Vanishing Fermi to Gamow-Teller Mixing Ratio of the β^+ Decay of ^{58}Co

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Since 1956, fourteen experimentally-deduced values of the Fermi to Gamow-Teller mixing ratio $y = C_V M_F / C_A M_{GT}$ were published, varying from 0.05 to -0.36 with most values consistent with $y=0$, particularly those from recent measurements. Our calculation, using the Nilsson model, yields $y = 2.07 \times 10^{-4}$, which can be taken as zero. Our result is therefore consistent with time-reversal invariance. Furthermore, the vanishing value of y arises both from the ΔT selection rule and the ΔK selection rule.

Introduction

The Fermi to Gamow-Teller mixing ratios, $y = C_V M_F / C_A M_{GT}$, in isospin-forbidden beta decays ($J \neq 0$, $\Delta J = 0$, $\Delta T = \pm 1$ and no parity change) have been extensively studied [1] because of their fundamental significance. The validity of time-invariance [2] in nuclear beta decay can be tested if y is known, since the time-reversal violating amplitude is proportional to $y/(1+y^2)$. Furthermore, non-zero values of y imply the existence of isospin impurity in the nuclear states, which may arise either from the charge dependence of nuclear forces or from electromagnetic effects [3].

Since 1956, fourteen measurements [4–17] have been made on the asymmetry coefficient A for $2^+ \xrightarrow{\beta^+} 2^+ \xrightarrow{\gamma} 0^+$ of the $^{58}\text{Co} \rightarrow ^{58}\text{Fe}$ decay with a view to obtaining y . However, such measurements by either polarized nuclei or β - γ circular polarization correlations in unpolarized nuclei are difficult, and different workers tend to obtain rather different values. Figure 1 gives fourteen independent measurements of y over the years. These experimentally-deduced values of y vary from roughly 0.05 to -0.36 with most values consistent with $y=0$, particularly those from recent measurements. The aim of this paper is to obtain a theoretical value of y and discuss the value so obtained in relation to time-reversal invariance.

Calculation and Results

The Fermi nuclear matrix element M_F is related [1] to y through the relation

$$|M_F| = \left(\frac{2 \text{ ft (superallowed)}}{\text{ft (decay under study)}} \right)^{\frac{1}{2}} \frac{y}{(1+y^2)^{\frac{1}{2}}} \quad (1)$$

It has been shown [18] that, if a nucleus is deformed, calculations using shell-model wave functions are incorrect because of inhibitions due to the ΔK selection rule in β -decay. However, calculations [19–21] using the Nilsson model [22] with a one-body spheroidal Coulomb potential to obtain M_F seem to give reasonably agreement between theory and experiment, and we shall use the same approach.

We assume that the deformed nucleus ^{58}Co has the rotational band $K=2$ and the deformed ^{58}Fe has $K=0$ as shown in Fig. 2, where $|G\rangle$, $|P\rangle$, $|A\rangle$ and $|T_<\rangle$ are the ground state, the parent state, the analogue state and the anti-analogue state, respectively. By the K -selection rule $\Delta K \leq 1$ for beta transitions, the beta matrix elements with $\Delta K=2$ vanish. The experimentally observed decay is due to the admixture of other K bands to the $K=2$ ground state of ^{58}Co and to the $K=0$ excited state of ^{58}Fe . Assuming axially symmetric prolate deformation, in the initial state

$$\begin{aligned} |i\rangle = & |J=2, M, K=2, T=2, T_z=-2\rangle \\ & + \bar{a}_0 |J=2, M, K=0, T=2, T_z=-2\rangle \\ & + \bar{a}_2 |J=2, M, K=2, T=3, T_z=-2\rangle \\ & + \bar{a}_0 \bar{a}_0 |J=2, M, K=2, T=3, T_z=-2\rangle \\ & + \dots \end{aligned} \quad (2)$$

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Fig. 1. Plot of all experimental values of the Fermi to Gamow-Teller mixing ratios y that have been reported. The numbers that label the data points refer to references.

and in the final state

$$\begin{aligned}
 |f\rangle = & |J=2, M, K=0, T=3, T_z=-3\rangle \\
 & + a_2 |J=2, M, K=2, T=3, T_z=-3\rangle \\
 & + a_0 |J=2, M, K=0, T=4, T_z=-3\rangle \\
 & + a_2 \alpha_2 |J=2, M, K=2, T=4, T_z=-3\rangle \\
 & + \dots
 \end{aligned} \quad (3)$$

$$M_{GT}^2 = \frac{1}{2J+1} \sum_{\mu, M_i, M_f} |\langle J, M_f, K_f, T_f, T_{zf} | D_{GT}(\mu) | J, M_i, K_i, T_i, T_{zi} \rangle|^2. \quad (9)$$

where $\bar{a}_0$ is the admixture amplitude of the $K=0$ in the initial state while a_2 that of the $K=2$ in the final state. $\bar{\alpha}_0, \bar{\alpha}_2, \dots$ are isospin impurity amplitudes in the initial state and $\alpha_0, \alpha_2, \dots$ are those in the final state.

The Fermi matrix element is

$$M_F = \langle f | T_- | i \rangle = \sqrt{6} (\bar{a}_0 \bar{\alpha}_0 + a_2 \bar{\alpha}_2), \quad (4)$$

where

$$\bar{\alpha}_0 = - \frac{\langle J=2, M, K=0, T=2, T_z=-2 | V_c | J=2, M, K=0, T=3, T_z=-2 \rangle}{\Delta E}, \quad (5)$$

$$\bar{\alpha}_2 = - \frac{\langle J=2, M, K=2, T=2, T_z=-2 | V_c | J=2, M, K=2, T=3, T_z=-2 \rangle}{\Delta E} \quad (6)$$

Fig. 2. Partial decay scheme of ^{58}Co .

with ΔE as the separation energy and V_c is the one-body spheroidal coulomb potential given by

$$\begin{aligned}
 V_c &= \frac{(Z-1)e^2}{R} \left\{ \frac{3}{2} - \frac{1}{2} (r/R)^2 \right\} + a(r/R)^2 Y_{20} \quad \text{for } r < R, \\
 V_c &= \frac{(Z-1)e^2}{r} + a(R/r)^3 Y_{20} \quad \text{for } r > R,
 \end{aligned} \quad (7)$$

where R is the nuclear radius and a is related to the Bohr deformation parameter β by

$$a = \frac{3}{5} \beta (Z-1) \frac{e^2}{R}. \quad (8)$$

If we take [23, 24] $\beta \approx 0.1$, we obtain $\alpha_2 = 1.60 \times 10^{-3} \gg \bar{\alpha}_0$, which can be neglected.

The Gamow-Teller (GT) matrix element is calculated from the relation

Calculation yields $|M_{GT}| = 0.726 a_2$. Using $M_{GT} = C_V M_F / C_A y$, and (1), we obtain $|M_{GT}| = 0.03322$, from which $a_2 = 0.0457$.

This yields $M_F = 1.79 \times 10^{-4}$, from which we deduce $y = 2.07 \times 10^{-4}$, and such a small value of y can be regarded as zero experimentally. Hence our result is

consistent with time-reversal invariance. The vanishing value of γ indicates that the Fermi contribution is

strongly suppressed in this decay, not only due to ΔT selection rule but also due to ΔK selection rule.

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