# Lie groups of germs of analytic mappings

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**Abstract.** Let $X$ be a metrizable topological vector space over $\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$, $K \subseteq X$ be a non-empty compact subset, and $G$ be a Banach-Lie group over $\mathbb{K}$. In this paper, we turn the group $\Gamma(K, G)$ of germs around $K$ of $\mathbb{K}$-analytic $G$-valued mappings into a $\mathbb{K}$-analytic Baker–Campbell–Hausdorff Lie group.

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## Introduction

Besides groups of real analytic diffeomorphisms of compact manifolds ([19], [20]) it is important in connection with Lie pseudogroups associated with involutive systems of analytic partial differential equations to consider also Lie groups of germs of $\mathbb{K}$-analytic local diffeomorphisms around $0 \in \mathbb{K}^n$ fixing the origin, and generalizations of such groups (see [23] for $\mathbb{K} = \mathbb{C}$, [18] for $\mathbb{K} = \mathbb{R}$). Similarly, replacing globally defined mappings with germs, it is our goal here to consider not groups of smooth or real analytic Lie group-valued mappings (as usually done), but groups of germs of analytic mappings with values in Lie groups.

Throughout the introduction, let $X$ be a metrizable topological $\mathbb{K}$-vector space over $\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$, and $K \subseteq X$ be a non-empty compact subset. We are mostly interested in the case where $X$ is locally convex, but the constructions work just as well for general $X$.

**Groups of germs.** If $G$ is a Banach–Lie group over $\mathbb{K}$, we consider the group $\Gamma(K, G)$ of germs $[\gamma]$ around $K$ of $\mathbb{K}$-analytic mappings $\gamma : U \to G$ defined on open neighbourhoods $U \subseteq X$ of $K$. As our main result, we show that $\Gamma(K, G)$ can be made a $\mathbb{K}$-analytic Lie group modelled on the space of germs $\Gamma(K, L(G))$, equipped with its natural locally convex direct limit topology (Theorem 5.10). By construction, $\Gamma(K, G)$ will be a so-called Baker–Campbell–Hausdorff (BCH-) Lie group, *i.e.*, it has an exponential function inducing a local isomorphism of $\mathbb{K}$-analytic manifolds