Research Article

Mehran Karimi Zeverdejani and Yaghoub Tadi Beni*

Size-dependent vibration analysis of graphene-PMMA lamina based on non-classical continuum theory

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Abstract: This paper studies the free vibration of polymer nanocomposite reinforced by graphene sheet. In this work, the new size dependent formulation is presented for nanocomposites based on couple stress theory. For this purpose, the first shear deformation theory is applied. The effect of scale parameter is investigated based on anisotropic couple stress theory. Vibration equations of the composite lamina are extracted using Hamilton's principle. Numerical results are provided for Poly methyl methacrylate/graphene composite. Mechanical properties of the composite are obtained from molecular dynamics simulation. Based on eigenvalue procedure, an analytical solution is obtained for the natural frequency of composite lamina. In the results section, the effect of dimensional and physical parameters are investigated on lamina natural frequency. It is observed that graphene defects caused to diminish the lamina frequency. Furthermore, it is revealed that the increase in graphene volume fraction leads to natural frequency be greater.

Keywords: composite lamina, anisotropic, size effect, couple stress theory, graphene defects

1 Introduction

Today, composite materials have improved the performance in various mechanisms. Additionally, by exploiting the low volume fraction of the reinforcement, the mechanical properties of structures have been greatly improved and lead to the ideal efficiency of consumables [1–4]. Basically, designers are eager to build the structures with high

strength to weight ratio. Depend on the applications, there are various applicable types of reinforcements and matrices. Among the most widely used composites, one can be mentioned to metal-metal, metal-ceramics and polymer composites [5, 6]. Including the nanoscale reinforcements, it can be noted to the graphene sheet, carbon nanotube, and fullerene which the second and third elements made from twisting and rounding the graphene sheets, respectively [7]. Graphene is one of the thinner known materials in the nature that is about 0.34 nanometers thick. Graphene has special properties compared to other nano-reinforcements. Because the top and bottom surfaces of the graphene sheet surrounded in the matrix, it has an excellent surface contacts. Moreover, the price of graphene is reasonable [8, 9].

Thus far, several studies have been conducted on polymer composite reinforced by nanoparticles [10–13]. Shen *et al.* [14, 15] investigated vibration and buckling of functionally graded composite reinforced by graphene and carbon nanotube (CNT). Mirzaei and Kiani [16] studied vibration of functionally graded CNT reinforced composite cylindrical panels. Zhang *et al.* [17] investigated the vibration of triangle composite plate reinforced by CNT.

In addition to macro, Composites are also used in micro and nano scales. For instance, nano-scale composites utilize in the production of nanoelectromechanical systems (NEMS) and microelectromechanical systems (MEMS) [18]. In the nanoscale, due to certain physical conditions and the low ratio of molecular distance to the structural dimensions, the material scale parameter affects the mechanical behavior of the structure [19]. As a result, the size effect must be applied in constitutive equations. There are several size dependent continuum theories. In some of these theories, the scale parameter has a softening effect on the results, such as nonlocal Eringen's theories [20–22]. Barretta *et al.* [23] presented nonlocal integral model for Timoshenko beam model. Romano *et al.* [24] studied constitutive boundary condition of nano-beam based on non-local theory. Higher order nonlocal Euler Bernoulli beam model presented by Barretta *et al.* [25]. Acierno *et al.* [26]

*Corresponding Author: Yaghoub Tadi Beni: Faculty of Engineering, Shahrekord University, Shahrekord, Iran; Email: tadi@eng.sku.ac.ir

Mehran Karimi Zeverdejani: Mechanical engineering department, Shahrekord University, Shahrekord, Iran

measured mechanical properties of polymer nano composite based on both nonlocal theory and experimental approach. Also, the nonlocal elastoplastic model presented by De sciara [27]. In some other theories, the scale parameter has a hardening effect on the results, such as the strain gradient and couple stress theories [28–30]. Aifanties [31] presented update of scale dependent gradient theories. Reviewed study was conducted on the stress gradient theory for size-dependent response of structures by Askes and Gitman [32]. Furthermore, some studies conducted based on nonlocal strain gradient theory [33, 35]. Barretta and Sciarra [36] investigated constitutive boundary conditions of nano-beams according to nonlocal strain gradient theory. Vibration analysis of elastic beam was studied by Apuzzu *et al.* [37] based on nonlocal gradient theory. Canadija *et al.* [38] extended nonlocal gradient formulation for beam model.

For the first time, couple stress theory was presented by Mindlin in 1962 [39]. So far, many scientists have studied the behavior of nanoscale elements using this theory [40–44]. Chen and Yang [45] developed the couple stress theory for anisotropic behavior. Some scholars using the anisotropic couple stress theory have studied the mechanical behavior of anisotropic structures. Tadi *et al.* [46, 47] studied free vibration and buckling of CNT based on anisotropic size dependent shell model. Gao *et al.* [48] benefitting anisotropic couple stress model studied the static deformation of layered magneto electro-elastic plates under surface loading. Buckling analysis of orthotropic protein microtubules investigated by Tadi *et al.* [49]. Yang and He [50] investigated static analysis of orthotropic microplate based on modified couple stress theory. Tsiatas and Yiotis [51] studied the static, dynamic and buckling of the anisotropic skew plate. It should be noted that conventional size dependent models have been challenged in some new studies [52–54]. The kinematics of generalized micro-morphic continua is presented by Romano *et al.* [52]. They proposed the simplest non-redundant model by dropping the micro-curvature term from the Mindlin theory. Furthermore, Barbagallo *et al.* [53] showed that a general formula is not applicable for the standard Mindlin-Eringen-format of the anisotropic micro-morphic model. Furthermore, Neff *et al.* [54] presented the relaxed micro-morphic model and derive the set of appropriate conditions that have to be imposed on the constitutive parameters. While, some researchers used nonlocal and gradient based size dependent theories [55].

Given that mechanical testing and control of its process are very costly and complex, molecular simulation is a suitable method for determining the materials properties. Some researchers calculated mechanical properties of

nanocomposites by molecular dynamics simulations (MD). Han an Eliot [56] calculated mechanical properties of polymer composite reinforced by CNT using MD simulation. Lin *et al.* [57] using MD obtained mechanical properties of graphene-reinforced polymer composites. Moreover, Yunlong *et al.* [58] studied the mechanical properties of polymer composite reinforced by CNT and graphene sheet.

In this paper, new size dependent formulation is presented for the vibrational behavior of nano-composite. For sake of this aim, the anisotropic motion equations are extracted using Hamilton's principle. These equations are developed for the first shear deformation plate model (FSDT). Furthermore, the size effect is involved according to the couple stress theory. It is noted the present vibrational equations are reduced to different states in certain conditions:

- If the mechanical properties of the plate are identical in various directions, vibration equations of the size-dependent isotropic plate are obtained.
- If the scale parameter is ignored, vibration equations of the orthotropic plate are extracted for classical continuum mechanic.
- By combining the two above assumptions, vibration equations are derived for an isotropic macro plate.

Indeed, current work presents the new formulation for orthotropic composite plates which considers orthotropic and size sensitivity behavior of composite plates, simultaneously. Mechanical properties are also evaluated based on the molecular simulation. For the case study, the vibration of all edges simply supported (SSSS) Poly methyl methacrylate (PMMA) lamina reinforced by graphene sheet are investigated. Elastic properties of lamina reinforced by defected graphene are gained by molecular dynamics simulation. In the results section, the effect of physical and geometric parameters on the natural frequency of lamina has been investigated. Also, different vibrational mode shapes of lamina have been shown.

2 Size-dependent formulation for composite plate

In this work, the lamina is modeled as FSDT plate. The displacement components of this model are defined as follows:

$$\begin{aligned} u &= u_0(x, y, t) + z\phi_x(x, y, t) \\ v &= v_0(x, y, t) + z\phi_y(x, y, t) \\ w &= w_0(x, y, t) \end{aligned} \quad (1)$$

Figure 1: Schematic of the X-Z and Y-Z views of Graphene reinforced lamina.

In the above equation u , v and w are total displacements related to x , y and z directions, respectively. u_0 , v_0 , and w_0 indicate the in-plane displacements (at the $z=0$). Also, ϕ_x and ϕ_y represent a rotation around y and x -axes. Schematic of the lamina is illustrated in Figure 1. Based on energy methods, Hamilton's principle is used to develop the motion equations. According to this theory, the variation of the system energy in a period of time is equal to zero as follows:

$$\delta \int_0^t [T - U] dt = 0 \quad (2)$$

In the above equation, U expresses lamina strain energy, and T indicates the lamina kinetic energy. Based on the couple stress theory, the strain energy of a body is obtained through the following integration:

$$U = \frac{1}{2} \int_{\Omega} [\sigma : \varepsilon + m : \chi] dV \quad (3)$$

Where ε , σ , χ and m represent strain tensor, Cauchy's stress tensor, rotation gradient tensor, and higher order stress tensor, respectively. Based on the couple stress formulation, the strain and rotation gradient tensors are defined as follows:

$$\varepsilon = \frac{1}{2} [\text{grad}(\vec{u}) + (\text{grad}(\vec{u}))^T] \quad (4)$$

$$\chi = \frac{1}{2} \left[\text{grad} \left(\frac{1}{2} \text{curl}(\vec{u}) \right) + \left(\text{grad} \left(\frac{1}{2} \text{curl}(\vec{u}) \right) \right)^T \right] \quad (5)$$

In the following sections, the calculation process of classic and higher-order stresses using ε and χ values is explained. For orthotropic materials, the relationship between stress and strain is expressed as follows:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xz} \\ \sigma_{yz} \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{21} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{55} & 0 & 0 \\ 0 & 0 & 0 & Q_{66} & 0 \\ 0 & 0 & 0 & 0 & Q_{44} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xz} \\ \varepsilon_{yz} \\ \varepsilon_{xy} \end{Bmatrix} \quad (6)$$

$$= \begin{bmatrix} \frac{E_{xx}}{1-\nu_{xy}\nu_{yx}} & \frac{\nu_{xy}E_{yy}}{1-\nu_{xy}\nu_{yx}} & 0 & 0 & 0 \\ \frac{\nu_{yx}E_{xx}}{1-\nu_{xy}\nu_{yx}} & \frac{E_{yy}}{1-\nu_{xy}\nu_{yx}} & 0 & 0 & 0 \\ 0 & 0 & 2G_{yz} & 0 & 0 \\ 0 & 0 & 0 & 2G_{xz} & 0 \\ 0 & 0 & 0 & 0 & 2G_{xy} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xz} \\ \varepsilon_{yz} \\ \varepsilon_{xy} \end{Bmatrix}$$

Besides, the relationship between higher-order stress and rotation gradient is defined as follows [45–47]:

$$m_{ij} = l_i^2 G_{ij} \chi_{ij} + l_j^2 G_{ji} \chi_{ji} \quad (7)$$

In the above equation, l_i represents the material length scale parameter. By substituted Eq. (1) into Eq. (4), components of lamina strain are derived as follows:

$$\varepsilon_{xx} = u_{0,x} + z\phi_{x,x} \quad (8)$$

$$\varepsilon_{yy} = v_{0,y} + z\phi_{y,y} \quad (9)$$

$$\varepsilon_{xz} = \frac{1}{2} (\phi_x + w_{0,x}) \quad (10)$$

$$\varepsilon_{xy} = \frac{1}{2} (v_{0,x} + u_{0,y} + z(\phi_{y,x} + \phi_{x,y})) \quad (11)$$

$$\varepsilon_{yz} = \frac{1}{2} (\phi_y + w_{0,y}) \quad (12)$$

As well as, by substituting Eq. (1) into Eq. (5), components of lamina rotation gradient tensor are obtained as follows:

$$\chi_{xx} = \frac{1}{2} [w_{0,xy} - \phi_{y,x}] \quad (13)$$

$$\chi_{yy} = \frac{1}{2} [\phi_{x,y} - w_{0,xy}] \quad (14)$$

$$\chi_{zz} = \frac{1}{2} [\phi_{y,x} - \phi_{x,y}] \quad (15)$$

$$\chi_{xy} = \frac{1}{4} [w_{0,yy} - \phi_{y,y} + \phi_{x,x} - w_{0,xx}] \quad (16)$$

$$\chi_{xz} = \frac{1}{4} [v_{0,xx} - u_{0,xy} + z\phi_{y,xx} - z\phi_{x,xy}] \quad (17)$$

$$\chi_{zy} = \frac{1}{4} [v_{0,xy} - u_{0,yy} + z\phi_{y,xy} - z\phi_{x,yy}] \quad (18)$$

According to Eqs. (3), (6) and (7), and by using components of strain and rotation gradient tensors which were obtained in the Eqs. (8) to (18), the strain energy of composite is obtained as follows:

$$U = \iiint_V \left\{ \frac{E_{xx}}{1-\nu_{xy}\nu_{yx}} (u_{0,x} + z\phi_{x,x})^2 \right. \quad (19)$$

$$\begin{aligned}
& + \frac{v_{xy}E_{yy}}{1 - v_{xy}v_{yx}} (u_{0,x} + z\phi_{x,x}) (v_{0,y} + z\phi_{y,y}) \\
& + \frac{E_{yy}}{1 - v_{xy}v_{yx}} (v_{0,y} + z\phi_{y,y})^2 \\
& + \frac{v_{yx}E_{xx}}{1 - v_{xy}v_{yx}} (u_{0,x} + z\phi_{x,x}) (v_{0,y} + z\phi_{y,y}) \\
& + G_{xz}(\phi_x + w_{0,x})^2 + G_{xy}(v_{0,x} + u_{0,y} + z(\phi_{y,x} + \phi_{x,y}))^2 \\
& + G_{yz}(\phi_y + w_{0,y})^2 + \frac{1}{2}l_x^2 G_{xx}(w_{0,xy} - \phi_{y,x})^2 \\
& + \frac{1}{2}l_y^2 G_{yy}(\phi_x, y - w_{0,xy})^2 \\
& + \frac{1}{8}G_{xy}(w_{0,yy} - \phi_{y,y} + \phi_{x,x} - w_{0,xx})^2 (l_x^2 + l_y^2) \\
& + \frac{1}{2}l_z^2 G_{zz}(\phi_{y,x} - \phi_{x,y})^2 \\
& + \frac{1}{8}G_{xz}(v_{0,xx} - u_{0,xy} + z\phi_{y,xx} - z\phi_{x,xy})^2 (l_x^2 + l_z^2) \\
& + \frac{1}{8}G_{yz}(v_{0,xy} - u_{0,yy} + z\phi_{y,xy} \\
& - z\phi_{x,yy})^2 (l_z^2 + l_y^2) \} dV
\end{aligned}$$

After determining the lamina strain energy, the kinetic energy of composite is calculated. Using the time derivative on the FSDT displacement vector, the plate kinetic energy is computed as follows:

$$T = \frac{\rho}{2} \iiint_V \left\{ u_{0,t}^2 + z^2 \phi_{x,t}^2 + 2z\phi_{x,t}u_{0,t} + v_{0,t}^2 + z^2 \phi_{y,t}^2 + 2z\phi_{y,t}v_{0,t} + w_{0,t}^2 \right\} dV \quad (20)$$

By substituting Eqs. (19) and (20) into Eq. (2), and using calculus of variation approach, the size-dependent vibration equations of the orthotropic lamina are developed as follows:

$$C_1 u_{0,xx} + C_2 v_{0,xy} + C_3 u_{0,yy} + C_4 v_{0,xxx} + C_5 u_{0,xyy} + C_6 v_{0,xyy} + C_7 u_{0,yyy} + C_8 u_{0,tt} = 0 \quad (21)$$

$$C_9 v_{0,yy} + C_{10} u_{0,xy} + C_{11} v_{0,xx} + C_{12} v_{0,xxx} + C_{13} u_{0,xxx} + C_{14} v_{0,xyy} + C_{15} u_{0,xyy} + C_{16} v_{0,tt} = 0 \quad (22)$$

$$\begin{aligned}
& C_{17} \phi_{x,x} + C_{18} w_{0,xx} + C_{19} \phi_{y,y} + C_{20} w_{0,yy} \\
& + C_{21} w_{0,xyy} + C_{22} \phi_{y,xy} + C_{23} \phi_{x,xy} + C_{24} \phi_{y,yyy} \\
& + C_{25} w_{0,yyy} + C_{26} \phi_{x,xxx} + C_{27} w_{0,xxx} + C_{28} w_{0,tt} = 0
\end{aligned} \quad (23)$$

$$\begin{aligned}
& C_{29} \phi_{x,xx} + C_{30} \phi_{y,xy} + C_{31} \phi_{x,yy} + C_{32} \phi_x + C_{33} w_{0,x} \\
& + C_{34} w_{0,xy} + C_{35} w_{0,xxx} + C_{36} \phi_{y,xxx} + C_{37} \phi_{x,xyy} \\
& + C_{38} \phi_{y,yyy} + C_{39} \phi_{x,yyy} + C_{40} \phi_{x,tt} = 0
\end{aligned} \quad (24)$$

$$C_{41} \phi_{y,yy} + C_{42} \phi_{x,xy} + C_{43} \phi_{y,xx} + C_{44} \phi_y + C_{45} w_{0,y} \quad (25)$$

$$\begin{aligned}
& + C_{46} w_{0,xy} + C_{47} w_{0,yy} + C_{48} \phi_{y,xxx} + C_{49} \phi_{x,xyy} \\
& + C_{50} \phi_{y,xyy} + C_{51} \phi_{x,yyy} + C_{52} \phi_{y,tt} = 0
\end{aligned}$$

In the above equations, C_{ij} 's are reliant on the mechanical properties of the lamina which provided in appendix A.

3 Molecular dynamics simulation

This section may be divided by subheadings. It should provide a concise and precise description of the experimental results, their interpretation as well as the experimental conclusions that can be drawn. As mentioned in the introduction section, use of the experimental procedure for extract the mechanical properties of nanoscale composites are very complicated and difficult, and one of the most suitable methods is use of molecular dynamics. The molecular dynamics method is one of the powerful methods of molecular mechanics which is designed based on numerical integration of Newton's equations on the molecular domain of a body. In order to perform molecular dynamics simulation, first, the initial conditions of the system including the speed and the primary location of the particles are determined. Then, by selecting a suitable time interval, the new atomic positions will be predicted according to the previous position and initial velocity. Intermolecular interactions using a function the potential is expressed. In fact, the potential function is the main input in the molecular dynamics simulation. The potential function of N-particle systems is given by the following equation:

$$\begin{aligned}
\phi(r) = & \sum_i \phi_1(r_i) + \sum_i \sum_{j \neq i} \phi_2(r_i, r_j) \\
& + \sum_i \sum_{j \neq i} \sum_{k \neq j} \phi_3(r_i, r_j, r_k) + \dots
\end{aligned} \quad (26)$$

Where ϕ_i is represented the various kind of interactions for individual particles. The force applied to each particle is calculated from the following equation:

$$F_i = \frac{\partial \phi}{\partial r_i} \quad (27)$$

Also, according to Newton's second law, the following basic relation is extracted for each particle

$$F_i = \frac{\partial \phi}{\partial r_i} = m_i \ddot{r}_i \quad (28)$$

Using the above equation, we can calculate the acceleration of each particle and then using numerical integration, in a very small time interval, the new position and speed of the particles have been achieved. In the present

Figure 2: (a) Stone Walls defect (SW), (b) Vacancy Defect (VD), (c) molecular model of the lamina.

Table 1: Mechanical properties of the composite with 0.19 volume fraction for various graphene defects.

Type of graphene-reinforced in PMMA	$E_{11}(\text{GPa})$	$E_{22}(\text{GPa})$	$G_{12}(\text{GPa})$
Perfect graphene	104.1	107.7	22.7
Graphene with 1.6% vacancy	98.1	100.5	20.8
Graphene with 5% vacancy	93.2	92.6	18.8
Graphene with 1.6% stone wales	103.5	97	21.4
Graphene with 5% stone wales	91.9	86.9	19.7

work, the mechanical properties of the composite are calculated using Material Studio software and effects of vacancy (VD) and stone wales (SW) defects of graphene are explored on the stiffness of lamina. The stone wales and vacancy defects along the molecular model of the lamina are shown in Figure 2. Composite is modeled based on COMPASS forcefield. In the beginning, the molecular model of the lamina is subjected to NVT ensemble in 450°K . Then, it is exposed to NPT ensemble by 0.1 GPa pressure. In the end, by applying finite strain, mechanical properties of the lamina are determined according to Table 1.

4 Solution method

Based on Navier's solution method, for simply supported lamina, the displacement components are considered as follows:

$$u_0 = A_1 \cos\left(\frac{m\pi x}{L}\right) \cos\left(\frac{n\pi y}{L}\right) e^{i\omega t} \quad (29)$$

$$v_0 = A_2 \sin\left(\frac{m\pi x}{L}\right) \sin\left(\frac{n\pi y}{L}\right) e^{i\omega t}$$

$$w_0 = A_3 \sin\left(\frac{m\pi x}{L}\right) \cos\left(\frac{n\pi y}{L}\right) e^{i\omega t}$$

$$\phi_x = A_4 \cos\left(\frac{m\pi x}{L}\right) \cos\left(\frac{n\pi y}{L}\right) e^{i\omega t}$$

$$\phi_y = A_5 \sin\left(\frac{m\pi x}{L}\right) \sin\left(\frac{n\pi y}{L}\right) e^{i\omega t}$$

In the above equation, ω is the lamina natural frequency. Also, A_1 , A_2 , A_3 , A_4 , and A_5 represent the vibration amplitude for each of the variables. Moreover, m and n represent the longitudinal mode number and circumferential mode number, respectively. By substituting Eq. (29) into the motion equations, the resulted equations are converted to following eigenvalue problem:

$$([K] - [M\omega^2]) \{A_i\} = 0 \quad (30)$$

In the above equation, K and M are stiffness and mass matrixes. After solving Eq. (30), the lamina natural frequency is obtained, which will be discussed in detail in the results section.

Figure 3: Variation of lamina frequency with α parameter: a) vacancy defect b) stone wales defect.

Figure 4: Variation of lamina natural frequency with α parameter for different graphene volume fraction.

5 Results and discussions

The aim beyond of the present work is to study the free vibration of Graphene-reinforced lamina. Predict the oscillation behavior of nano-structures can be useful for the control of their performance. Polymeric composites reinforced by graphene sheets have a good physical properties and can be used in nanoelectromechanical systems. In this section, according to the procedure that presented before, the vibration of simply supported lamina composite is investigated using the orthotropic FSDT model. Various numerical examples are proposed to investigate the influence of geometrical and material properties on the vibration behavior of lamina. In order to study the vibrational behavior

of current composite model, natural frequencies are presented for the various geometrical parameters, α parameter represents ratio of length to lamina thickness (a/h). β parameter also signifies width to lamina thickness (b/h). Furthermore, λ parameter is defined as a non-dimensional length scale parameter (l/h). Besides, the material length scale parameter is assumed as $l_1 = l_2 = l_3 = l$. By literature review, the value of length scale l is calibrated for graphene sheet by 2.46 nm length and width. Arash and Wang [59] using MD simulation demonstrated that the natural frequency of the aforementioned graphene is equal to 77.6 GHz. By comparison this frequency with the solution of equation (30), length scale parameter is calibrated in 0.118 nm.

In the Figure 3, influences of graphene defects on lamina natural frequency are studied. The mechanical properties of the composites reinforced by defected graphene are given in Table 1. According to the following graphs, it is clear that vacancy (VD) and stone wales (SW) defects lead to decrease lamina frequency. Furthermore, it is obvious that the impact of graphene defects is more severe for lower α parameter.

In Figure 4, the variation of the lamina frequency related to α parameter is shown. As seen in this figure, by increasing of α parameter lamina natural frequency is reduced. It is also clear that by increasing the volume fraction of graphene, the natural frequency of lamina has grown that for α smaller than 60 this increase is more significant. It should be noted mechanical properties for the various volume fraction of reinforcement have been used from Shen study [40].

In Figure 5 the simultaneous effects of α and β dimensionless parameters are illustrated. According to this fig-

Figure 5: Variation of the natural frequency with α for different β parameters.

Figure 6: Variation of the natural frequency with λ for different graphene volume fraction.

ure, the highest lamina natural frequency has been obtained for smaller α and β parameters.

In Figure 6, the effect of the material length scale parameter is investigated on the results. It is observed that by increasing in λ parameter vibrational frequency of lamina increases which this result confirms the hardening effect of couple stress scale parameter. In addition, it is observed that the rate of frequency increase is more severe for composites with higher volume fractions of the graphene. Figure 7, provides the various vibration mode shapes of the lamina. In this figure, different mode shapes along with their corresponding frequencies are observed which $\alpha = 50$, $\beta = 50$, $\lambda = 1$ and $V_G = 0.03$ are considered.

Figure 7: Various vibrational mode shapes of the lamina.

6 Conclusion

In this paper, a new model was presented for nanoscale composites. For this purpose, the composite was modeled as FSDT plate model. Scale sensitivity was considered according to couple stress theory. Using the variations principles governing equations of anisotropic plate were derived. In order to a case study, free vibration of graphene-reinforced polymer lamina was examined. Mechanical properties of lamina reinforced by defected graphene were obtained by molecular dynamic simulation.

After solving the lamina free vibration equations, the results demonstrated that the increase in graphene volume fraction leads to increase in lamina frequency. Furthermore, it was determined that although the graphene volume fraction increases composite frequency, the intensity of the effect of this parameter for lamina with $\alpha > 0.01$ condition is stronger. In addition, it was founded that the impact of raising the scale parameter on the lamina frequency is stronger for higher graphene volume fraction.

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Appendix A

$$C_1 = -Q_{11}h$$

$$C_2 = -\left(Q_{12} + Q_{66}l_z^2\right)h$$

$$C_3 = -Q_{66}h$$

$$C_4 = \frac{G_{xz}h(l_z^2 + l_x^2)}{8}$$

$$C_5 = -C_4$$

$$C_6 = -\frac{G_{yz}h(l_y^2 + l_z^2)}{8}$$

$$C_7 = -C_6$$

$$C_8 = \rho h$$

$$C_9 = -Q_{22}h$$

$$C_{10} = C_2$$

$$C_{11} = C_3$$

$$C_{12} = -C_4$$

$$C_{13} = C_4$$

$$C_{14} = C_7$$

$$C_{15} = C_6$$

$$C_{16} = C_8$$

$$C_{17} = -Q_{44}h$$

$$C_{18} = C_{17}$$

$$C_{19} = Q_{55}h$$

$$C_{20} = C_{19}$$

$$C_{21} = \frac{h}{2} \left(G_{xx}l_x^2 + G_{yy}l_y^2 - \frac{G_{xy}(l_x^2 + l_y^2)}{2} \right)$$

$$C_{22} = \frac{h}{2} \left(\frac{G_{xy}(l_x^2 + l_y^2)}{2} - G_{xx}l_x^2 \right)$$

$$C_{23} = \frac{h}{2} \left(\frac{G_{xy}(l_x^2 + l_y^2)}{2} - G_{yy}l_y^2 \right)$$

$$C_{24} = \frac{G_{xy}h(l_x^2 + l_y^2)}{8}$$

$$C_{25} = -C_{24}$$

$$C_{26} = C_{24}$$

$$C_{27} = C_{25}$$

$$C_{28} = C_8$$

$$C_{29} = -\frac{h}{4} \left(\frac{Q_{11}h^2}{3} + \frac{G_{xy}(l_x^2 + l_y^2)}{2} \right)$$

$$C_{30} = \frac{h}{2} \left(\frac{G_{xy}(l_x^2 + l_y^2)}{4} + G_{zz}l_z^2 - \frac{Q_{12}h^2}{6} - \frac{Q_{66}h^2}{6} \right)$$

$$C_{31} = \frac{h}{2} \left(G_{zz}l_z^2 + G_{yy}l_y^2 + \frac{Q_{66}h^2}{6} \right)$$

$$C_{32} = Q_{44}h$$

$$C_{33} = C_{32}$$

$$C_{34} = \frac{h}{2} \left(G_{yy}l_y^2 - \frac{G_{xy}(l_x^2 + l_y^2)}{4} \right)$$

$$C_{35} = \frac{G_{xy}h(l_x^2 + l_y^2)}{8}$$

$$C_{36} = -\frac{G_{xz}h^3(l_x^2 + l_z^2)}{96}$$

$$C_{37} = -C_{36}$$

$$C_{38} = -\frac{G_{yz}h^3(l_y^2 + l_z^2)}{96}$$

$$C_{39} = -C_{38}$$

$$C_{40} = \frac{\rho h^3}{12}$$

$$C_{41} = -\frac{h}{4} \left(\frac{Q_{22}h^2}{3} + \frac{G_{xy}(l_x^2 + l_y^2)}{2} \right)$$

$$C_{42} = \frac{h}{2} \left(\frac{G_{xy}(l_x^2 + l_y^2)}{4} + G_{zz}l_z^2 - \frac{Q_{12}h^2}{6} - \frac{Q_{66}h^2}{6} \right)$$

$$C_{43} = -\frac{h}{2} \left(G_{zz}l_z^2 + G_{xx}l_x^2 + \frac{Q_{66}h^2}{6} \right)$$

$$C_{44} = Q_{55}h$$

$$C_{45} = C_{44}$$

$$C_{46} = \frac{h}{2} \left(G_{xx}l_x^2 - \frac{G_{xy}(l_x^2 + l_y^2)}{4} \right)$$

$$C_{47} = \frac{G_{xy}h(l_x^2 + l_y^2)}{8}$$

$$C_{48} = \frac{G_{xz}h^3(l_x^2 + l_z^2)}{96}$$

$$C_{49} = -C_{48}$$

$$C_{50} = \frac{G_{yz}h^3(l_y^2 + l_z^2)}{96}$$

$$C_{51} = -C_{50}$$

$$C_{52} = C_{40}$$