

Two-dimensional oblique stagnation-point flow towards a stretching surface in a viscoelastic fluid

Research Article

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Abstract: In this paper, the steady two-dimensional stagnation-point flow of a viscoelastic Walters' B' fluid over a stretching surface is examined. It is assumed that the fluid impinges on the wall obliquely. Using similarity variables, the governing partial differential equations are transformed into a set of two non-dimensional ordinary differential equations. These equations are then solved numerically using the shooting method with a finite-difference technique.

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Keywords: stagnation-point • viscoelastic fluid • stretching sheet • steady • shooting method

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1. Introduction

In the history of fluid dynamics, considerable attention has been given to the study of stagnation-point flows because of their importance in many engineering applications. In some situations, flow is stagnated by a solid wall, while in others a free stagnation-point or line exists interior to the fluid domain. The theoretical study of the flow of a viscous (Newtonian) fluid near the two-dimensional stagnation point on a semi-infinite wall has been first done by Hiemenz [1] in 1911. In a stagnation-point, a rigid or a stretching wall occupies the entire horizontal axis and the fluid domain is $y > 0$. The fluid impinges on the wall

either orthogonally or obliquely.

The flow of a fluid over a stretching plate is important in many industrial processes. The extrusion of plastic sheets, fabrication of adhesive tapes and application of coating layers onto rigid substrates are some of the examples. Polymer sheets are manufactured by continuous extrusion of the polymer from a die to a windup roller. The thin polymer sheet constitutes a continuously moving surface with a non-uniform velocity though the ambient fluid [2]. Crane [3] and McCormack and Crane [4] investigated the steady two-dimensional flow of an incompressible fluid over a stretching sheet which moves in its own plane with a velocity varying linearly with the distance from a fixed point due to the application of a uniform stress. Afterwards, several authors [5–9] have investigated various aspects of this problem such as the effects of surface mass transfer, thermal conductivity or wall

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temperature (heat flux). McLeod and Rajagopal [10] have studied the uniqueness of the exact solution of the flow of a Newtonian fluid due to a stretching boundary. Chiam [11] studied the steady two-dimensional stagnation-point flow of a viscous incompressible fluid towards a stretching surface when it is stretched in its own plane with a velocity proportional to the distance from the stagnation point. Mahapatra and Gupta [12] studied the heat transfer in a stagnation-point flow towards a stretching sheet when it is stretched in its own plane with a velocity proportional to the distance from the stagnation point. Lok et al. [13] investigated the non-orthogonal stagnation-point flow towards a stretching sheet and they found that the free stream obliqueness is the shift of the stagnation point toward the incoming flow and it depends on the inclination angle. Reza and Gupta [14], and Mahapatra et al. [15] studied the steady two-dimensional oblique stagnation-point flow and heat transfer of a Newtonian fluid towards a stretching surface.

In recent years, considerable attention has been given to non-Newtonian fluids because of their importance in industrial applications. The equations of motion of such fluids are highly non-linear and one order higher than the Navier-Stokes equations. For this reason, one will require boundary conditions in addition to the non-slip condition to have a well-posed problem. Only in some special cases where the higher order nonlinear terms in these equations can be neglected thereby reducing their order, are the “no-slip” condition sufficient to yield unique solutions. In general, Rajagopal [16], Rajagopal and Gupta [17], Rajagopal [18] and Rajagopal and Kaloni [19] have shown that the absence of this additional boundary condition leads to non-unique solutions for problems involving the flow of second grade fluids in a bounded domain. Therefore, the “no-slip” condition is insufficient to solve the equations of motions of second-grade fluids completely when the higher-order non-linearities in these equations cannot be ignored. Rajagopal et al. [20] studied steady flow of a second-order fluid past a stretching sheet. The temperature distribution of a steady flow of a second-order fluid was investigated by Bhattacharyya et al. [21]. Mahapatra and Gupta [22] considered the stagnation-point flow of a viscoelastic fluid towards a stretching surface. In their work the fluid impinges the wall orthogonally.

In this paper, the steady two-dimensional stagnation-point flow of a viscoelastic Walters’ B’ fluid over a stretching surface is examined. It is assumed that the fluid impinges on the wall obliquely. This flow appears when a viscous fluid jet impinges on a wall obliquely. In particular, we investigate the behaviour of the Walters’ B’ fluid near the wall for different values of the viscoelastic parameter. Using similarity variables, the governing partial

differential equations are transformed into a set of two non-dimensional ordinary differential equations. These equations are then solved numerically using the shooting method with a finite-difference technique. It is worth mentioning to this end that there are various analytical techniques to solve nonlinear differential equations arising in physics and engineering. Thus, various perturbation and non-perturbation techniques such as small parameter method and homotopy perturbation method (HAM), which has acquired a lot of credence in tackling nonlinear problems arising in the literature involving Newtonian and non-Newtonian fluids (see Hayat et al. [23–28]).

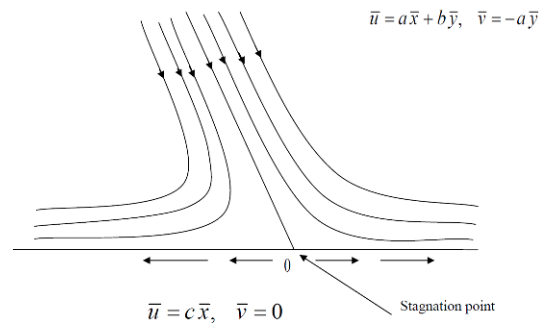


Figure 1. A sketch of the physical plane.

2. Basic equations

Consider the steady two-dimensional flow of a viscoelastic Walters’ B’ fluid near a non-orthogonal stagnation point at a stretching flat plate coinciding with the line $\bar{y} = 0$, the flow being confined to $\bar{y} > 0$. Cartesian coordinates (\bar{x}, \bar{y}) fixed in space are taken, the \bar{x} -axis being along the plate and the \bar{y} -axis normal to it, respectively as shown in Figure 1. The steady two-dimensional flow of a viscoelastic Walters’ B’ fluid is described by the following equations, see Beard and Walters [29],

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \tag{1}$$

$$\begin{aligned} & \bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial \bar{x}} + \nu \nabla^2 \bar{u} \\ & + \frac{k_0}{\rho} \left\{ \frac{\partial}{\partial \bar{x}} \left[2\bar{u} \frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + 2\bar{v} \frac{\partial^2 \bar{u}}{\partial \bar{x} \partial \bar{y}} + 4 \left(\frac{\partial \bar{u}}{\partial \bar{x}} \right)^2 \right. \right. \\ & + 2 \frac{\partial \bar{v}}{\partial \bar{x}} \left(\frac{\partial \bar{v}}{\partial \bar{x}} + \frac{\partial \bar{u}}{\partial \bar{y}} \right) \left. \right] + \frac{\partial}{\partial \bar{y}} \left[\left(\bar{u} \frac{\partial}{\partial \bar{x}} + \bar{v} \frac{\partial}{\partial \bar{y}} \right) \right. \\ & \left. \left. \cdot \left(\frac{\partial \bar{v}}{\partial \bar{x}} + \frac{\partial \bar{u}}{\partial \bar{y}} \right) + 2 \frac{\partial \bar{u}}{\partial \bar{x}} \frac{\partial \bar{u}}{\partial \bar{y}} + 2 \frac{\partial \bar{v}}{\partial \bar{x}} \frac{\partial \bar{v}}{\partial \bar{y}} \right] \right\}, \tag{2} \end{aligned}$$

$$\begin{aligned} & \bar{u} \frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{v}}{\partial \bar{y}} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial \bar{y}} + \nu \nabla^2 \bar{v} \\ & + \frac{k_0}{\rho} \left\{ \frac{\partial}{\partial \bar{x}} \left[\left(\bar{u} \frac{\partial}{\partial \bar{x}} + \bar{v} \frac{\partial}{\partial \bar{y}} \right) \left(\frac{\partial \bar{v}}{\partial \bar{x}} + \frac{\partial \bar{u}}{\partial \bar{y}} \right) \right] \right. \\ & + 2 \frac{\partial \bar{u}}{\partial \bar{x}} \frac{\partial \bar{u}}{\partial \bar{y}} + 2 \frac{\partial \bar{v}}{\partial \bar{x}} \frac{\partial \bar{v}}{\partial \bar{y}} \left. \right] + \frac{\partial}{\partial \bar{y}} \left[2 \bar{u} \frac{\partial^2 \bar{v}}{\partial \bar{x} \partial \bar{y}} + 2 \bar{v} \frac{\partial^2 \bar{v}}{\partial \bar{y}^2} \right. \\ & \left. \left. + 4 \left(\frac{\partial \bar{v}}{\partial \bar{y}} \right)^2 + 2 \frac{\partial \bar{u}}{\partial \bar{y}} \left(\frac{\partial \bar{v}}{\partial \bar{x}} + \frac{\partial \bar{u}}{\partial \bar{y}} \right) \right] \right\}, \end{aligned} \quad (3)$$

where \bar{u} and \bar{v} are the velocity components along the \bar{x} and \bar{y} axes, respectively, \bar{p} is the fluid pressure, ρ is the density, $\nu = \frac{\mu}{\rho}$ is the kinematic viscosity, k_0 is a measure of the viscoelasticity of the fluid and ∇^2 is the Laplacian in Cartesian coordinates (\bar{x}, \bar{y}) .

Continuity equation (1) implies the existence of a streamfunction $\bar{\psi}(\bar{x}, \bar{y})$ such that

$$\bar{u} = \frac{\partial \bar{\psi}}{\partial \bar{y}}, \quad \bar{v} = -\frac{\partial \bar{\psi}}{\partial \bar{x}}. \quad (4)$$

Substitution of (4) in equations (2) and (3) and elimination of pressure from the resulting equations using $\bar{p}_{\bar{x}\bar{y}} = \bar{p}_{\bar{y}\bar{x}}$ yields

$$\frac{\partial(\bar{\psi}, \nabla^2 \bar{\psi})}{\partial(\bar{x}, \bar{y})} + \frac{k_0}{\rho} \frac{\partial(\bar{\psi}, \nabla^4 \bar{\psi})}{\partial(\bar{x}, \bar{y})} + \nu \nabla^4 \bar{\psi} = 0. \quad (5)$$

The boundary conditions at the wall are given by

$$\bar{\psi} = 0, \quad \frac{\partial \bar{\psi}}{\partial \bar{y}} = c\bar{x} \quad \text{at} \quad \bar{y} = 0, \quad (6)$$

where $c > 0$ is a constant which has the units of inverse time.

Following Stuart [30], Tamada [31] and Dorrepaal [32], it can be assumed that the streamfunction $\bar{\psi}$ far from the wall has the form

$$\bar{\psi} = a\bar{x}\bar{y} + \frac{1}{2}b\bar{y}^2 \quad \text{as} \quad \bar{y} \rightarrow \infty, \quad (7)$$

where a , and b are constants.

We now introduce the following non-dimensional variables

$$x = \bar{x} \sqrt{\frac{c}{\nu}}, \quad y = \bar{y} \sqrt{\frac{c}{\nu}}, \quad \psi = \frac{\bar{\psi}}{\nu}. \quad (8)$$

Substituting (8) into equation (5), we obtain

$$\frac{\partial(\psi, \nabla^2 \psi)}{\partial(x, y)} + We \frac{\partial(\psi, \nabla^4 \psi)}{\partial(x, y)} + \nabla^4 \psi = 0, \quad (9)$$

where $We = \frac{k_0 c}{\rho \nu}$ is the Weissenberg number and the boundary conditions (6) and (7) can be written as

$$\psi = 0, \quad \frac{\partial \psi}{\partial y} = x \quad \text{at} \quad y = 0, \quad (10)$$

$$\psi = \frac{a}{c} x y + \frac{1}{2} \gamma y^2 \quad \text{as} \quad y \rightarrow \infty, \quad (11)$$

where $\gamma = b/c$ represents the shear in the free stream. A physical quantity of interest is the skin friction which can be written in non-dimensional form as

$$\begin{aligned} \tau_w = & \left\{ \frac{\partial^2 \psi}{\partial y^2} - \frac{\partial^2 \psi}{\partial x^2} + We \left[\frac{\partial \psi}{\partial y} \left(\frac{\partial^3 \psi}{\partial x \partial y^2} - \frac{\partial^3 \psi}{\partial x^3} \right) \right. \right. \\ & - \frac{\partial \psi}{\partial x} \left(\frac{\partial^3 \psi}{\partial y^3} - \frac{\partial^3 \psi}{\partial x^2 \partial y} \right) + 2 \frac{\partial^2 \psi}{\partial x \partial y} \frac{\partial^2 \psi}{\partial y^2} \\ & \left. \left. + 2 \frac{\partial^2 \psi}{\partial x^2} \frac{\partial^2 \psi}{\partial x \partial y} \right] \right\}_{y=0}. \end{aligned} \quad (12)$$

3. Solutions

The boundary conditions (11) suggest that the streamfunction $\psi(x, y)$ has the form

$$\psi(x, y) = x F(y) + G(y), \quad (13)$$

where the functions $F(y)$ and $G(y)$ are referring as the normal component and tangential component of the flow respectively.

Substituting equation (13) into (9), we obtain, after one integration, the following ordinary differential equations

$$\begin{aligned} & F''' + F F'' + \frac{a^2}{c^2} F - F'^2 \\ & + We (F F^{(iv)} - 2 F' F''' + F''^2) = 0, \end{aligned} \quad (14)$$

$$\begin{aligned} & G''' + F G'' - F' G' \\ & + We (F G^{(iv)} - F' G''' + F'' G'' - F''' G') = A \gamma \end{aligned} \quad (15)$$

subject to

$$F(0) = 0, \quad F'(0) = 1, \quad F'(\infty) = \frac{a}{c}, \quad (16)$$

$$G(0) = G'(0) = 0, \quad G''(\infty) = \gamma. \quad (17)$$

An analysis of equation (14) implies that $F(y)$ behaves as $F(y) = (a/c)y + A$ as $y \rightarrow \infty$, where $A = A(We, a/c)$ is a constant that accounts for the boundary layer displacement. The value of $A = A(We, a/c)$ is determined

We	$a/c = 0.1$	$a/c = 0.5$	$a/c = 1.1$	$a/c = 1.2$
0.0	-0.9693	-0.6672	0.1642	0.3377
0.001	-0.9699	-0.6678	0.16449	0.3381
0.01	-0.9747	-0.6728	0.16651	0.3426
0.05	-0.9971	-0.6967	0.17646	0.3646
0.07	-1.0089	-0.7095	0.18214	0.3773
0.1	-1.0271	-0.7299	0.19177	0.3992
0.2	-1.0955	-0.8101	0.2392	0.5139
0.3	-1.1777	-0.9141	0.35198	0.8499

Table 1. Numerical values of $F''(0)$ for different values of We and $\frac{a}{c}$.

by solving numerically equation (14) subject to boundary conditions (16).

Equation (14) subject to boundary conditions (16) is solved numerically using the shooting method with a finite difference technique. Numerical values of $F''(0)$ for different values of the elasticity of the fluid, We and a/c are shown in Table 1. These values are in good agreement with the results obtained by Lok et al. [13] when $We = 0.0$ and the values reported by Mahapatra and Gupta [22]. In their work, Mahapatra and Gupta [22] used a perturbation technique to solve equation (14) subject to (16). Figure 2 shows the profiles of F' for various values of We when $a/c = 1.2$. Figure 3 depicts the profiles of F' for various a/c when $We = 0.1$. We notice that as the elasticity of the fluid increases, the velocity near the wall increases when $a/c > 1$ and decreases when $0 < a/c < 1$. We also observed that the velocity near the wall increases as a/c is increasing.

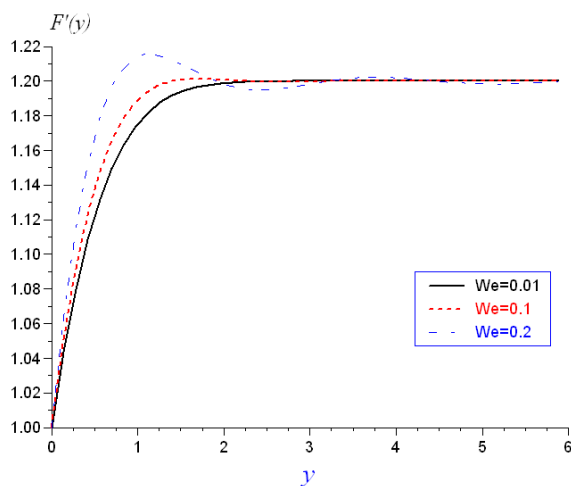


Figure 2. Variation of $F'(y)$ with $a/c = 1.2$ and various We .

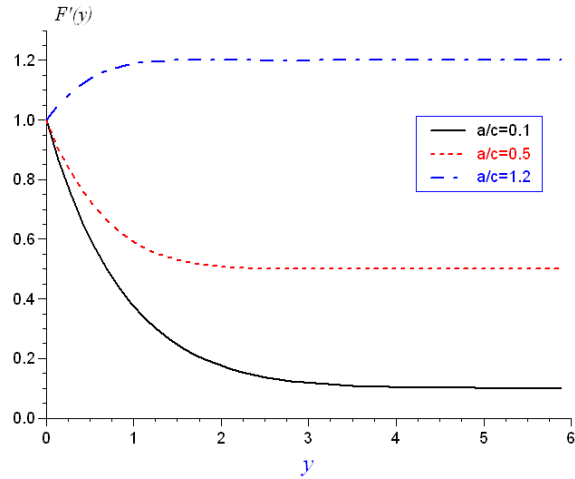


Figure 3. Variation of $F'(y)$ with $We = 0.1$ and various a/c .

Introducing a new variable

$$G'(y) = \gamma H(y), \tag{18}$$

equation (15) with boundary conditions (17) can be written as

$$H'' + F H' - F' H + We (F H''' - F' H'' + F'' H' - F''' H) = A, \tag{19}$$

$$H(0) = 0, \quad H'(\infty) = 1. \tag{20}$$

The boundary value problem for $H(y)$ is solved numerically using finite differencing and the numerical results for $H'(0)$ are given in Table 2 for different values of We . Figure 4 depicts the profiles of $H'(y)$ for various values of We when $a/c = 1.2$.

We	$a/c = 0.1$	$a/c = 0.5$	$a/c = 1.1$	$a/c = 1.2$
0.0	32.7505	7.1765	0.3430	
0.01	32.6015	7.1851	0.1850	-0.5600
0.05	31.9474	7.2134	0.1567	-0.6220
0.1	30.9900	7.2313	0.1167	-0.7123
0.2	28.5862	7.1863	0.0184	-0.9572
0.3	25.5095	7.0054	-0.1133	-1.4116

Table 2. Numerical values of $H'(0)$ for different values of We and $\frac{a}{c}$.

Employing (13) and (18) in (12), the non-dimensional skin friction can be written as

$$\tau_w = x(1 + 3We)F''(0) + (1 + 2We)\gamma H'(0), \tag{21}$$

where numerical values of $F''(0)$ and $H'(0)$ are given in Table 1 and 2 respectively.

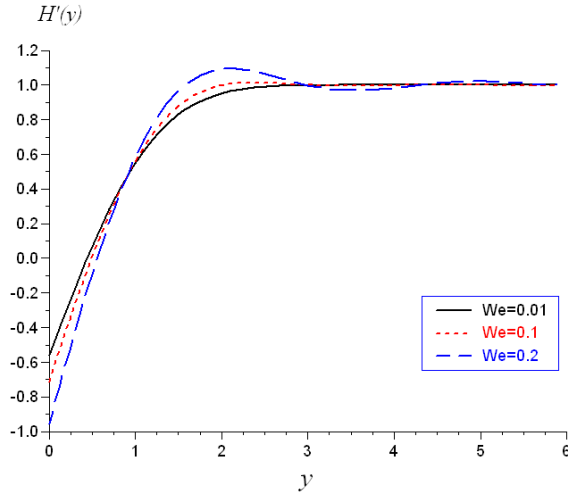


Figure 4. Variation of $H'(y)$ with $a/c = 1.2$ and various We .

4. Discussion and conclusions

The oblique stagnation-point flow of a viscoelastic fluid towards a stretching surface was studied. It was found that, as the Weissenberg number is increasing the velocity and the skin friction near the wall are increasing. The streamlines for the oblique flow can be calculated using equation (18) for different values of We . In particular, the streamline $\psi = 0$ meets the wall at $x = x_s$ (point of stagnation and zero skin friction) where, from (21), x_s is given by

$$x_s = -\frac{(1 + 2We)\gamma H'(0)}{(1 + 3We)F''(0)}, \quad (22)$$

where numerical values of $F''(0)$ and $H'(0)$ are given in Tables 1 and 2.

The Maclaurin series for $F(y)$ and $G(y)$ near the wall $y = 0$ are given by

$$F(y) = y + \frac{1}{2}y^2F''(0) + \frac{1}{6}y^3F'''(0), \quad (23)$$

$$G(y) = \frac{1}{2}\gamma y^2H'(0) + \frac{1}{6}y^3\gamma H''(0). \quad (24)$$

Thus, the streamfunction $\psi(x, y)$ near the wall is given by

$$\begin{aligned} \psi(x, y) = & xy + \frac{1}{2}xy^2F''(0) + \frac{1}{6}xy^3F'''(0) \\ & + \frac{1}{2}y^2\gamma H'(0) + \frac{1}{6}y^3\gamma H''(0) + \dots \end{aligned} \quad (25)$$

which can be rewritten as

$$\psi(x, y) = y \left[x + \frac{1}{2}\gamma H'(0)y + O(xy) \right]. \quad (26)$$

Thus near the wall, the dividing streamline $\psi = 0$ has the equation

$$x + \frac{1}{2}\gamma y H'(0) = 0 \quad (27)$$

and its slope m_s near the wall is given by

$$m_s = -\frac{2}{\gamma H'(0)}. \quad (28)$$

Letting m_∞ to be the slope of the dividing streamline far from the wall, we find that $m_\infty = -\frac{2a}{c}$. Thus, the ratio $R = \frac{m_s}{m_\infty}$ is found to be

$$R = \frac{m_s}{m_\infty} = \frac{1}{\frac{a}{c}H'(0)}. \quad (29)$$

From equation (29), we conclude that the slope ratio depends on the Weissenberg number We but is independent of the angle of incidence γ of the dividing streamline at infinity. The same conclusion is true for the Newtonian fluid as reported by Dorrepaal [32].

Table 1 shows that as We is increasing, $F''(0)$ is decreasing and as a/c increases then $F''(0)$ is also increasing. It can be seen from Table 1 that when $a/c < 1$, the magnitude of $F''(0)$ decreases with increasing a/c for all values of We considered. But when $a/c > 1$, $F''(0)$ increases with increasing a/c for all values of We considered and this is consistent with the fact that there is progressive thinning of the viscoelastic boundary layer with increase in a/c . Table 2 shows that as We and a/c are increasing then $H'(0)$ is decreasing.

Figure 2 shows the dimensionless velocity profiles of $F'(y)$ for $a/c = 1.2$ and various values of We while Figure 3 depicts the dimensionless velocity profiles of $F'(y)$ for $We = 0.1$ and various values of a/c . Figure 2 shows that when $a/c < 1$, the dimensionless horizontal velocity $F'(y)$ at a point decreases with increase in We . We observe from Figure 3 that as the stretching parameter a/c is increasing the velocity is also increasing. We can also notice that the viscoelastic flow has a boundary layer structure for values of $a/c > 1$ and the thickness of this boundary layer decreases with increase in a/c . Physically, this is explained due to the fact that a fixed value of c (such that $a/c > 1$) implies increase in the straining motion near the stagnation-point region resulting in increased acceleration of the external stream. This leads to thinning of the boundary layer with increase in a/c . The same is true for

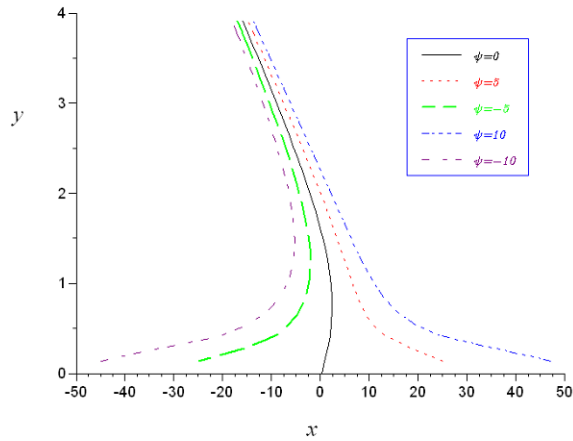


Figure 5. Streamline pattern for oblique flow when $We = 0.1$, $\alpha/c = 1.2$ and $\gamma = 20$.

the Newtonian fluid as reported by Mahapatra and Gupta [12]. Further, it is worth noticing from Figure 3 that when $a/c < 1$, the flow has an inverted boundary layer structure. This results in from the fact that when $a/c < 1$, the stretching velocity cx of the surface exceeds the stagnation velocity ax of the external stream. It is interesting to note that when $a/c = 1$, equation (14) subject to boundary conditions (16), has an exact analytical solution $F(y) = y$. This leads to $u = ax$ and $v = -ay$. From this, one can infer that when $a/c = 1$, the velocity distribution near the stretching surface is the same as that of the inviscid flow (flow away from the surface), so that no boundary layer is formed near the surface. Figure 4 depicts the profiles of $H'(y)$ for various values of We when $a/c = 1.2$.

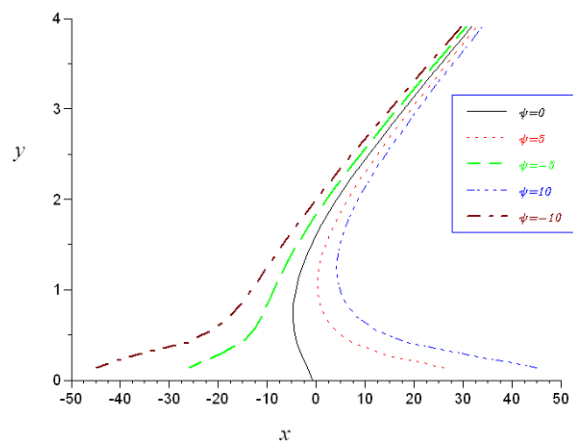


Figure 6. Streamline pattern for oblique flow when $We = 0.1$, $\alpha/c = 1.2$ and $\gamma = -40$.

The streamline pattern for the oblique flow for $We = 0.1$,

$\frac{\alpha}{c} = 1.2$ and $\gamma = 20$ is shown in Figure 5. Figure 6 depicts the streamline pattern for $We = 0.1$, $\frac{\alpha}{c} = 1.2$ and $\gamma = -40$.

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