diffusion coefficient brings about a decrease in the concentration profile and the nanofluid Lewis number.

Figure 13: Effect of parameter Nt on the concentration profile. Figure 16: Effect of parameter Lb on the microorganism profile. Figure 14: Effect of parameter Ln on the concentration profile.

Figure 15 shows the effect of Peclet number (Pe) on the microrotation distribution $\chi(\eta)$. The Peclet number is the prominent factor to study the microorganisms swimming in the fluid. The Peclet number is defined as the ratio of maximum cell swimming speed to diffusion of microorganisms. Diffusion is the process in which a

substance moves from an area of high concentration to an area of low concentration. It explains the movement of the substances in the fluid. It is found that diffusivity of microorganisms is decreased in the case of an augmentation in Pe . As a result, the microrotation distribution declines. Figure 16 depicts the effect of the bioconvection Lewis number (Lb) on the microrotation distribution. Similar to Figure 14, an augmentation in Lb results in a decrease in the diffusivity of microorganisms, which results in the reduction of the motile density profile.

Figure 17 portrays the effect of microorganism concentration difference parameter σ on the motile density profile. It is observed that by increasing the value of σ , the concentration of microorganisms in ambient fluid is decreased. Figure 18 delineates the effect of the regular Lewis number (Le) on the solute profile $\gamma(\eta)$. The Lewis number is defined as the ratio of thermal diffusivity to mass diffusivity. As seen in Figure 13, the Lewis number is related to the Brownian diffusion coefficient. It is observed that a positive variation in

Figure 15: Effect of parameter Pe on the microorganism profile. Figure 17: Influence of parameter σ on the microorganism profile.

Figure 18: Effect of parameter Le on the solute profile. Figure 19: Effect of parameter Ld on the solute profile.

Brownian diffusion leads to a decrease in the concentration of particles. Thus, a positive variation in the Lewis number (Le) leads to a decrease in the solute profile. Figure 19 portrays the relationship between the Dufour Lewis number (Ld) and the solute profile $\gamma(\eta)$. The Dufour Lewis number depicts the influence of temperature gradient on the

Figure 20: Effect of parameter Nb on the Sherwood number.

concentration field. It is perceived that the concentration gradient excites the flow with an enhancement in the thermal energy, which results in an increase in the solute profile. Figure 20 depicts the effect of mass fraction field on Nb for the distinguished values of the nanofluid Lewis number (Ln). It is also observed that due to the random collision of molecules, the heat transfer process escalates and nanoparticle diffusion reduces, which results in an increment in the Sherwood number.

Figure 21: Effect of parameter Nt on the Sherwood number.

Figure 21 elucidates the performance of Nt on the mass fraction field for various values of Ln . It is found that in the presence of the thermophoretic force, the nanoparticles present close to the hot boundary have been shifted towards the cold fluid, which decreases the thermal boundary layer and heightens the nanofluid Lewis number. An upsurge in Nt escalates nanofluid Lewis number (Ln) and further leads to an augmentation in the mass fraction field. Figure 22 presents the effect of microorganism concentration difference parameter σ on the density number of microrotation distribution for different values of Peclet number (Pe). A positive

Figure 22: Effect of σ on the microorganism density profile.

Figure 23: Effect of parameter L_d on the solutal Sherwood number.

Figure 24: Effect of parameter Le on the solutal Sherwood number.

variation in σ lessens the thickness of the boundary layer and leads to an increment in the concentration of the motile gyrotactic microorganisms. Figure 23 elucidates the conduct of the Dufour Lewis number (L_d) on the solutal Sherwood number for different values of the Prandtl number. The Lewis number is defined as the ratio of thermal diffusivity to momentum diffusivity. It is observed that an enhancement in Lewis number drives more heat within the fluid, which brings about an augmentation in the Prandtl number. It is noteworthy that a positive variation in the Dufour Lewis parameter leads to an augmentation in the solutal Sherwood number. Figure 24 elaborates the effect of Lewis number (Le) on the solutal Sherwood number. It has been observed that the solutal Sherwood number increases with an augmentation in the Lewis number.

5 Concluding remarks

This article elaborates the effects of nanoparticles and double-diffusive convection along with motile gyrotactic

microorganisms on the non-Newtonian fluid past a stretching sheet. To our knowledge, no model has been developed so far to see the impact of gyrotactic microorganisms and double-diffusive convection simultaneously on the non-Newtonian hyperbolic tangent nanofluid, and furthermore, a numerical technique (Keller box) has been used to achieve the numerical solution of the problem. A comparison with the previous literature was made to check the reliability of our proposed numerical scheme. The results are quite promising. Some of the key findings of the present investigation are as follows:

- An improvement in the Weissenberg number (We) leads to a decrease in the velocity profile.
- The mass fraction field shows an opposite behaviour as a result of variation in the nanofluid Lewis number (Ln).
- A positive variation in the Peclet number (Pe) leads to a decrease in the solute profile.
- The microrotation distribution profile declines with an improvement in the bioconvection Lewis number (Lb) and microorganism concentration difference parameter σ .
- The solute profile is decreased with an enhancement in the regular Lewis number (Le).

Nomenclature

a	stretching rate
B_0	magnetic field strength
C_∞	ambient concentration
C_∞	ambient solute concentration at the wall
C_f	skin friction coefficient
C_p	specific heat
C_w	solute concentration at the wall
D_B	Brownian diffusion
D_m	diffusivity of the microorganisms
D_T	thermophoresis diffusion
g	gravity
G_T	Grashof number
k	thermal conductivity
Lb	bioconvection Lewis number
L_d	Dufour Lewis number
Le	Lewis number
Ln	nanofluid Lewis number
M	magnetic parameter
n	power law index
N_∞	ambient density of the motile microorganisms

Nb	Brownian motion
Nd	modified Dufour parameter
Nt	thermophoresis parameter
N_w	density of the motile microorganisms at the wall
Pe	Peclet number
Pr	Prandtl number
q_m	wall mass flux
q_w	wall heat flux
Re	Reynolds number
T_∞	ambient temperature
T_w	wall temperature
u, v	velocity components
U_w	stretching velocity along the x -axis
W_c	cell swimming speed
We	Weissenberg number
α	thermal diffusivity
γ	solute profile
μ	dynamic viscosity
ν	kinematic viscosity
Λ	mixed convection parameter
ρ	density of the fluid
ρ_m	density of the microorganisms
ρ_p	density of the nanoparticles
σ	microorganism concentration difference
σ_1	electric conductivity
τ_w	wall shear stress
ϕ	volume fraction of the nanoparticles

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