Research Article

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Equivalent stiffness prediction and global buckling analysis using refined analytical model of composite laminated box beam

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Abstract: The analytical model applicable to calculate the equivalent stiffnesses of composite box beam has been refined. The calculation of equivalent stiffness coefficients of composite laminated box beam is simplified and the connection between shear-deformable beam theory and classical laminate theory is established. The equivalent stiffness analytic formulas expressed by beam cross-section geometry and laminate stiffness coefficients are obtained. These analytical formulas are suitable for composite laminated box beam with circumferential uniform stiffness, and accounts for bending-transverse shear and torsion-tensile coupling effect. The correctness and precision of refined analytical model is verified by test and finite element method, respectively. The influences of the lay-ups on the elastic coupling of composite structures and their causes are studied. The variation of the equivalent stiffnesses of the laminated box beams with different lay-ups is predicted. The global buckling analysis of composite laminated box beam considering the transverse shear deformation is carried out. The formula of the global buckling critical load is obtained combining with the theoretical formulas of equivalent stiffnesses. The influences of the lay-ups, shear deformation and slenderness ratio on the global buckling critical load are studied.

Keywords: composite laminated box beam; bending stiffness; torsional stiffness; shear-deformable beam theory; laminate stiffness coefficients; global buckling

1 Introduction

Composite laminated box beams are usually used in bridge engineering [1–3], wind turbine blade [4], launch vehicle instrumentation [5], helicopter rotors and aircraft wings [6]. When designing the composite laminated box beam to resist bending, torsion, pressure, etc., it is necessary to calculate its equivalent stiffnesses first. There are non-classical effects like warping, transverse shear strain and 3-D elastic effect in characterizing elastic coupling deformation of composite laminated box beam. Accordingly, calculating the equivalent stiffnesses of composite laminated box beam is a crucial but complicated work, which has received increased attention like the analytical model.

Shina *et al.* [7] defined the section shape function describing the local cross-section deformation, and established a high-order beam theory for the thin-walled box beam of composite materials by numerical method. Teter *et al.* [8] studied the detection and localization of cracks in two different configuration composite laminated box beam by numerical modal analysis; Ghafari *et al.* [9] used Rayleigh-Ritz's dimensionality reduction method to carry out two-dimensional cross-sectional analysis of composite beams; Malkaca *et al.* [10] studied the effect of joint flexibility on the vibration characteristics of composite box manipulators. The methods for calculating the equivalent stiffness of composite laminated box beam can be divided into two types: classical beam theory and shear-deformable beam theory. The classical beam theory adopts the Euler-Bernoulli beam model based on the flat-section assumption and the one-dimensional stress-strain relationship under the pure bending load. The equivalent stiffness of each single layer of the composite laminate is superimposed and summed [11, 12]. This method does not implement overall force analysis of structures and ignores the interlayer interaction and the non-classical effects. The deviation caused by neglecting the shear deformation is large because the shear modulus of composite material is small. Vo and Lee [13, 14] used the theory of first-order shear-deformable beam to establish the tensile-bending-

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shear-torsion coupling static analytical model of composite thin-walled box beam, but the elastic coupling effects are neglected when calculating the equivalent stiffnesses. Literature [15, 16] studied the influences of non-classical effects on the static response and natural frequency of composite laminated box beam based on shear-deformable beam theory. Kim and White [17, 18] comprehensively considered the 3-D elastic and warping effects of composite structures, and established the equivalent stiffness matrix of composite thick-walled box beam with thickness-to-height ratio greater than 0.1. Shadmehri [19] calculated the equivalent bending stiffness of composite thin-walled closed section structures based on the displacement field of thin-walled beam, but the calculation of the converted stiffness coefficients in the simplification process of the constitutive equation is complicated. Zhang [20] used the thick-walled beam theory and the single-layer 3-D constitutive equation proposed by Kim and White to calculate the equivalent bending stiffness of the composite thick-walled tube. Geuchy and Hoa [21] calculated the equivalent bending stiffness of a thick-walled tube with a balanced anti-symmetric composite layer using the stress-strain field of a composite tube under pure bending.

In order to solve the problem that calculating the equivalent stiffnesses of composite laminated box beam is complicated, inaccurate, and difficult to analyze in depth, the analytical model of the first-order shear-deformable beam established in the literature [13, 14, 19] is refined in this paper. Considering the 3-D elastic effect, the two-dimensional reduced modulus components of the composite single layer are used to represent the three-dimensional reduced modulus components, which simplifies the calculation of the equivalent stiffness coefficients. The analytical formulas of the equivalent torsional and bending stiffnesses of composite laminated box beam are derived respectively. These formulas are expressed by beam cross-section geometric dimensions and the laminate stiffness coefficients, which makes calculation simple and precise.

The refined analytical model of composite laminated box beam can be used for equivalent stiffnesses prediction and global buckling analysis. The influences of lay-ups and the ply angle on the elastic coupling effects, and on the variation of equivalent stiffnesses are studied respectively. The global buckling analysis of composite laminated box beam is carried out under the consideration of transverse shear deformation. The effects of shear deformation, lay-ups and slenderness ratio on global buckling critical load are studied. The works provide theoretical guidance for engineering design of composite laminated box beam.

2 Analytical model of composite laminated box beam

2.1 Displacement field

The Cartesian coordinate system (x, y, z) is taken as global coordinate, the coordinate origin is located in the cross-section centroid; the curvilinear coordinate system (z, s, n) is used as local coordinate, and the origin is located in the midline of cross section. n, s are the normal and the tangential direction of the middle line respectively, as shown in Figure 1. The composite material is a typical anisotropic material, and the shear modulus is a small amount compared with the elastic modulus. Therefore, the shear deformation under the lateral load cannot be neglected. The flat section assumption in the classical beam theory is also no longer applicable. The analytic model of the composite laminated box beam adopts the first-order shear-deformable beam theory, considering the transverse shear deformation, the primary and secondary warping effects and the non-uniform torsional effect. The assumptions made are as follows:

- (1) The cross section does not undergo in-plane deformation;
- (2) The transverse shear strains γ_{xy} and γ_{xz} are evenly distributed in the cross section;
- (3) The torsion angle φ is a function of the longitudinal coordinate;

Figure 1: Geometry and coordinate system of composite laminated box beam

- (4) In addition to considering the main warping displacement along the contour of the centerline, the secondary warping displacement deviating from the contour of the centerline is also considered;
- (5) Only small deformations in the linear elastic range occur.

According to the above assumptions, the displacement field of an arbitrary point on the cross-section is assumed to be [13, 14]

$$\begin{cases} u(x, y, z) = u_0(z) - y\varphi(z) \\ v(x, y, z) = v_0(z) + x\varphi(z) \\ w(x, y, z) = w_0(z) + \theta_x(z)[y(s) - n\frac{dx}{ds}] \\ + \theta_y(z)[x(s) + n\frac{dy}{ds}] - \varphi'(z)[F_w(s) + na(s)] \end{cases} \quad (1)$$

where

$$\begin{cases} \theta_x(z) = \gamma_{yz}(z) - v_0'(z) \\ \theta_y(z) = \gamma_{xz}(z) - u_0'(z) \\ a(s) = -y(s)\frac{dy}{ds} - x(s)\frac{dx}{ds} \\ F_w s = \int_0^s [r_n(s) - \psi(s)] ds \end{cases} \quad (2)$$

u, v, w are the displacements along the x, y , and z directions of the coordinate axis. $\theta_x(z), \theta_y(z)$ and $\varphi(z)$ are the rotation angles around the coordinate axis x, y , and z , respectively. $u_0(z), v_0(z)$ and $w_0(z)$ are rigid body displacements in three directions. φ is the angle between the n and the x direction (see Figure 2). $\gamma_{xy}(z)$ and $\gamma_{xz}(z)$ are two transverse shear strains. If the transverse shear and the out-of-plane warping are neglected, $\gamma_{xy} = \gamma_{xz} = 0$, the model is simplified to the Euler-Bernoulli beam model without considering shear deformation. $a(s)$ is the height of the right

triangle in the geometric relationship. $F_w(s)$ is the generalized fan coordinate. $\psi(s)$ is the warping function. For box section with a wall thickness of T , height of h , and width of b :

$$\begin{cases} \psi(s) = \frac{b_1 b_2}{b_1 + b_2} \\ b_1 = h - T \\ b_2 = b - T \end{cases} \quad (3)$$

2.2 Geometric equation

Based on the assumptions (1), (5) and the displacement field, there are only three strains existing, namely $\epsilon_z, \gamma_{sz}, \gamma_{nz}$. The axial strain, ϵ_z , can be obtained as:

$$\epsilon_z(z, s, n) = \epsilon_z^0(z, s) + n\epsilon_z^1(z, s) \quad (4)$$

where $\epsilon_z^0(z, s)$ and $\epsilon_z^1(z, s)$ are strain caused by primary and secondary warping, respectively, which are calculated by:

$$\begin{cases} \epsilon_z^0(z, s) = w_0'(z) + \theta_y'(z)x(s) \\ + \theta_x'(z)y(s) - \varphi''(z)F_w(s) \\ \epsilon_z^1(z, s) = \theta_y'(z)\frac{dy}{ds} + \theta_x'(z)\frac{dx}{ds} - \varphi''(z)a(s) \end{cases} \quad (5)$$

The circumferential shear strain, γ_{sz} , can be expressed as:

$$\begin{aligned} \gamma_{sz}(x, s, n) &= \gamma_{zy}\frac{dx}{ds} + \gamma_{zx}\frac{dy}{ds} \\ &= \frac{dx}{ds}[u_0' + \theta_y(z)] + \frac{dy}{ds}[v_0' + \theta_x(z)] + \psi(s)\varphi'(z) \end{aligned} \quad (6)$$

The transverse shear strain, γ_{nz} , is shown as:

$$\begin{aligned} \gamma_{nz}(z, s, n) &= \gamma_{zx}\frac{dy}{ds} - \gamma_{yz}\frac{dx}{ds} \\ &= \frac{dy}{ds}[u_0' + \theta_y(z)] - \frac{dx}{ds}[v_0' + \theta_x(z)] \end{aligned} \quad (7)$$

2.3 Equivalent constitutive equation

The geometric relationship between (x, y, z) and (z, s, n) of arbitrary point on the midline is:

$$dx = ds \cos \phi, \quad dy = ds \sin \phi \quad (8)$$

The normal stress σ_z , shear stresses τ_{sz} and τ_{nz} are integrated along the cross section, and the expressions of the internal force and moment can be obtained as:

$$\begin{aligned} N_z &= \int_A \sigma_z ds dn \\ V_x &= \int_A (\tau_{sz} \cos \phi + \tau_{nz} \sin \phi) ds dn \end{aligned} \quad (9)$$

Figure 2: Displacement field of composite laminated box beam

$$\begin{aligned}
V_y &= \int_A (\tau_{sz} \sin \phi - \tau_{nz} \cos \phi) ds dn \\
M_x &= \int_A \sigma_z (y - n \cos \phi) ds dn \\
M_y &= \int_A \sigma_z (x + n \sin \phi) ds dn \\
M_z &= \int_A \tau_{sz} \psi(s) ds dn \\
M_\omega &= \int_A \sigma_z [F_w(s) + na(s)] ds dn
\end{aligned}$$

where N_z is the axial force; V_x and V_y are the shear forces in the x and y directions, respectively; M_x , M_y , and M_z are the bending moments around the x , y , and z axis, respectively; M_ω is the bimoment generated by the constrained torsional normal stress. The equivalent constitutive equation of composite laminated box beam can be obtained by simplifying Eq. (9):

$$\begin{Bmatrix} N_z \\ M_y \\ M_x \\ V_x \\ V_y \\ M_\omega \\ M_z \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} & a_{17} \\ & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} & a_{27} \\ & & a_{33} & a_{34} & a_{35} & a_{36} & a_{37} \\ & & & a_{44} & a_{45} & a_{46} & a_{47} \\ & & & & a_{55} & a_{56} & a_{57} \\ & & & & & a_{66} & a_{67} \\ \text{sym} & & & & & & a_{77} \end{bmatrix} \begin{Bmatrix} w_0' \\ \theta_y' \\ \theta_x' \\ u_0' + \theta_y \\ v_0' + \theta_x \\ \phi'' \\ \phi' \end{Bmatrix} \quad (10)$$

a_{ij} are called the equivalent stiffness coefficients and the method of calculating a_{ij} is shown in Section 2.

2.4 Equivalent stiffness calculation

Circumferentially uniform stiffness (CUS) is a typical composite tube configuration with angular symmetry of symmetric wall. The composite laminated box beam of CUS configurations can be formed by lamination or winding method. The CUS configuration can reduce the number of times of cutting lamina, the initial defects are small, and the global mechanical properties are superior. Therefore, this paper focused on the stiffness performance of composite laminated box beam with CUS configuration. According to the symmetry of $\theta(y) = \theta(-y)$ of the CUS configuration,

the internal force-displacement relationship, Eq. (10), can be simplified as:

$$\begin{Bmatrix} N_z \\ M_z \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{17} \\ a_{17} & a_{77} \end{bmatrix} \begin{Bmatrix} w_0' \\ \phi' \end{Bmatrix} \quad (11)$$

$$\begin{Bmatrix} M_y \\ M_x \\ V_x \\ V_y \\ M_\omega \end{Bmatrix} = \begin{bmatrix} a_{22} & 0 & 0 & a_{25} & 0 \\ 0 & a_{33} & a_{34} & 0 & 0 \\ 0 & a_{34} & a_{44} & 0 & 0 \\ a_{25} & 0 & 0 & a_{55} & 0 \\ 0 & 0 & 0 & 0 & a_{66} \end{bmatrix} \begin{Bmatrix} \theta_y' \\ \theta_x' \\ v_0' + \theta_y \\ w_0' + \theta_x \\ \phi'' \end{Bmatrix} \quad (12)$$

It can be found that for the composite laminated box beam of CUS configurations, the stiffness coefficient a_{17} characterizing the tensile-torsional coupling effect and the stiffness coefficients a_{25} , a_{34} characterizing the bending-shear coupling effect are not zero. Assume that the box beam works under pure bending, the equivalent bending stiffnesses of the x -axis and y -axis are obtained by inverting Eq. (12):

$$\begin{cases} [EI]_x = a_{33} - \frac{a_{34}^2}{a_{44}} \\ [EI]_y = a_{22} - \frac{a_{25}^2}{a_{55}} \end{cases} \quad (13)$$

Assume that it works under pure torsion, the equivalent torsional stiffness is obtained by inverting Eq. (11):

$$[GJ] = a_{77} - \frac{a_{17}^2}{a_{11}} \quad (14)$$

The calculation method of the equivalent stiffness coefficients a_{ij} in Eqs. (13) and (14) are expressed in Section 2.

3 Refined calculation of the equivalent stiffness coefficients

The existing analytical model of composite laminated box beam is based on the theory of first-order shear-deformable beam, considering the true deformation state of the member, but the calculation of the equivalent stiffness coefficients in Eqs. (11) and (12) is particularly complicated. The calculation of the equivalent stiffness coefficients will be refined in the following with the classical laminate theory.

The constitutive equation of an orthotropic lamina referenced to local coordinate system can be written as:

$$\begin{Bmatrix} \sigma_z \\ \sigma_s \\ \sigma_n \\ \tau_{sn} \\ \tau_{nz} \\ \tau_{sz} \end{Bmatrix}_k = \begin{bmatrix} \overline{C}_{11} & \overline{C}_{12} & \overline{C}_{13} & 0 & 0 & \overline{C}_{16} \\ \overline{C}_{12} & \overline{C}_{22} & \overline{C}_{23} & 0 & 0 & \overline{C}_{26} \\ \overline{C}_{13} & \overline{C}_{23} & \overline{C}_{33} & 0 & 0 & \overline{C}_{36} \\ 0 & 0 & 0 & \overline{C}_{44} & \overline{C}_{45} & 0 \\ 0 & 0 & 0 & \overline{C}_{45} & \overline{C}_{55} & 0 \\ \overline{C}_{16} & \overline{C}_{26} & \overline{C}_{36} & 0 & 0 & \overline{C}_{66} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_z \\ \varepsilon_s \\ \varepsilon_n \\ \gamma_{sn} \\ \gamma_{nz} \\ \gamma_{sz} \end{Bmatrix}_k \quad (15)$$

where $\overline{C}_{ij}$ are the stiffness coefficients under the off-axis coordinate system. In the classical laminate theory, the stress $\sigma_n = 0$ in the thickness direction of the laminate, the Eq. (15) can be reduced to:

$$\begin{Bmatrix} \sigma_z \\ \sigma_s \\ \tau_{sz} \end{Bmatrix}_k = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_z \\ \varepsilon_s \\ \gamma_{sz} \end{Bmatrix}_k \quad (16)$$

where $\overline{Q}_{ij}$ are the two-dimensional reduced modulus components of the composite single layer under plane stress state, which are calculated by:

$$\begin{cases} \overline{Q}_{ij} = \overline{C}_{ij} - \frac{\overline{C}_{i3}\overline{C}_{3j}}{\overline{C}_{33}} (i, j = 1, 2, 6) \\ \overline{Q}_{ij} = \overline{C}_{ij} (i, j = 4, 5) \end{cases} \quad (17)$$

Considering the Poisson effect of layers, the three strain components $\varepsilon_n, \varepsilon_s, \gamma_{sn}$ perpendicular to the cross section cannot be ignored, but the corresponding stress component $\sigma_n = \sigma_s = \tau_{sn} = 0$ [17, 18]. The Eq. (15) can be further reduced as:

$$\begin{Bmatrix} \sigma_z \\ \tau_{sz} \\ \tau_{nz} \end{Bmatrix} = \begin{bmatrix} C_{11}^* & C_{12}^* & 0 \\ C_{12}^* & C_{22}^* & 0 \\ 0 & 0 & C_{33}^* \end{bmatrix} \begin{Bmatrix} \varepsilon_z \\ \gamma_{sz} \\ \gamma_{nz} \end{Bmatrix} \quad (18)$$

where C_{ij}^* are the three-dimensional reduced modulus components of the composite single layer, which are calculated by:

$$\begin{cases} C_{11}^* = \overline{C}_{11} + \frac{2\overline{C}_{12}\overline{C}_{13}\overline{C}_{23} - \overline{C}_{12}\overline{C}_{12}\overline{C}_{33} - \overline{C}_{13}\overline{C}_{13}\overline{C}_{22}}{\overline{C}_{22}\overline{C}_{33} - \overline{C}_{23}\overline{C}_{23}} \\ C_{12}^* = \overline{C}_{16} + \frac{\overline{C}_{12}\overline{C}_{23}\overline{C}_{36} - \overline{C}_{12}\overline{C}_{26}\overline{C}_{33} + \overline{C}_{13}\overline{C}_{23}\overline{C}_{26} - \overline{C}_{13}\overline{C}_{22}\overline{C}_{36}}{\overline{C}_{22}\overline{C}_{33} - \overline{C}_{23}\overline{C}_{23}} \\ C_{22}^* = \overline{C}_{66} + \frac{2\overline{C}_{26}\overline{C}_{23}\overline{C}_{36} - \overline{C}_{26}\overline{C}_{26}\overline{C}_{33} - \overline{C}_{36}\overline{C}_{22}\overline{C}_{36}}{\overline{C}_{22}\overline{C}_{33} - \overline{C}_{23}\overline{C}_{23}} \\ C_{33}^* = \overline{C}_{55} - \frac{\overline{C}_{45}^2}{\overline{C}_{44}} \end{cases} \quad (19)$$

The literature [19, 20] derived the expressions of a_{ij} based on the three-dimensional calculation modulus calculated

by Eq. (19). These expressions need to calculate the converted stiffness coefficients K_{ij} firstly, and the calculation of K_{ij} is more complicated. If Eq. (17) is brought into Eq. (19), Eq. (19) can be further refined as:

$$\begin{cases} C_{11}^* = \overline{Q}_{11} - \frac{\overline{Q}_{12}^2}{\overline{Q}_{22}} \\ C_{12}^* = \overline{Q}_{16} - \frac{\overline{Q}_{12}\overline{Q}_{26}}{\overline{Q}_{22}} \\ C_{22}^* = \overline{Q}_{66} - \frac{\overline{Q}_{26}^2}{\overline{Q}_{22}} \\ C_{33}^* = \overline{Q}_{55} - \frac{\overline{Q}_{45}^2}{\overline{Q}_{44}} \end{cases} \quad (20)$$

Eq. (20) shows that the three-dimensional reduced modulus components can be calculated by two-dimensional reduced modulus components. Substitute the Eq. (20) into (18), and then the result is substituted into Eq. (9), the expressions of a_{ij} in Eqs. (13) and (14) can be obtained as:

$$a_{11} = \int_s \left(A_{11} - \frac{A_{12}^2}{A_{22}} \right) ds a_{17} \quad (21)$$

$$= \int_s \psi s \left(A_{16} - \frac{A_{12}A_{26}}{A_{22}} \right) ds$$

$$a_{22} = \int_s \left[x^2 A_{11} - \frac{A_{12}^2}{A_{22}} + 2 \left(B_{11} - \frac{A_{12}B_{12}}{A_{22}} \right) x \sin \phi \right.$$

$$\left. + \left(D_{11} - \frac{B_{12}^2}{A_{22}} \right) \sin^2 \phi \right] ds$$

$$a_{25} = \int_s \left[\left(A_{16} - \frac{A_{12}A_{26}}{A_{22}} \right) x \sin \phi \right.$$

$$\left. + \left(B_{16} - \frac{A_{26}B_{12}}{A_{22}} \right) \sin^2 \phi \right] ds$$

$$a_{33} = \int_s \left[y^2 A_{11} - \frac{A_{12}^2}{A_{22}} - 2y \left(B_{11} - \frac{A_{12}B_{12}}{A_{22}} \right) \cos \phi \right.$$

$$\left. + \left(D_{11} - \frac{B_{12}^2}{A_{22}} \right) \cos^2 \phi \right] ds$$

$$a_{34} = \int_s \left[\left(A_{16} - \frac{A_{12}A_{26}}{A_{22}} \right) y \cos \phi \right.$$

$$\left. - \left(B_{16} - \frac{A_{26}B_{12}}{A_{22}} \right) \cos^2 \phi \right] ds$$

$$a_{44} = \int_s \left[\left(A_{66} - \frac{A_{26}^2}{A_{22}} \right) \cos^2 \phi + \left(A_{55} - \frac{A_{45}^2}{A_{44}} \right) \sin^2 \phi \right] ds$$

$$a_{55} = \int_s \text{Biggl} \left(A_{66} - \frac{A_{26}^2}{A_{22}} \right) \sin^2 \phi$$

$$\left. + \left(A_{55} - \frac{A_{45}^2}{A_{44}} \right) \cos^2 \phi \right] ds$$

$$a_{77} = \int_s \left(A_{66} - \frac{A_{26}^2}{A_{22}} \right) \psi^2(s) ds$$

where A_{ij} , B_{ij} and D_{ij} matrices are extensional, coupling and bending stiffnesses in classical laminate theory, respectively, defined by:

$$\begin{cases} \int \bar{Q}_{ij}(1, n, n^2) dn = (A_{ij}, B_{ij}, D_{ij}) \\ (i, j = 1, 2, 6) \\ A_{ij} = \frac{5}{4} \sum_{k=1}^n \bar{C}_{ij}^{(k)} \left[n_k - n_{k-1} - \frac{4}{3} \frac{(n_k^3 - n_{k-1}^3)}{T^3} \right] \\ (i, j = 4, 5) \end{cases} \quad (22)$$

The explicit forms of the equivalent stiffness coefficients a_{ij} for the composite laminated box section are given in the Appendix A.

It can be seen from Eqs. (21) and (22) that the refined analytical model of composite material laminated box beam is used to calculate the equivalent stiffness coefficients has the following advantages:

- (i) represented by the beam cross-section geometric dimensions and the laminate stiffness coefficients A_{ij} , B_{ij} , D_{ij} , and the calculation is simple;
- (ii) combining the theory of classical laminate with the theory of shear-deformable beam, many conclusions on the lay-ups characteristics of laminates can be used to study the variation of equivalent stiffnesses of composite laminated box beam.

4 Verification of refined analytical model

4.1 Test verification

In order to verify the correctness of the refined analytical model of the shear-deformable beam, the three-point bending and the torsion test were carried out on the specimens in Group B and T, respectively. The specimens are made of T300/QY8911, the mechanical properties parameters are shown in Table 1. The lay-ups and geometric parameters of specimens are shown in and Table 2, respectively.

In the engineering, the layered equivalent superposition method based on classical laminate theory is often used to calculate the equivalent stiffnesses of composite thin-walled beam structures:

$$\langle EI \rangle_x = \sum_{k=1}^n E_k \frac{b_{k+1} h_{k+1}^3 - b_k h_k^3}{12} \quad (23)$$

Table 1: Material mechanical properties parameters of T300/QY8911

Material property	Value
E_1/GPa	135
$E_2 = E_3/GPa$	8.8
$G_{12} = G_{13}/GPa$	4.47
G_{23}/GPa	3.0
$\nu_{12} = \nu_{13}$	0.33
ν_{23}	0.33

$$\langle GJ \rangle = \frac{2b_1^2 b_2^2}{b_1 + b_2} \sum_{k=1}^n \bar{Q}_{66}^k t_k \quad (24)$$

where t_k is the thickness of a single layer; E_k is the longitudinal elastic modulus of a single layer, which is calculated by:

$$E_k = \frac{E_1}{\cos^4 \theta + \left(\frac{E_1}{G_{12}} - 2\nu_{12} \right) \cos^2 \theta \sin^2 \theta + \frac{E_1}{E_2} \sin^4 \theta} \quad (25)$$

where b_k and h_k are the width and height of the k -th single layer section, θ is the ply angle.

The equivalent stiffnesses are calculated by the layered equivalent superposition method and the refined analytical model using shear-deformable beam theory respectively, and compared with the experimental results.

4.1.1 Three-point bending test

A three-point bending loading test was carried out on the specimens of Group B. The test was conducted on the INSTRON 5982 universal testing machine, as shown in Figure 3(a). A lateral concentrated load P was applied on the upper middle surfaces of the span, and the loading speed was 1 mm/min. The strain ϵ at mid-span of the specimens was monitored [20, 21]. It is known from the traditional material mechanics that the relationship between P and ϵ is:

$$P = \frac{8K\epsilon}{L'h} \quad (26)$$

where L' is the distance between the two supported points, h is the height of the section, K is the bending stiffness of the specimens. In order to avoid inaccurate results caused by the excessive local deformation on the upper bottom surface, the test were only carried out in the elastic range. The initial and final state of the three-point bending test are shown in Figures 3(b) and (c)(take specimen BCS4 as an example).

Table 3 compares the theoretical and experimental results of the equivalent bending stiffness of four specimens

Table 2: The lay-ups and geometric dimensions parameters of specimens

Group	Specimen	Lay-ups	b/mm	h/mm	T/mm	L/mm
B	BCS1	[30]6	20	20	1.00	200
	BCS2	[0/90]3			0.98	200
	BCS4	[30/−30/30]s			1.01	200
	BCS5	[02/±30/±60]			1.02	200
T	TCS1	[30]6			1.00	200
	TCS3	[04/±45]			0.99	200
	TCS4	[30/−30/30]s			1.01	200
	TCS5	[02/±30/±60]			1.02	200

Figure 3: Three-point bending test process of specimen BCS4

Figure 4: Theoretical and experimental $P - \epsilon$ curves of the four specimens in Group B.

Table 3: Comparison of theoretical and experimental results of equivalent bending stiffness

No.	Experiment/Theory	Bending stiffness/ (N·mm ²)	De/%
BCS1	Experiment	54580205	–
	$\langle EI \rangle_x$	45536750	–16.6
	$[EI]_x$	58547250	7.3
BCS2	Experiment	162759101	–
	$\langle EI \rangle_x$	169383531	4.1
	$[EI]_x$	168580503	3.6
BCS4	Experiment	74004918	–
	$\langle EI \rangle_x$	45536750	–38.5
	$[EI]_x$	78930367	6.7
BCS5	Experiment	139189698	–
	$\langle EI \rangle_x$	125002347	–10.2
	$[EI]_x$	143815866	3.3

Table 4: Comparison of theoretical and experimental results of equivalent torsional stiffness.

No.	Experiment/Theory	Twisting stiffness/ (N·mm ²)	De/%
TCS1	Experiment	44087964	–
	$\langle G \rangle$	106484765	141.5
	$[G]$	45815345	3.9
TCS3	Experiment	92078987	–
	$\langle G \rangle$	108910048	18.2
	$[G]$	96489393	4.7
TCS4	Experiment	159784264	–
	$\langle G \rangle$	165162237	3.3
	$[G]$	165162237	3.3
TCS5	Experiment	126678364	–
	$\langle G \rangle$	144476626	14.1
	$[G]$	129924277	2.6

(a)

(b)

Figure 5: Torsion test process of specimen BCS4.

in Group B. Figure 4(a)-(d) are theoretical and experimental $P - \epsilon$ curves of the four specimens in Group B.

It can be seen from Table 3 and Figure 4 that the maximum error of results calculated by the layered equivalent superposition method is -38.5% , while the maximum error calculated by the shear deformation beam theory is only 7.3% . This is because the layered equivalent superposition method is based on the one-dimensional constitutive equation of composite materials, the transverse shear deformation and the bending-shear coupling effects are not considered, while the shear-deformable beam theory is based on three-dimensional constitutive equation and the actual deformation state of the component is considered.

4.1.2 Torsion test

The torsion test was carried out on the specimens in Group T. The torsion testing machine can record the relative torsion angle φ between the two chucks. The relationship between

tween φ and torque M_z is:

$$\varphi = \frac{180^\circ}{\pi} \frac{M_z L}{K_G} \quad (27)$$

where L is the total length of the specimens, K_G is the torsional stiffness of the specimens. The torsion test was carried out in the linear elastic range of the specimens. The specimens did not undergo nonlinear strength failure of the material. The initial and final state of the torsion test are shown in Figures 5(a) and (b) (take specimen TCS4 as an example). Table 4 compares the theoretical and experimental results of the equivalent bending stiffness of four specimens in Group T. Figure 6(a)-(d) are the $M_z - \varphi$ curves of four specimens in Group T.

It can be seen from Table 4 and Figure 6 that the maximum error of the results calculated by the layered equivalent superposition method is up to 141.53% . The reason for error is that the layered equivalent superposition method ignores the cross-sectional warping deformation and the torsional-tensile coupling. The maximum error calculated by the analytical model of shear-deformable beam is 4.78% , and the calculation accuracy can meet the requirements of engineering design. For the TCS4 specimen, the results calculated by the two methods are the same, the reason is that TCS4's lay-up is balanced oblique symmetric, the effect of lay-ups on the equivalent stiffnesses of composite laminated box beam will be discussed in Section 4.1.

4.2 Finite element verification

In order to analyze the calculation accuracy of the theoretical model, a composite laminated box beam with the cross section width $b=164 \text{ mm}$, height $h=230 \text{ mm}$, span $l=2600 \text{ mm}$ and wall thickness $T=10 \text{ mm}$ was selected as a numerical example. The number of layers is 50, single layer thickness is 0.2 mm . The material of the example is the same as the specimens in Section 3.1. The finite element method for calculating the equivalent bending stiffness of composite structures is given by:

$$[EI]_{x\text{FEM}} = \frac{N_z I_x}{A \epsilon_z} \quad (28)$$

where N_z is the applied axial tensile load, I_x is section moment of inertia respect to the x -axis, A is the cross-sectional area, ϵ_z is the axial strain. Establish a finite element model of the example in ABAQUS. C3D8R three-dimensional solid element is selected [17, 18]. The C3D8R element has 8 nodes, each node has 6 degrees of freedom. Set 5 and 300 elements in the thickness and length direction, respectively. Material properties are assigned to four

Figure 6: Theoretical and experimental M_z - ϕ curves of the four specimens in Group T.

walls using the “Composite layup” function in ABAQUS. Apply a completely fixed constraint on the right end face, establish a reference point RP1 at the left end, and establish a coupling constraint between the RP1 and the left end face. Apply an axial tensile load $N_z=7480$ N on the RP1, and ensure that the composite laminated box beam only undergoes axial deformation within the linear elastic range. After the calculation is completed, a strain cloud image can be obtained, and the axial strain ϵ_z is extracted.

In the similar way, replace tensile load $N_z=7480$ N with the torque $M_z=10$ N·m on RP1, and the historical output of the torsion angle, ϕ , is established. The equivalent torsional stiffness of the composite laminated box beam can be calculated as:

$$[GJ]_{FEM} = \frac{M_z l}{\phi} \quad (29)$$

The finite element model of composite laminated box beam is shown in Figure 7. Assume that the numerical example adopts a uniform angle lay-ups, and the equivalent bending stiffness and equivalent torsional stiffness of the

Figure 7: Finite element model of composite laminated box beam.

example are calculated by the finite element method, the layered equivalent superposition method and the shear deformation beam theory respectively. The results are shown in Figures 8 and 9.

It can be seen from Figure 8 that compared with the layered equivalent superposition method, the equiv-

Figure 8: The equivalent bending stiffness of composites laminated box beam calculated by three methods.

Figure 9: The equivalent torsional stiffness of composites laminated box beam calculated by three methods.

alent bending stiffness calculated by the refined analytical model using shear-deformable beam theory is closer to the finite element simulation, which further verifies the correctness of the refined analytical model. For composite laminated box beam with uniform angle lay-ups, the layered equivalent superposition underestimates the equivalent bending stiffness in the range of $0^\circ < \theta \leq 50^\circ$, and the equivalent bending stiffness value decreases rapidly in this range; at $\theta=0^\circ$ and above 50° , the results calculated by the three methods are not much different, indicating that the non-classical effects like shear deformation, section warping etc. have little effect on overall deformation. It can be seen from Figure 9 that the equivalent torsional stiffness takes the maximum value at $\theta=45^\circ$. The layered equivalent superposition method seriously overestimates

Figure 10: Equivalent bending stiffness of composite laminated box beam in three kinds of lay-ups change with ply angle.

Figure 11: Equivalent torsional stiffness of composite laminated box beam in three kinds of lay-ups change with ply angle.

equivalent torsional stiffness, which is far from the results of refined analytical model and finite element simulation.

5 Prediction of equivalent stiffnesses of composite laminated box beam

5.1 Selection of lay-ups

In order to study the effect of the lay-ups on the equivalent stiffnesses, the numerical example in Section 2.2 is also taken as the research object. Under the premise that the number of layers also is 50, the three most commonly used lay-ups in engineering were selected: (i) the uniform angle,

$[\theta]_{50}$; (ii) the balanced oblique symmetric, $[(\theta/-\theta)_{12}/\theta]_s$; and (iii) the balanced anti-symmetric, $[\theta/-\theta]_{25}$. The equivalent bending stiffness and equivalent torsional stiffness of numerical example with these three different lay-ups were calculated by the layered equivalent superposition method and the shear-deformable beam theory, the results are shown in Figure 10 and Figure 11.

It can be seen from Figure 10 that for numerical example with three different lay-ups in this section, the results of the equivalent bending stiffnesses calculated by the layered equivalent superposition method are the same. The reason is that the equivalent longitudinal elastic modulus E_k of each single layer is an even function with respect to the ply angle θ . The actual situation is that the composite laminated box beam with balanced symmetrical or balanced anti-symmetric wall has higher equivalent bending stiffness in the range of $0^\circ < \theta < 45^\circ$, which is due to the layered equivalent superposition method ignores the inter-layer interaction and the tensile-bending coupling effect of composite structures.

It can be seen from Figure 11 that the equivalent torsional stiffness takes the maximum value at $\theta = 45^\circ$. If the number of layers is the same, equivalent torsional stiffness of composite laminated box beam with the balanced oblique symmetric and the balanced anti-symmetric lay-ups is higher. The results of the layered equivalent superposition method and the shear deformation beam theory are the same. This is because that the wall plate is balanced oblique symmetric lay-ups, it can be known from the classical laminate theory that the tensile-shear coupling coefficient A_{16} and A_{26} , the tensile-bending coupling stiffness coefficients B_{11} and B_{12} , the tensile-torsional coupling stiffness coefficient B_{16} and the shear coupling stiffness coefficient A_{45} of the balanced oblique symmetric lay-ups are all equal to zero. According to Eq. (21), $a_{25} = a_{34} = a_{17} = 0$, the Eqs. (13) and (14) are reduced to:

$$[EI]_x = a_{33} \quad (30)$$

$$= \left(A_{11} - \frac{A_{12}^2}{A_{22}} \right) \left(\frac{1}{2} b_2 b_1^2 + \frac{1}{6} b_1^3 \right) + 2D_{11} b_2$$

$$[GJ] = a_{77} = \frac{2b_1^2 b_2^2}{b_1 + b_2} A_{66} \quad (31)$$

That is to say, the composite laminated box beam with the balanced oblique symmetric laminated wall has no bending-shear coupling and tensile-torsional coupling effect, so the equivalent bending stiffness and the equivalent torsional stiffness are larger even if the number of layers is constant.

Compared with the balanced oblique symmetric lay-ups, the existence of tensile-torsional coupling stiffness co-

Table 5: Five different kinds of lay-ups

Type	Lay-ups	No. of plies
CRL1	$[0/((\pm\theta)_m/O_4)_n]_s$	$N=2 \times [1+(2m+4) \times n]=50$
CRL2	$[90/(O_4/(\pm\theta)_m)]_s$	$N=2 \times [1+(2m+4) \times n]=50$
CRL3	$[0/(90_4/(\pm\theta)_m)_n]_s$	$N=2 \times [1+(2m+4) \times n]=50$
CRL4	$[90/(\pm 45)_n/(\pm\theta)_m]_s$	$N=2 \times [1+2(m+n)]=50$
CRL5	$[90/(\pm 45/(\pm\theta)_m)_n]_s$	$N=2 \times [1+(2m+2) \times n]=50$

efficient B_{16} of the balanced anti-symmetric lay-ups leads to a slightly smaller equivalent bending stiffness of the composite laminated box beam, but B_{16} will decrease as the number of layers is increased. When the number of layers is 50, $B_{16} \approx 0$, the equivalent bending stiffness and the equivalent torsional stiffness of composite laminated box beam with the balanced oblique symmetric and the balanced anti-symmetric are approximately equal.

In summary, in the case of the same number of layers, the composite laminated box beam with the balanced oblique symmetric or the balanced anti-symmetric lay-ups can obtain a greater equivalent stiffness. According to the design principle of the laminate, the balanced oblique symmetric laminate can also avoid the local out-of-plane warping deformation caused by tensile-bending and the bending-torsion coupling of the asymmetric laminate. Moreover, so as to reduce the intra-layer cracking and edge separation between the two directional layers [22], the number of layers of same ply angle in laminates is often less than 4 in practical engineering applications. In order to study the influences of the ply angle and the lay-ups on equivalent bending stiffness and equivalent torsional stiffness of composite laminated box beam, the example in 3.2 was also taken as the object. The five kinds of lay-ups shown in Table 5 were selected on the basis of satisfying the design principle of laminates mentioned above. Calculate the equivalent bending stiffness and equivalent torsional stiffness using Eqs. (13) and (14), the results are shown in Figure 12.

5.2 Prediction of equivalent stiffness under five lay-ups

It can be seen from Figure 12 that the equivalent stiffness of the composite laminated box beam has extremely high designability. Dividing the values of $[EI]_x$ and $[GJ]$ in each figure in Figure 12 by the section bending moment of inertia I_x or the torsional moment of inertia J_d respectively, the equivalent elastic modulus E_{eq} and the equivalent shear modulus G_{eq} of the composite laminated box beam under

Figure 12: Equivalent bending and torsional stiffnesses of composite laminated box beam in five lay-ups change with ply angle.

five kinds of lay-ups can also be predicted. The change of E_{eq} and G_{eq} is the same as that of the curves in Figure 12, but the numerical values and units are different, which is not repeated here.

Compared with the balanced anti-symmetric lay-up, the equivalent bending stiffness of the CRL1 and CRL2 lay-ups are larger, and the equivalent torsional stiffness is smaller; the CRL1 lay-ups can obtain the maximum equivalent bending stiffness at any angle. However, the CRL1 lay-up is continuously laid 8 layers of 0° at the mid-plane of the wall laminate, which does not meet the design principle of the laminate, and should be avoided in engineering. The equivalent bending stiffness of the CRL2 lay-up is greater than the balanced anti-symmetric lay-up in the range of $0^\circ < \theta \leq 12^\circ$, which can be used for the composite structures that require high equivalent bending stiffness.

The 90° layers ratio of the CRL3 lay-up is high, and the equivalent bending stiffness and equivalent torsional stiffness are lower than the balanced oblique symmetric lay-up. The CRL3 lay-ups can fully exert the hoop tightening effect of the 90° layer. The CRL4 and CRL5 lay-ups can

obtain greater equivalent torsional stiffness than the balanced oblique symmetric lay-ups, and the equivalent torsional stiffness is not sensitive to the change of θ . Within the range of $0^\circ < \theta < 45^\circ$, the value of the equivalent bending stiffness is larger, so if both the equivalent bending stiffness and equivalent torsional stiffness of the composite laminated box beam have maximum design requirements, θ should be less than 45° .

6 Global buckling analysis of composite laminated box beam

The cross-section of composite laminated box beam has biaxial symmetry, and bending instability generally occurs under axial compression load. It can be known from the traditional material mechanics that the critical load of bending instability is closely related to the bending stiffness and slenderness ratio of structures. Therefore, the theoretical calculation formula of the global buckling critical

load can be derived by combining the analytical formulas of equivalent stiffnesses.

6.1 Global buckling equilibrium differential equation

For composite laminated thin-walled box beam, the effect of transverse shear deformation should be fully considered when establishing the global buckling calculation model. For simple supported beams, let w_M and w_S be the deflections caused by the bending moment and the shear force respectively, the total deflection is:

$$w = w_M + w_S \quad (32)$$

Find the second derivative of Eq. (32), the formula for calculating the curvature can be defined by:

$$\frac{d^2 w}{dz^2} = \frac{d^2 w_M}{dz^2} + \frac{d^2 w_S}{dz^2} \quad (33)$$

The curvature produced by the bending moment is:

$$\frac{d^2 w_M}{dz^2} = \frac{Fw}{[EI]_y} \quad (34)$$

where F is the axial compression load, $[EI]_y$ is the equivalent bending stiffness with respect to the y -axis, calculated by Eq. (13). The shear angle can be calculated according to Shearing Hooke's law:

$$\gamma = \frac{dw_S}{dz} = k_s \frac{Fw'}{G_{eq}A} \quad (35)$$

where A is the cross-sectional area, k_s is the shear section coefficient, for a box cross-sections, $k_s = 2$; G_{eq} is the equivalent shear modulus, which can be obtained by dividing the equivalent torsional stiffness $[GJ]$ by the torsional moment of inertia J_d , $[GJ]$ is calculated by Eq. (14). The additional curvature produced by the shear deformation is:

$$\frac{d^2 w_S}{dz^2} = k_s \frac{Fw''}{G_{eq}A} \quad (36)$$

Substitute Eqs. (36) and (34) into Eq. (33), the differential equation of the deflection curve considering both the bending moment and the shear force can be defined by:

$$\frac{d^2 w}{dz^2} + \frac{G_{eq}A}{G_{eq}A - k_s F} \frac{F}{[EI]_y} w = 0 \quad (37)$$

The general solution of Eq. (37) is:

$$\begin{cases} w = C_1 \sin kz + C_2 \cos kz \\ k^2 = \frac{G_{eq}A}{G_{eq}A - k_s F} \frac{P}{[EI]_y} \end{cases} \quad (38)$$

where C_1, C_2 are constant. The boundary condition of the simply supported beam is:

$$\begin{cases} z = 0, w = 0 \\ z = l, w = 0 \end{cases} \quad (39)$$

Substitute Eq. (39) into Eq. (38), the formula for calculating the global buckling is given by:

$$F_{cr} = \frac{1}{1 + \frac{k_s \pi^2}{G_{eq}A} \frac{[EI]_y}{l^2}} \frac{\pi^2}{l^2} [EI]_y \quad (40)$$

By introducing the length factor μ_y , the formula for calculating the global buckling critical load of composite laminated box beam with y -axis instability under different boundary conditions is obtained as:

$$F_{cr} = \alpha \frac{\pi^2}{(\mu_y l)^2} [EI]_y \quad (41)$$

where α is the shear effect coefficient, defined by:

$$\alpha = \frac{1}{1 + \frac{k_s \pi^2}{G_{eq}A} \frac{[EI]_y}{(\mu_y l)^2}} \quad (42)$$

It can be found that $\alpha \leq 1$, and if $\alpha = 1$, $F_{cr} = P_y$, the result of the Eq. (41) is the same as the Euler critical load formula in traditional material mechanics.

6.2 Analysis of influencing factors of global buckling critical load

6.2.1 Effect of shear deformation

The Eq. (42) can be reduced to:

$$\alpha = \frac{1}{1 + \frac{k_s \pi^2 E_{eq}}{G_{eq} \lambda_y^2}} \quad (43)$$

where λ_y is the slenderness ratio of y axis; E_{eq} is the equivalent elastic modulus, which can be obtained by dividing the equivalent bending stiffness $[EI]_y$ by the moment of inertia I_y . It can be found that if the span of the composite laminated box beam is fixed, the shear effect coefficient is related to the ratio of E_{eq} and G_{eq} . The numerical examples in Section 3.2 with balanced oblique symmetric $[(\theta/-\theta)_{12}/\theta]_s$ lay-up is selected, the shearing effect coefficient of the composite laminated box beam under the simple supported and cantilever conditions are calculated by the Eq. (43), and compared with the isotropic material box beam, as shown in Figure 13. It can be seen that the effect of transverse shear deformation for isotropic material box beam is less than 5%; for composite laminated

Figure 13: Shear effect coefficients of simply supported beam and cantilever beam change with ply angle.

Figure 14: Global buckling load of composite box beam calculated by the two methods change with ply angle.

box beam, transverse shear deformation will reduce the global buckling critical load by 27% at $\theta=0^\circ$. The different of calculating the global buckling critical load with consideration of shear deformation or not is shown in Figure 14. Compared with the cantilever beam, the simply supported beam is more affected by the transverse shear deformation; at $\theta = 10^\circ$, the global buckling critical load of the simply supported beam takes the maximum value; in the range of $0^\circ < \theta < 20^\circ$, neglecting the transverse shear deformation to calculate the global buckling critical load of the simply supported beam will produce a large error, the maximum error can reach more than 20%.

Figure 15: Global buckling load of composite laminated box beam in 3 different lay-ups.

6.2.2 Effect of the lay-ups

It can be seen from Eqs. (40) that F_{cr} is related to $[EI]_y$, $[GJ]$ of composite laminated box beam if the box beam geometry is fixed. As can be seen from Section 4, it is possible to obtain a larger $[EI]_y$, $[GJ]$ by increasing the ratio of 0° and $\pm 45^\circ$ laminates, and laying 0° and $\pm 45^\circ$ monolayers on the outside of the wall as much as possible. According to the above conclusions, combined with the design principle of laminate ply, select three lay-ups: (i) $[\pm\theta]_{25}$; (ii) $[90/(0_4/\pm\theta)_4]_s$; (iii) $[90/(\pm 45)_{11}/\pm\theta]_s$. The F_{cr} of the numerical example in the three lay-ups are calculated by Eqs. (40) and Euler formula respectively, and the results are shown in Figure 15. It can be seen that increasing the 0° layers ratio can significantly improve F_{cr} . Regardless of the lay-ups, the reduction of F_{cr} is most obvious in the range of $0^\circ < \theta < 45^\circ$; increasing the 45° layers ratio reduces the effect of lateral shear deformation, but also reduces F_{cr} .

6.2.3 Effect of slenderness ratio

Select the three lay-ups of section 5.2.2, calculate the F_{cr} by Eq. (40). The change of F_{cr} with the slenderness ratio λ_y and the ply angle θ is shown in Figure 16. It can be seen that the F_{cr} of the lay-ups $[90/(0_4/\pm\theta)_4]_s$ is the largest. $0^\circ < \theta < 40^\circ$, the three layers are arranged in order of F_{cr} from large to small: $[90/(0_4/\pm\theta)_4]_s$, $[\pm\theta]_{25}$, $[90/(\pm 45)_{11}/\pm\theta]_s$; $40^\circ \leq \theta < 90^\circ$, the order is: $[90/(0_4/\pm\theta)_4]_s$, $[90/(\pm 45)_{11}/\pm\theta]_s$, $[\pm\theta]_{25}$. $\lambda_y > 80$, the difference of F_{cr} under the three lay-ups is less than 5%, indicating that the enhancement of F_{cr} by increasing the number of 0° layers

Figure 16: Global buckling critical load of composite laminated box beam in three different lay-ups change with the ply angle and the slenderness ratio.

Figure 17: $1/\alpha$ of composite laminated box beam in three different lay-ups change with the ply angle and the slenderness ratio

is weakened with the increase of slenderness ratio. Therefore, for large-to-fine-ratio composite laminated box beam, the λ_y should be reduced by increasing the cross-sectional size and increasing the thickness of the wall, thereby increasing F_{cr} .

The shear effect coefficient α is calculated by the Eq. (43), and the change of $1/\alpha$ with λ_y and θ is as shown in Figure 17. It can be seen from that the three types of lay-ups are ordered according to the degree of effect of shear deformation as: $[90/(0_4/\pm\theta)_4]_s$, $[\pm\theta]_{25}$, $[90/(\pm 45)_{11}/\pm\theta]_s$; α increases to 1 with the increase of λ_y and θ . $\lambda_y > 80$, $1/\alpha < 1.05$, the effect of transverse shear deformation is small, which can be neglected in engineering.

7 Conclusion remarks

- (1) The analytical model of shear-deformable beam considering true stress state of composite laminated box beam is established and refined, and the equivalent bending and torsional stiffness are calculated by the stiffness coefficients in classical laminate theory. This method is simple, accurate, and suitable for CUS configuration composite laminated box beams with arbitrary lay-ups.
- (2) The shear-bending coupling and tensile-torsional coupling of composite laminated box beam reduce the equivalent bending stiffness and equivalent torsional stiffness, respectively. There are no elastic coupling effects for composite laminated box beam of the balanced oblique symmetric, the balanced anti-symmetric and the orthogonal lay-ups.
- (3) The deflection caused by transverse shear deformation must be considered in the global buckling analysis of composite laminated box beam. In order to obtain a larger equivalent bending stiffness, the layer angle is generally taken within $0^\circ < \theta < 45^\circ$, and the shear deformation has a greater effect on the global buckling critical load in this range. Compared with the cantilever beam, the simply supported beam is greatly affected by shear deformation.
- (4) F_{cr} of composite laminated box beam with $[90/(0_4/\pm\theta)_4]_s$ lay-ups is the largest. With the increase of the slenderness ratio, the enhancement of the F_{cr} by increasing the number of 0° layer is weakened. The effect of shear deformation can be ignored if the slenderness ratio is greater enough.

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Appendix A

$$a_{11} = 2b_1 + b_2 \left(A_{11} - \frac{A_{12}^2}{A_{22}} \right)$$

$$a_{17} = 2b_1 b_2 \left(A_{16} - \frac{A_{12} A_{26}}{A_{22}} \right)$$

$$a_{22} = A_{11} - \frac{A_{12}^2}{A_{22}} \left(\frac{1}{2} b_1 b_2^2 + \frac{1}{6} b_2^3 \right) + 2 \left(B_{11} - \frac{A_{12} B_{12}}{A_{22}} \right) b_1 b_2 + 2 \left(D_{11} - \frac{B_{12}^2}{A_{22}} \right) b_1$$

$$a_{25} = \left(A_{16} - \frac{A_{12} A_{26}}{A_{22}} \right) b_1 b_2 + 2 \left(B_{16} - \frac{A_{26} B_{12}}{A_{22}} \right) b_1$$

$$a_{33} = \left(A_{11} - \frac{A_{12}^2}{A_{22}} \right) \left(\frac{1}{2} b_2 b_1^2 + \frac{1}{6} b_1^3 \right) + 2 \left(B_{11} - \frac{A_{12} B_{12}}{A_{22}} \right) b_1 b_2 + 2 \left(D_{11} - \frac{B_{12}^2}{A_{22}} \right) b_2$$

$$a_{34} = - \left(A_{16} - \frac{A_{12} A_{26}}{A_{22}} \right) b_1 b_2 - 2 \left(B_{16} - \frac{A_{26} B_{12}}{A_{22}} \right) b_2$$

$$a_{44} = 2 \left(A_{55} - \frac{A_{45}^2}{A_{44}} \right) b_1 + 2 \left(A_{66} - \frac{A_{26}^2}{A_{22}} \right) b_2$$

$$a_{55} = 2 \left(A_{55} - \frac{A_{45}^2}{A_{44}} \right) b_2 + 2 \left(A_{66} - \frac{A_{26}^2}{A_{22}} \right) b_1$$

$$a_{77} = \frac{2b_1^2 b_2^2}{b_1 + b_2} \left(A_{66} - \frac{A_{26}^2}{A_{22}} \right)$$