Chapter 7

Tilting approximations and the finitistic dimension conjectures

In this chapter we present applications of tilting approximations to computing finitistic dimensions of rings and algebras.

The simple, but key fact is that the little finitistic dimension of a right noetherian ring is finite, if and only if there is a (possibly infinitely generated) tilting module T_f such that $T_f^{\perp_\infty} = (\mathcal{P}^{<\omega})^\perp$ (see Theorem 7.1.10 below). The surprising phenomenon here is that even in the artin algebra case, we cannot in general take T_f finitely generated, so the infinite–dimensional tilting theory developed above comes up as a natural tool.

Our first application concerns (non–commutative) Iwanaga–Gorenstein rings. In Theorem 7.1.12 we prove that, if R is n–Iwanaga–Gorenstein, then $\operatorname{fin\,dim} R = \operatorname{Fin\,dim} R = n$.

In the second application (Theorem 7.2.4), for a right artinian ring R, we provide a formula for computing $\operatorname{fin\,dim} R$ involving only approximations of the (finitely many) simple modules.

Our third application yields a simple proof of a result going back to Auslander–Reiten, Huisgen–Zimmermann and Smalø saying that $\operatorname{fin\,dim} R = \operatorname{Fin\,dim} R < \infty$ in case R is an artin algebra such that $\mathcal{P}^{<\omega}$ is contravariantly finite.

This chapter is based on [14] and [18].

7.1 Finitistic dimension conjectures and the tilting module T_f

In this section we will introduce the finitistic dimension conjectures going back to Bass [35], and then deal with their relations to tilting approximations. In particular, we will introduce the tilting module T_f and study its properties. An explicit computation of T_f for Iwanaga–Gorenstein rings will yield a proof of the Bass conjectures in that particular case (see Theorem 7.1.12).